How is the Janus cosmological model a simple variant of the models of S. Hossenfelder, T. Damour and I. Kogan.

Jean-Pierre Petit, Gille d'Agostini

Keywords :bimetrics models, Janus Cosmological Model,

Abstract We show how the heuristic choice of taking, in an attempt at bimetric modeling, equal Einstein constants, meant that T. Damour and Kogan, in 2002, then S. Hossenfelder in 2008, missed the benefit of this approach, that is to say the Janus Cosmological Model, which derives from theirs by simply opting for a choice of equal and opposite cosmological constants, which eliminates the runaway effect and leads to an extremely fruitful interaction scheme, with regard to the agreement with the observations.

Introduction :

The Janus cosmological model has been the subject of numerous publications since its inception, more than thirty years ago, and more recently in [1] and [2]. The approach can be summarized in a simple project: that of introducing elements of negative mass and energy into the cosmological model. However, this had been considered impossible in general relativity since 1957 [3]. The reasoning is simple. General relativity is based on Einstein's equation. In the second member is the source of the gravitational field. Assuming that this field is created by a positive mass, the solution to the equation is presented in the form of a unique solution, specified by Karl Schwarzschild in 1916 ([4], [5]) which results in an attraction of the witness particle, whatever its nature, its mass (positive or negative). Similarly, if the field is created by a negative mass, the equation produces only a single metric solution, generating geodesics, which will once again be followed by all types of masses, positive or negative. The shape of these masses then suggests repulsion. The force laws that follow from this tentative introduction are:

- Positive masses attract each other
- Negative masses repel each other - Positive masses attract negative masses
- Negative masses repel positive masses.

Such a scheme, if only because the interaction between masses of opposite signs violates the principle of action-reaction, justified the conclusion as early as 1957 that the introduction of negative masses into the model of general relativity was simply impossible. This is absolutely correct. If one absolutely wanted to add these masses to the model, it was necessary, not to invalidate the model of general relativity, the Einstein equation, but to consider this as a first step in the construction of a larger and more ambitious model, which necessarily had to be bimetric, so that the masses of positive sign follow the geodesics of one metric $g_{\mu\nu}$, and the negative masses follow the geodesics of a second metric $\bar{g}_{\mu\nu}$.

The first bimetric model, that of T.Damour and I.Kogan.

This first one emerged in 2002 [6]. It did not immediately aim at introducing a second metric and started from an action:

(1)

$$S = \int d^4x \sqrt{-g_L} \left(M_L^2 R(g_L) - \Lambda_L \right) + \int d^4x \sqrt{-g_L} L(\Phi_L, g_L) + \int d^4x \sqrt{-g_R} \left(M_R^2 R(g_R) - \Lambda_R \right) + \int d^4x \sqrt{-g_R} L(\Phi_R, g_R) - \mu^4 \int d^4x \, (g_R \, g_L)^{1/4} \, V(g_L, g_R) \, .$$

$R_{(g_L)}$ is the Ricci scalar derived from the "left" metric

$R_{(g_R)}$ is the Ricci scalar derived from the "right" metric

Λ_L and Λ_R are two cosmological constants.

$L(\Phi_L , g_L)$ is the Lagrangian of the "left" matter

$L(\Phi_R , g_R)$ is the Lagrangian of the "right" matter

The components of the action reveal the elementary hypervolumes $\sqrt{-g_L}\, d^4x$ and $\sqrt{-g_R}\, d^4x$ as well as an equivalent hypervolume $(\, g_L \, g_R)^{1/4}$. A variational calculus, not explicitly stated, leads the authors to the following system of two field equations:

(2)

$$2\, M_L^2 \left(R_{\mu\nu}(g^L) - \frac{1}{2}\, g^L_{\mu\nu}\, R(g^L) \right) + \Lambda_L \, g^L_{\mu\nu} = t^L_{\mu\nu} + T^L_{\mu\nu}$$

$$2\, M_R^2 \left(R_{\mu\nu}(g^R) - \frac{1}{2}\, g^R_{\mu\nu}\, R(g^R) \right) + \Lambda_R \, g^R_{\mu\nu} = t^R_{\mu\nu} + T^R_{\mu\nu}$$

$T^L_{\mu\nu}$ and $T^R_{\mu\nu}$ are the field tensors of the two species, L and R.

$t^L_{\mu\nu}$ and $t^R_{\mu\nu}$ are interaction tensors between the two species, L and R.

In order to unify the notations and to be able to make a comparison with the Janus system, where the metrics are noted: $g_{\mu\nu}$ et $\bar{g}_{\mu\nu}$, we will take the constants M_L et M_R equal to unity and will give the two cosmological constants Λ_L et Λ_R ero values. We will adopt the notations:

(3) $$T^L_{\mu\nu} = T_{\mu\nu} \qquad T^R_{\mu\nu} = \bar{T}_{\mu\nu}$$

(4) $$t^L_{\mu\nu} = K_{\mu\nu} \quad t^{LR}_{\mu\nu} = \bar{K}_{\mu\nu}$$

This system becomes:

(3)

$$R_{\mu\nu} - \frac{1}{2} \mathrm{R}\, g_{\mu\nu} = T_{\mu\nu} + K_{\mu\nu}$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} R\, \bar{g}_{\mu\nu} = \bar{T}_{\mu\nu} + \bar{K}_{\mu\nu}$$

It is clear that the authors, without explicitly stating it, gave the "Einstein constants" equal values. By reintroducing these constants, their action would be written:

(4)

$$S = \int d^4x \sqrt{-g_L} \left(M_L^2 R(g_L) - \Lambda_L \right) - \chi^L \int d^4x \sqrt{-g_L}\, L(\Phi_L, g_L) + \int d^4x \sqrt{-g_R} \left(M_R^2 R(g_R) - \Lambda_R \right) - \chi^R \int d^4x \sqrt{-g_R}\, L(\Phi_R, g_R) - \mu^4 \int d^4x \, (g_R\, g_L)^{1/4}\, V(g_L, g_R) \, .$$

And the system would become:

(5)

$$2\, M_L^2 \left(R_{\mu\nu}(g^L) - \frac{1}{2} g^L_{\mu\nu} R(g^L) \right) + \Lambda_L\, g^L_{\mu\nu} = \chi^L \left(t^L_{\mu\nu} + T^L_{\mu\nu} \right)$$

$$2\, M_R^2 \left(R_{\mu\nu}(g^R) - \frac{1}{2} g^R_{\mu\nu} R(g^R) \right) + \Lambda_R\, g^R_{\mu\nu} = \chi^R \left(t^R_{\mu\nu} + T^R_{\mu\nu} \right)$$

By using our ratings and opting for:

(6) $$\chi_R = -\chi_L = -\chi$$

We get :

(7)

$$R_{\mu\nu} - \frac{1}{2} \mathrm{R}\, g_{\mu\nu} = \chi \left(T_{\mu\nu} + K_{\mu\nu} \right)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} R\, \bar{g}_{\mu\nu} = -\chi \left(\bar{T}_{\mu\nu} + \bar{K}_{\mu\nu} \right)$$

Which corresponds to the first formulation of the Janus model [12]. Considering that the masses m of the first population are the masses of general relativity and that the masses $\bar{m} < 0$ are negative masses, we obtain the system of equations of the Janus model, which is therefore a variant of the Damour and Kogan model. The presence of this minus sign in front of the terms of the second member of the second equation, as shown in [1] eliminates the runaway effect and, with simple assumptions, in Newtonian approximation:

(8) For $\mu \neq 0$ $K^{\mu}_{\mu} \cong 0$ and $\bar{K}^{\mu}_{\mu} \cong 0$

(9) $K^0_0 < 0$ and $\bar{K}^0_0 > 0$

Whence the following interaction laws :

- Positive masses attract each other according to Newton's law
- Negative masses attract each other according to Newton's law
- Masses of opposite signs repel each other according to "anti-Newton" law

This has sometimes not been well understood ([7], [7]). For the benefit gained, see [1]. Article [2] provides an additional basis for the model, in the group domain, allowing, by exploiting Andrei Sakharov's idea [9], to confer an identity on this second species, which then advantageously replaces, in terms of effects, the "dark matter-dark energy" pair. These invisible components of the universe are then identified as negative-mass antihydrogen and antihelium.

2008 : **Second bimetric model, that of S.Hossenfelder** ([10], [11]).

In [10] the author specifies his wish to model a form of antigravitation. There is again here, as a starting point, an action:
(10)

$$S = \int d^4x \left[\sqrt{-g}\ ({}^{(g)}R/8\pi G + L(\psi) + \sqrt{-h}\ (P_{\underline{h}}\underline{L}\left(\underline{\phi}\right)\right] + \int d^4x \left[\sqrt{-\underline{h}}\ ({}^{(\underline{h})}R/8\pi G + \underline{L}\left(\underline{\phi}\right) + \sqrt{-g}\ (P_g L(\psi)\right]$$

The terms ${}^{(g)}R/8\pi G$ and ${}^{(\underline{h})}R/8\pi G$ are the Ricci scalars, derived from the two metrics $g_{\mu\nu}$ and $h_{\mu\nu}$. The terms $L(\phi)$ and $L(\psi)$ are the Lagrangians of the two materials. The terms $d^4x\sqrt{-h}\ (P_{\underline{h}}\underline{L}\left(\underline{\phi}\right)$ and $d^4x\sqrt{-g}\ (P_g L(\psi)$ are the contributions to the action that give the interaction tensors. L'action est alors exclusivement construite sur la base des hypervolumes élémentaires The action is then constructed exclusively on the basis of elementary hypervolumes$\sqrt{-\ g}\ d^4x$ and $\sqrt{-\ h}\ d^4x$. This shows that there are several possible Lagrangian derivations [1]. We have added the brackets, which were missing in the article, a simple oversight in the original article. In the action the factor 1/8\piG\ ,\ which appears in both components of this one explicitly shows that the author has opted for two identical Einstein constants. The resulting system of field equations is then:
(11)

$$ {}^{(g)}R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu}\, {}^{(g)}R_{\kappa\nu} = T_{\kappa\nu} - \underline{V} \sqrt{\frac{\underline{h}}{g}}\ a_{\underline{\nu}}^{\nu}\ a_{\nu}^{\underline{\nu}}\ \underline{T}_{\underline{\nu\kappa}} $$

$$ {}^{(\underline{h})}R_{\underline{\nu}\,\underline{\kappa}} - \frac{1}{2}\ h_{\underline{\nu}\,\underline{\kappa}}\, {}^{(\underline{h})}R_{\kappa\nu} = T_{\underline{\nu}\,\underline{\kappa}} - W \sqrt{\frac{g}{\underline{h}}}\ a_{\underline{k}}^{\kappa}\, a_{\underline{\nu}}^{\nu}\, T_{\kappa\nu} $$

In a simplified version, with our notations, this system becomes:

(12) $$R_{\mu\nu} - \frac{1}{2}\,\mathrm{R}\, g_{\mu\nu} = \chi \left(T_{\mu\nu} + \sqrt{\frac{\bar{g}}{g}}\ K'_{\mu\nu} \right)$$

(13) $$\bar{R}_{\mu\nu} - \frac{1}{2}\, R\, \bar{g}_{\mu\nu} = -\ \chi \left(\bar{T}_{\mu\nu} + \sqrt{\frac{g}{\bar{g}}}\ \bar{K}'_{\mu\nu} \right)$$

That is, the Janus model in its 2024 version [1]. We see once again how the heuristic choice of taking the two Einstein constants as equal caused the author to completely miss the potential benefits of bimetric models. The factors corresponding to the square root of the ratio of the determinants of the two metrics stem from the choices made, focusing on elementary volumes $\sqrt{-g}$ et $\sqrt{-\bar{g}}$. Incidentally, it should be noted that it is this formulation which allows the construction of an exact unsteady solution [12], with homogeneous and isotropic configurations, the mathematical condition of compatibility, a variant of the Bianchi conditions, resulting in a generalized conservation of energy.

(14) $$E = \rho\, c^2 a^3 + \bar{\rho}\, c^2 \bar{a}^3 = Cst$$

A solution which, moreover, fits with observational data [13]. One of the particularities of a Janus model, which makes it so fruitful, is the total asymmetry between its two components, on all levels, the negative species being dominant and driving both the dynamics of the expansion (it is this which is responsible for its acceleration[12]) and the mechanisms of gravitational instability, by imposing on the positive mass its lacunar structure [1], by ensuring the confinement of the galaxies and, through its interaction, by being responsible for the spiral structure. The origin of this asymmetry will be explained in a future publication.

Conclusion :

This bimetric approach, which has demonstrated its fecundity, suggests a leap that many theorists are still hesitant to make, as it seems dizzying to them to have to consider leaving the prison that the model of general relativity represents today, and integrating a second, adjacent cell into a larger apartment. The topological aspects are also new elements, entirely centered on the concepts of covering complete symplectic groups.

References.

[1] J.P.Petit, F.Margnat, H.Zejli : A bimetric cosmological model on Andreï's twin universe approach. Th European Physical Journal. Vol. 84 :N°1126 (2024)
[2] RMC J.P.Petit, H.Zejli : Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, n001, 2024.
[3] H. Bondi: Negative mass in General Relativity : Negative mass in General Relativity. Rev. of Mod. Phys., Vol 29, N°3, july1957
[4] K. Schwarzschild : Über das Gravitationsfeld Messenpunktes nach der Einsteinschen Theorie. Sit. Deut. Akad. Wiss. 1916
[5] K. Schwarzschild : Über das Gravitionalsfeld einer Kugel Aus incompressibler Flüssigkeit nach der Einsteinschen Theorie. Sitzung der phys. Math. Klasse v.23 märz 1916
[6] Damour T. , Kogan I I. Effective Lagrangians and universality classes of nonlinear bigravity Phys. Rev. D **66** (2002) 104024. hep-th/0206042.
[7] T.Damour : « A propos du Modèle Cosmologique Janus ». Website of the Institut des Hautes Etudes Scientifique, 2019, in french.
https://www.ihes.fr/~damour/publications/JanusJanvier2019-1.pdf
[8] T.Damour : « Incohérence physique et mathématique du modèle cosmologique Janus ». 2022, in french. . Website of the Institut des Hautes Etudes Scientifique, 2019, in french. https://www.ihes.fr/~damour/publications/JanusDecembre2022.pdf
[9] A.D.Sakharov , (1980). Cosmological Model of the Universe with a Time Vector

[10] S. Hossenfelder : A bimetric Theory with Exchange Symmetry. Phys. Rev. D78, 044015, 2008 and arXiv : 0807.2838v1 (gr-qc)17 july 2008
[11] S.Hossenfelder. Static Scalar Field Solutions in Symmetric Gravity. 2016 arXiv [gr-qc] 1603.07075v2
[12] J.P.Petit, G.D'Agostini : Cosmological Bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe. Modern Physics Letters
A, Vol.29 ; N° 34, **2014** ; Nov 10th DOI :10.1142/So21773231450182X
[13] G. DAgostini and J.P.Petit : Constraints on Janus Cosmological model from recent observations of supernovae type Ia, Astrophysics and Space Science, (**2018**), 363:139.https://doi.org/10.1007/s10509-018-3365-3