

SHOCK WAVE CANCELLATION IN GAS BY LORENZ FORCE ACTION

J.P. PETIT and B. LEBRUN

Centre National de la Recherche Scientifique, France

Abstract :

Some theoretical and experimental work has been done between 1950 and 1972 about plasma acceleration towards two main directions : simulation of reentry process with high enthalpy wind tunnels and space propulsion. The first was abandoned about 1972 when one decided that the reentry phenomenon was understood enough. The second purpose was given up when the long duration space journeys were abandoned (missions to Mars). In fact one can show that for a given mission duration there is an optimum specific impulse value. Circumterrestrial or lunar missions did not need any longer sophisticated large specific impulse MHD devices.

But MHD propulsion could offer some specific interest for high mach numbers flights for, as will be shown here, shock waves could possibly be cancelled. At the present level, this work is just a fundamental research. But France has recently decided to support it through an original french MHD project. We present here the first numerical simulations results, based on characteristic theory. They will be used to run shock tube experiments in hot argon (in order to achieve a high electrical conductivity and to avoid the ionization instability due to non equilibrium conditions). A 300,000 dollars governmental grant has been given recently to the Laboratoire de Thermodynamique de Rouen, which will take in charge the experimental features.

1. INTRODUCTION

Previous experiments have been carried out (1 and 2) in shock tube, showing that strong plasma accelerations could be obtained without thermal blocking. When a strong Joule effect occurs, locally, in a supersonic flow, this may cause the birth of a strong front shock wave (1 and 2). If this energy is too big, the slowing down due to the pressure gradient may balance or excess the $J \times B$ acceleration. A simple theoretical analysis shows that acceleration is possible if the following criterium is satisfied :

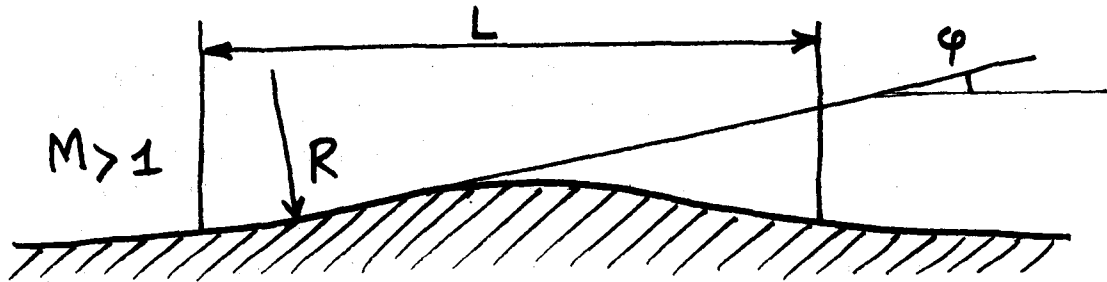
$$\gamma = \frac{C_p}{C_v} \quad \boxed{\frac{\sigma B^2 L}{\rho V (\gamma - 1)} > 1} \quad (1)$$

Private adresses of the authors :

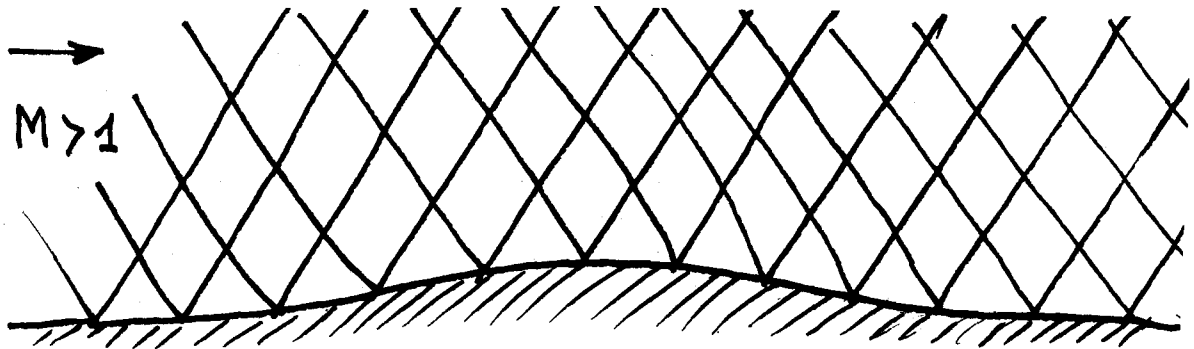
J.P.PETIT 9 rue Aude 13100 Aix-en-Provence, France.

B.LEBRUN : Lou Cigaloun, Boulevard Kennedy, 13090 Aix-en-Provence, France.

This adimensional quantity is similar to the classical MHD interaction parameter. The $(\gamma - 1)$ coefficient shows that real gas effects help, by internal energy storage, to reduce the pressure jump. This fits quite well with the (1 and 2) references which are, as far as we know, the only experiments in which thermal blocking was investigated. In a first step we can derive a quasi one dimensional analysis. Consider a wall with some sort of bump:



Developping the classical approach with the Lorenz forces and the the Joule effect one can derive the variations of the gas parameters : velocity, pressure, density, absolute temperature. If the characteristic lines, coming from the model, are kept parralel to the upstream ones, the shock phenomenon will be avoided.



Call α the Mach angle and suppose φ defines the direction of the tangent to the wall with respect to the laboratory frame of reference. The parallelism will be kept if :

$$\frac{dM}{M} = \sqrt{M^2 - 1} d\varphi \quad (2)$$

Using the previous equations we get :

$$\text{JB} = \frac{M^2(\gamma - 1)}{2\sqrt{M^2 - 1}} \rho V^2 \frac{d\varphi}{dx} + \frac{J^2}{\sigma V} (\gamma - 1) \left(1 + \frac{\gamma M^2}{2}\right) \quad (3)$$

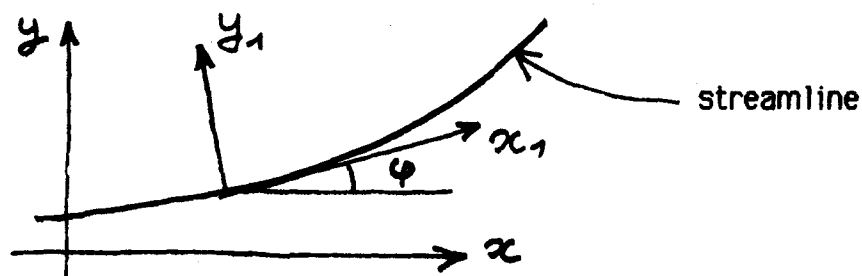
Note that $d\phi / dx$ is nothing but the inverse of the local curvature radius R . This is the value of the local Lorentz force that will make possible to avoid shock birth in the vicinity of the wall. All that can be connected to some classical shock tube experiments and provides all the desirable magnitude orders for physical parameters.

Test gas	: Argon
Gas temperature T_g	: 10 000°K
Volumic mass ρ	: 0.05 Kg/m ³
Flow Mach number M	: 1,5
Gas velocity V	: 2500 m/s
Electrical conductivity σ	: 3500 Mhos/m
Specific heats ratio γ	: 1,275
Curvature ratio at the wall	: 0,2 m
MHD channel lenght	: 0,1 m
Test duration	: 100 μ s
Magnetic field B	: 2 Teslas
Current density	: 200 Amp/cm ²
Hall parameter $\beta < 1$	

Note that the $J B$ forces depends on the sign of $d\phi / dx$. Roughly speaking (if we neglect the pressure gradient due to the Joule heating) we must accelerate in the converging sections ($d\phi / dx > 0$) and slow down in the divergent one ($d\phi/dx < 0$). Slowing down the gas implies to convert an appreciable amount of its kinetic energy into electrical energy, i.e. these sections act as generators. Such as the global energy expense appears to be the DIFFERENCE between two terms and in fact represents the Joule losses.

2. STEADY BIDIMENSIONAL ANALYSIS

We start from the classical Euler flow equations. One can show that in shoch tube with hot argon, with some moderate current density J , the electron temperature T_e can be close to the gas temperature T_g , as was confirmed by the experiments (1 and 2). Furthermore the Joule effect is weak, such as we can neglect it and consider the flow as a quasi isentropic one. Consider now the following frame of reference, based on the flow line :



the mass conservation becomes :

$$\frac{\partial \rho u}{\partial x_1} + \rho u \frac{\partial \varphi}{\partial y_1} = 0 \quad (4)$$

and the momentum conservation :

$$\rho u \frac{\partial u}{\partial x_1} + \frac{\partial p}{\partial x_1} = J_{y_1} B = F_{x_1} \quad (5)$$

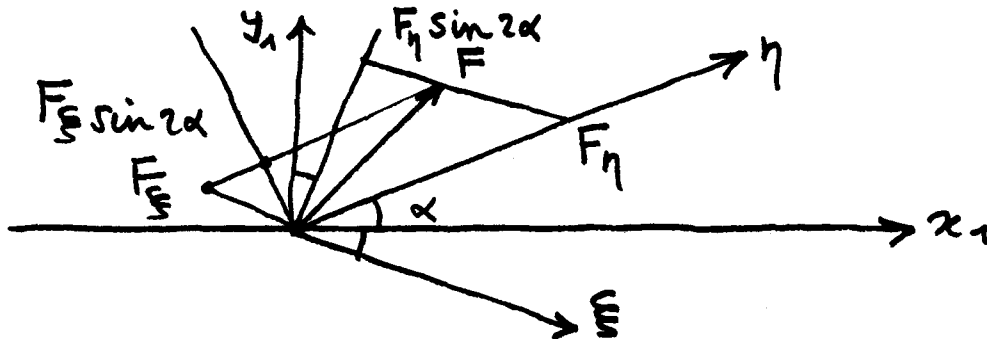
$$\rho u^2 \frac{\partial \varphi}{\partial x_1} + \frac{\partial p}{\partial y_1} = -J_{x_1} B = F_{y_1} \quad (6)$$

introduce now the characteristic lines η and ξ . In this new frame the equations become :

$$\left\{ \begin{array}{l} \sqrt{M^2 - 1} \frac{\partial p}{\partial \xi} - \rho u^2 \frac{\partial \varphi}{\partial \xi} = -\rho u^2 \frac{\partial \varphi}{\partial \xi} = -\sin \alpha F_{x_1} - \cos \alpha F_{y_1} \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} \sqrt{M^2 - 1} \frac{\partial p}{\partial \eta} + \rho u^2 \frac{\partial \varphi}{\partial \eta} = -\sin \alpha F_{x_1} + \cos \alpha F_{y_1} \end{array} \right. \quad (8)$$

let us project the Lorenz force :



we get :

$$\left\{ \begin{array}{l} \frac{\partial}{\partial \xi} (P - \varphi) = -F \frac{\sin^2 \alpha}{\gamma P} = d\lambda_L \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial \eta} (P + \varphi) = -F \frac{\sin^2 \alpha}{\gamma P} = d\mu_L \end{array} \right. \quad (10)$$

where P is the Buseman number as defined by :

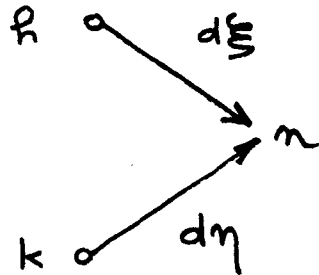
$$dP = \frac{\sin \alpha \cos \alpha}{\gamma P} dp$$

$$P = -\alpha - \sqrt{\frac{\gamma+1}{\gamma-1}} \operatorname{Arctg} \left[\sqrt{\frac{\gamma-1}{\gamma+1}} \operatorname{Ctg} \alpha \right] + \operatorname{Cst.} \quad (11)$$

λ and μ are the wellknown epicycloidal coordinates. In absence of Lorenz force they remain constant along the associated characteristics. The introduction of the force cause a variation of these epicycloidal coordinates. By integration we get :

$$\begin{cases} P + \varphi = \lambda & P = P(\text{Mach}) = \lambda + \mu & (12) \\ P - \varphi = \mu & \varphi = \mu - \lambda & (13) \end{cases}$$

The flow can be derived step by step, following the characteristic lines.



$$\begin{cases} \lambda_n = \lambda_p - \frac{\sin^2 \alpha}{\gamma P} F_{\perp \xi} d\xi & (14) \\ \mu_n = \mu_k - \frac{\sin^2 \alpha}{\gamma P} F_{\perp \eta} d\eta & (15) \end{cases}$$

3. HOW TO KEEP PARALLELISM

Assuming the flow pattern to be uniform at a distance from the wall we derive a simple criterium giving the required spatial extension of the force field:

$$\frac{JBh}{\rho V^2} = \text{tg } \varphi \quad (16)$$

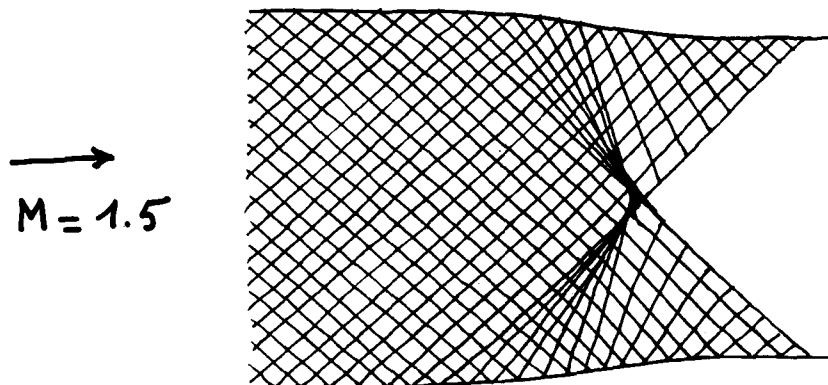
h being the width of the force field. This relation means that the energy provided to the flow must be proportional to the velocity deviation. This can be written as :

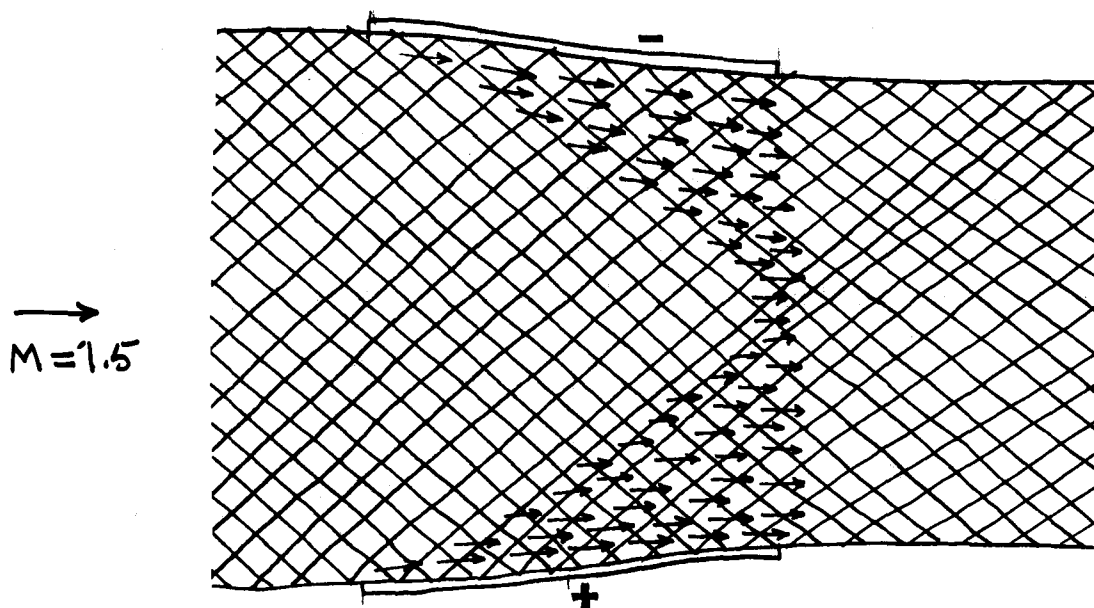
$$\boxed{\frac{JBR}{\rho V^2} = \sqrt{M^2 - 1}} \quad (17)$$

with the experimental values, as defined before, we find, for an example, for a ten degrees deviation $h = 0.03 \text{ m}$.

4. NUMERICAL RESULTS :

We consider first an internal solution and a converging section in a shock tube. If the Lorenz force is absent the characteristic lines get focussed as shown on next figure.

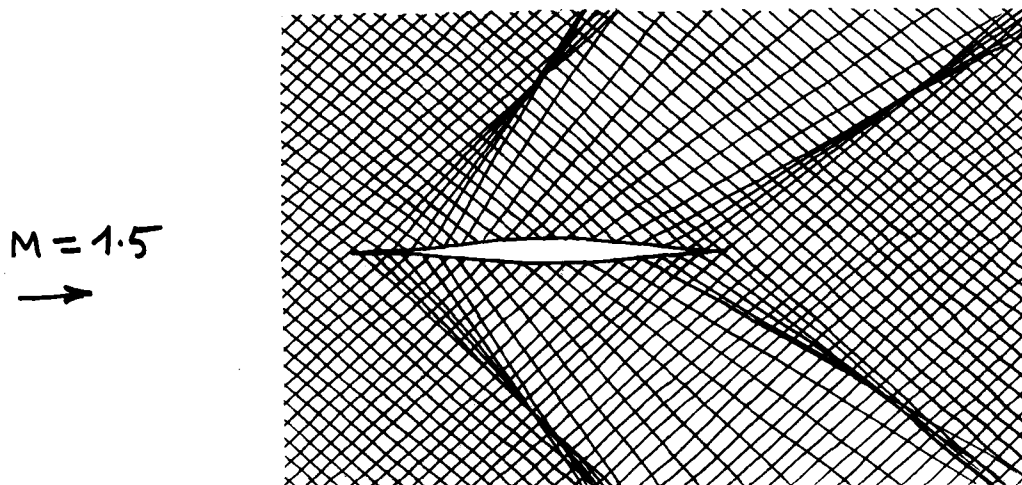




Converging section with Lorenz force action. Two continuous electrodes. The force field is shown. The magnetic field acts in the two triangular areas.

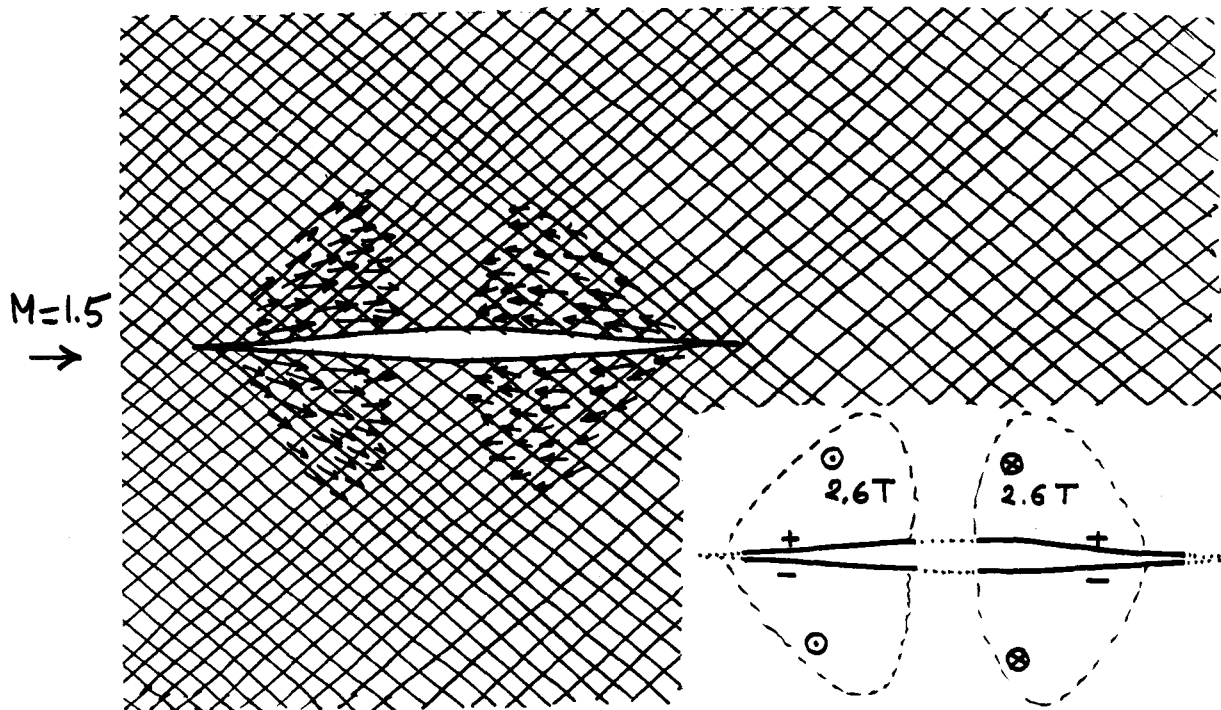
The triangular area shows the desirable localization of the $J \times B$ action (non uniform transverse B field). We assume a constant electrical conductivity over the whole channel. The E field is derived from solving the Poisson' equation.

Consider now an external solution. A thin model is now imbeded in an infinite supersonic gas flow with same upstream conditions, as described in the introduction. If the Lorenz force are zero the cutting of the characteristics reveals the shock waves birthplaces. See next figure on the left. The inlet Mach number is 1,5.



Shock wave formation around a thin model

Introduce now a convenient $J \times B$ field (see next page). They are two pairs of electrodes, shown on the foot figure. The field value is 2.6 Teslas and its orientation is shown to. Its intensity shapes the force field (arrows). The plasma is accelerated in the front part of the model and slowed down in the bottom part.



Shock wave cancellation around a thin model, by Lorentz force field. $M = 1.5$

He hope the experimental results in the shock tube, carried out with classical and wellknown techniques, will confirm these numerical results. Of course this is close to perturbation method, for the involved body is thin. We intend to extend later this to blunt objects. In this case some supersonic regions will appear for an example near by the stagnation point. A continuous change from supersonic to subsonic regime is required which could be made possible by long range upstream Lorentz force action, as shown previously in hydraulic simulations (3).

6. FUTURE EXTENSION TO NON EQUILIBRIUM CONDITIONS.

If the experiments would confirm the theory it would mean that if we would live on a planet whose atmosphere would correspond to these severe gaz conditions we could fly a machine at supersonic regime without creating any shock wave (the Lorentz force also may cancel turbulence, as shown in (3)). One would like to know if it could be possible to operate in standard atmosphere. This requires non equilibrium ionization. For such purpose we would like to low the gas temperature in the shock tube. Immediatly we will have to face the Velikhov instability that will tend to cause strong electron density fluctuations. In the previous international meeting we had suggested a method for instability cancelling by magnetic confinement (4). Suppose we introduce some fluctuation of the B field over space. In the high B area the Hall parameter will be high which will increase the local resistivity of the plasma such as the electrons will tend to flow along the minimum B pathes..

Along these paths the electric current density J will be increased and the electron density and temperature subsequently rise, such as some Coulomb regime could be locally achieved, that would diminish the local Hall parameter value. Then we would get a non uniform Hall parameter field. If its value would be locally lower than the critical one (close to 2 in Coulomb dominated plasma), along minimum B paths such "streamers" would be stable, surrounded by unstable high Hall parameter plasma. Note that this plasma stabilization by magnetic confinement is fairly different from the fusion confinement system.

As shown in (4) this works well in rarefied gases. Similar experiments in dense plasmas, provided by the shock tube, are planned and belong to the French MHD research project. However numerical computations show that shock wave cancellation could be achieved in cold air, if we achieve a minimum electric conductivity in the vicinity of the object. Let us take :

Characteristic length $L = 10 \text{ m}$

Volumic mass $\rho = 1.3 \text{ Kg/m}^3$

Gas velocity $V = 660 \text{ m/s}$

Magnetic field $B = 4 \text{ Teslas}$

Gas temperature : 300°K

Electric current density $J = 1.5 \cdot 10^4 \text{ Amp/m}^2$

Following the previously defined criterium, the thermal blocking could be avoided if the electric conductivity would be close to one Mho/m, which is quite moderate and could be achieved by "natural" non equilibrium operation (due to strong E field) or through microwave emission from the wall (controlled ionization).

In addition one can show that a strong B field rises the acceleration efficiency. The high β model geometry would be completely different for we can act $J \times B$ forces perpendicular to the wall. When the model enters the gas it appears convenient to "separate" the fluid in the vicinity of the stagnation point and to "close" it softly downstream, following the principle "keep the medium in the same conditions that you found when coming in. If the Lorenz force can balance the inertial force the constancy of the gas pressure could possibly be kept along the streamlines. This would lead to very unconventional solutions of the flow equations at supersonic regimes.

On another hand, high Hall parameter MHD converters are preferably disk shaped. Similar considerations show that disk shape aerodynes fits better for high β operating conditions.

This work is supported by the French CNRS (engineering department) and by the French Ministry of research and industry.

REFERENCES :

1) B.FONTAINE "Contribution à l'étude de l'action d'un champ magnétique et électrique transversal sur un courant supersonique d'argon ionisé. Cas d'une décharge pure. Cas de la conversion MHD". Thèse CNRS n° AO 7860 IMFM 1973

2) B.FORESTIER : "Etude de l'action des forces de Laplace sur un courant supersonique d'argon ionisé. Accélération. Thèse CNRS n° AO 7861 IMFM 1973.

3) J.P.PETIT : "Cancellation of the Velikhov instability by magnetic confinement". Eight international MHD meeting. Moscow 1983.

