2 Schwarzschild's letter to Einstein

Letter from K Schwarzschild to A Einstein dated 22 December 1915

The letter is presented in English owing to Professor Roger A. Rydin

Honored Mr. Einstein,

In order to be able to verify your gravitational theory, I have brought myself nearer to your work on the perihelion of Mercury, and occupied myself with the problem solved with the First Approximation. Thereby, I found myself in a state of great confusion. I found for the first approximation of the coefficient $g_{\mu\nu}$ other than your solution the following two:

$$g_{\rho\sigma} = -\frac{\beta x_\rho x_\sigma}{r^5} + \delta_{\rho\sigma}\left(\frac{\beta}{3r^3}\right); \quad g_{44} = 1$$

As follows, it had beside your α yet a second term, and the problem was physically undetermined. From this I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: It gave only a line element, which fulfills your conditions 1) to 4), as well as field- and determinant equations, and at the null point and only in the null point is singular.

If:

$$x_1 = r\cos\phi\cos\theta, \quad x_2 = r\sin\phi\cos\theta, \quad x_3 = r\sin\theta$$
$$R = (r^3 + \alpha^3)^{1/3} = r\left(1 + \frac{1}{3}\frac{\alpha^3}{r^3} + ...\right)$$

then the line element becomes:

$$ds^2 = \left(1 - \frac{\gamma}{R}\right)dt^2 - \frac{dR^2}{\left(1 - \frac{\gamma}{R}\right)} - R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

R, θ, ϕ are not "allowed" coordinates, with which one must build the field equations, because they do not have the determinant = 1, however the line element expresses itself as the best.

The equation of the orbit remains exactly as you obtained in the first approximation (11), only one must understand for x *not* $1/r$*, but* $1/R$*, which is a difference of the order of* 10^{-12}*, so it has practically the same absolute validity.*

The difficulty with the two arbitrary constants α and β*, which the First Approximation gave, resolves itself thereby, that* β *must have a determined value of the order of* α^4 *, so as* α *is given, so will the solution be divergent by continuation of the approximation.*

It is after all the clear meaning of your problem in the best order.

It is an entirely wonderful thing, that from one so abstract an idea comes out such a conclusive clarification of the Mercury anomaly.

As you see, it means that the friendly war with me, in which in spite of your considerable protective fire throughout the terrestrial distance, allows this stroll in your fantasy land.