nonzero divergence. If $m \neq 0$, then there are certain integrability conditions which must be satisfied by Eqs. (2)-(4). These may be solved for m as a function of Ω and its derivatives provided that either $\dot{\Delta}$ or $\ddot{\Omega}$ is nonzero. This expression for m may then be substituted back into the field equations giving conditions on Ω and its derivatives, from which further integrability conditions are extracted.

If both $\dot{\Delta}$ and $\ddot{\Omega}$ are zero, then we may transform the metric to a coordinate system in which Ω is pure imaginary and $P \not\equiv 1$, with $\dot{\Omega} = \dot{P} = 0$. The field equations then become

$$m = cu + A + iB,$$

where c is a real constant, and

$$P^{-2}\nabla[P^{-2}\nabla(\ln P)] = 2c, \quad \nabla = \partial^2/\partial\zeta\partial\zeta^*.$$

A, B, and Ω, which are all independent of u and r, are determined by

$$iB = \tfrac{1}{2}P^{-2}\nabla(P^{-2}\partial\Omega/\partial\zeta) - P^{-4}(\partial\Omega/\partial\zeta)\nabla(\ln P),$$

$$\nabla B = ic\partial\Omega/\partial\zeta,$$

$$(\partial/\partial\zeta)(A - iB) = c\Omega,$$

where $\zeta = \xi + i\eta$. If c is zero, then $\partial/\partial u$ is a Killing vector.

Among the solutions of these equations, there is one which is stationary ($c = 0$) and also is axially symmetric. Like the Schwarzschild metric, which it contains, it is Type D. Also, B is zero, and m is a real constant, the Schwarzschild mass. The metric is

$$ds^2 = (r^2 + a^2\cos^2\theta)(d\theta^2 + \sin^2\theta d\phi^2) + 2(du + a\sin^2\theta d\phi) \times (dr + a\sin^2\theta d\phi) - \left(1 - \frac{2mr}{r^2 + a^2\cos^2\theta}\right) \times (du + a\sin^2\theta d\phi)^2,$$

where a is a real constant. This may be transformed to an asymptotically flat coordinate system by the transformation

$$(r - ia)e^{i\phi}\sin\theta = x + iy, \quad r\cos\theta = z, \quad u = t + r,$$

the metric becoming

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + \frac{2mr^3}{r^4 + a^2z^2}(k)^2,$$

$$(r^2 + a^2)rk = r^2(xdx + ydy) + ar(xdy - ydx) + (r^2 + a^2)(zdz + rdt). \quad (5)$$

This function r is defined by

$$r^4 - (R^2 - a^2)r^2 - a^2z^2 = 0, \quad R^2 = x^2 + y^2 + z^2,$$

so that asymptotically $r = R + O(R^{-1})$. In this coordinate system the solution is analytic everywhere, except at $R = a$, $z = 0$.

If we expand the metric in Eq. (5) as a power series in m and a, assuming m to be of order two and a of order one, and compare it with the third-order Einstein-Infeld-Hoffmann approximation for a spinning particle, we find that m is the Schwarzschild mass and ma the angular momentum about the z axis. It has no higher order multipole moments in this approximation. Since there is no invariant definition of the moments in the exact theory, one cannot say what they are, except that they are small. It would be desirable to calculate an interior solution to get more insight into this.

---

*Supported in part by U. S. Air Force through the Aerospace Research Laboratory of the Office of Aerospace Research.

[1]J. N. Goldberg and R. K. Sachs, Acta Phys. Polon. 22, 13 (1962).

[2]I. Robinson and A. Trautman, Proc. Roy. Soc. (London) A265, 463 (1961).

[3]E. Newman, L. Tamburino, and T. Unti (to be published).