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Negative Mass in General Relativity

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Mechanics is considered in a universe containing negative mass. Demanding (i) conservation of momentum, (ii) principle of equivalence, (iii) no runaway motions, (iv) no Schwarzschild black holes, and (v) the inertial and active gravitational masses of a body shall have the same sign, we find that *all* mass must be negative. Some properties of such a universe are investigated. We show that a neutral spherical body of arbitrarily small size is possible, and observers external to it can communicate with each other by light rays without horizon problems. There are no cosmological models with a power-law big bang, and there is an abundance of nonsingular models. Like electric charges would attract each other, and unlike ones would repel. This could produce stars and galaxies held together by charge and not gravity. The investigation does not suggest any reason why mass in the real universe should be positive.

1. INTRODUCTION

As far as we know, mass is always and everywhere positive. Nevertheless, it is interesting to speculate on a universe containing negative masses, and many writers have done so.

The first speculations occurred in the nineteenth century and these are described in the book by Max Jammer [1]. Karl Pearson attributed the observed fast recession of a certain star to the fact that, having negative mass, it was being repelled from our region of space. Föppl in 1897 worked out an elaborate theory of negative masses, and Schuster in 1898 contemplated a universe containing negative mass.

The fundamental modern paper on negative mass is that of Bondi [2]. He points out that mass in classical mechanics really consists of three

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concepts: inertial mass, m_i, passive gravitational mass, m_p, and active gravitational mass, m_a. In Newton's theory these are taken as equal. In general relativity the principle of equivalence states that $m_i = m_p$, but m_a need not be equal to the others. All three masses are normally taken to be positive, but the theories do not compel this.

After Bondi's work, several papers about negative mass appeared [3–10]. Most of these investigated the interaction and possible coexistence of particles with masses of both signs.

My own interest in the topic arose from the simple observation, which must have been made by most relativists, that if mass were negative the Schwarzschild solution would contain no horizon and Schwarzschild black holes would not exist. To this extent the universe would be, at least superficially, a simpler place. I then asked myself what compensating disadvantages, if any, would such a hypothetical universe have; and could it be made sensible to us at all given the outlook we have acquired from our familiarity with positive mass?

If some of the masses m_i, m_p, m_a are allowed to be negative, a variety is possible in the laws of mechanics. The option I shall choose is to keep these and other physical laws as we know them; in particular, I retain the principle of equivalence and the equations of general relativity and electromagnetism. I suppose, however, that all active gravitational mass is negative, so that no uncharged spherical particles are black holes. *I am then led to a universe in which all three types of mass are negative.*

At this point it becomes clear that the universe I am considering has no practical relation to the one we live in. Indeed, what I am writing may be called science fantasy, and the busy reader is fully entitled to turn the page. My reason for continuing is to see whether the properties of the hypothetical universe suggest why the real universe contains only positive mass. My intention is well-summarized by Einstein's metaphorical phrase, "What interests me is whether God had any choice in the creation of the world."

The conclusion, briefly summarized, is that the negative mass universe is comprehensible, but black holes are not eliminated because they can occur in charged spherical particles.

The plan of the paper is as follows. Negative mass in classical mechanics is studied in Section 2, and there, assuming m_a negative, I am led to the supposition that m_i and m_p should be negative also. Section 3 deals with negative mass in the theory of relativity; here the exterior and interior Schwarzschild solutions are considered, and the paths of test particles and light rays in the surrounding vacuum. In Section 4 there is a brief investigation of relativistic cosmology for a universe of negative mass density. Section 5 gives the metric for a charged spherical particle of negative mass in Einstein–Maxwell theory, and the paper ends with discussion and conclusions in Section 6.

Professor W. H. McCrea has pointed out to me that many of the results of the paper can be obtained from the standard ones by keeping the masses positive but altering the sign of the gravitational constant, G. I prefer to keep G positive throughout and vary the signs of mass, because negative inertial mass produces unorthodox effects when nongravitational forces are present.

2. NEGATIVE MASS IN CLASSICAL MECHANICS

Consider two gravitating particles (1) and (2) with constant masses, and position vectors $\mathbf{r}_1, \mathbf{r}_2$. Newton's mechanics and law of gravitation give

$$\overset{(1)}{m}_i \ddot{\mathbf{r}}_1 = \frac{G(\mathbf{r}_2 - \mathbf{r}_1)\, \overset{(1)}{m}_p \overset{(2)}{m}_a}{|\mathbf{r}_2 - \mathbf{r}_1|^3} \tag{1}$$

$$\overset{(2)}{m}_i \ddot{\mathbf{r}}_2 = \frac{G(\mathbf{r}_1 - \mathbf{r}_2)\, \overset{(2)}{m}_p \overset{(1)}{m}_a}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \tag{2}$$

where an overdot means d/dt and G is the constant of gravitation, assumed positive. Adding, we obtain

$$\frac{d}{dt}(\overset{(1)}{m}_i \dot{\mathbf{r}}_1 + \overset{(2)}{m}_i \dot{\mathbf{r}}_2) = \frac{G(\mathbf{r}_2 - \mathbf{r}_1)(\overset{(1)}{m}_p \overset{(2)}{m}_a - \overset{(2)}{m}_p \overset{(1)}{m}_a)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} \tag{3}$$

and *for the conservation the momentum of the particles we require*

$$\overset{(1)}{m}_a / \overset{(1)}{m}_p = \overset{(2)}{m}_a / \overset{(2)}{m}_p = k \text{ (const.)} \tag{4}$$

This is equivalent to Newton's third law for the pair of particles (i.e., their actions on each other shall be equal and opposite). We shall henceforth suppose this to be true for all pairs of particles, so we have

$$\overset{(j)}{m}_a = k \overset{(j)}{m}_p \qquad \text{(for all } j) \tag{5}$$

Condition (4) *ensures also the conservation of energy* for gravitational motion of two particles. From (1) and (2) we have

$$\overset{(1)}{m}_i \ddot{\mathbf{r}}_1 \cdot \dot{\mathbf{r}}_1 + \overset{(2)}{m}_i \ddot{\mathbf{r}}_2 \cdot \dot{\mathbf{r}}_2$$

$$= G(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\overset{(1)}{m}_p \overset{(2)}{m}_a \dot{\mathbf{r}}_1 - \overset{(2)}{m}_p \overset{(1)}{m}_a \dot{\mathbf{r}}_2)\, |\mathbf{r}_2 - \mathbf{r}_1|^{-3}$$

and assuming (4) we can easily show that

$$\frac{d}{dt}(T+V)=0$$

where T, the total kinetic energy, is defined as usual, and the potential energy is

$$V=-\frac{1}{2}G(\overset{(1)}{m}_p\overset{(2)}{m}_a+\overset{(2)}{m}_p\overset{(1)}{m}_a)\,|\mathbf{r}_2-\mathbf{r}_1|^{-1} \tag{6}$$

The laws of conservation of momentum and energy are easily extended to any number of gravitating particles provided (5) is fulfilled.

From (3) and (4)

$$\overset{(1)}{m}_i\ddot{\mathbf{r}}_1+\overset{(2)}{m}_i\ddot{\mathbf{r}}_2=0 \tag{7}$$

Suppose now that inertial mass can exist with either sign, and envisage two particles of equal and opposite inertial mass, $\overset{(1)}{m}_i=-\overset{(2)}{m}_i$. Then (7) gives $\ddot{\mathbf{r}}_1=\ddot{\mathbf{r}}_2$, both accelerations being given by (1) or (2), and being nonzero if $\mathbf{r}_1\neq\mathbf{r}_2$. Thus the two particles, starting from rest, will follow each other with constant acceleration to infinity. The velocities increase without limit but momentum and energy are conserved because the inertial masses are equal and opposite.

The behavior is somewhat different if the inertial masses of the two particles are opposite in sign but not equal. Consider two particles whose inertial masses satisfy

$$\overset{(2)}{m}_i=-\gamma\overset{(1)}{m}_i$$

where γ is a positive constant not equal to unity. If the particles start from rest we find once again that they move off in the same direction, the velocity of $\overset{(2)}{m}$ being γ^{-1} times that of $\overset{(1)}{m}$. Unless the particles collide their velocities increase throughout the motion but tend to finite limits; these limits can be made arbitrarily large by allowing γ to approach unity. Momentum and energy are conserved.

This runaway motion still takes place if the particles are charged, unless the gravitational and electrical forces just balance. Indeed, any system of forces obeying Newton's third law allows (7).

I regard the runaway (or self-accelerating) motion described in the previous three paragraphs as so preposterous that I prefer to rule it out *by supposing that inertial mass is all positive or all negative,*[2] i.e.,

$$\overset{(j)}{m}_i > 0 \quad \text{or} \quad \overset{(j)}{m}_i < 0 \qquad (\text{for all } j) \tag{8}$$

Next I shall assume the principle of equivalence for all particles, i.e.,

$$\overset{(j)}{m}_i = \overset{(j)}{m}_p \qquad (\text{for all } j) \tag{9}$$

and also

$$\overset{(j)}{m}_a < 0 \qquad (\text{for all } j) \tag{10}$$

which has the effect of ruling out black holes.

Equations (5), (8), (9), and (10) then lead to two possibilities

$$\text{(a)} \quad \overset{(j)}{m}_i < 0, \quad \overset{(j)}{m}_p < 0, \quad \overset{(j)}{m}_a < 0 \qquad (\text{for all } j) \tag{11}$$

$$\text{(b)} \quad \overset{(j)}{m}_i > 0, \quad \overset{(j)}{m}_p > 0, \quad \overset{(j)}{m}_a < 0 \qquad (\text{for all } j) \tag{12}$$

Since we are contemplating a different universe there is no *a priori* basis for choosing between (11) and (12). Some justification for ruling out (12) will appear in the next section where we shall consider possible interiors for Schwarzschild particles of negative m_a. While admitting that (12) deserves further study, I shall choose (11) here. *Throughout the rest of the paper we shall envisage a universe in which all bodies have m_i, m_p and m_a negative.*

Let me summarize the assumptions which have led me to this world-outlook. I started with Newton's second law and his law of gravitation in forms (1) and (2). I then assumed (i) conservation of momentum (or the third law), (ii) no runaway motion, (iii) principle of equivalence, and (iv) no Schwarzschild black holes. This brought me to (11) and (12), of which I have chosen to consider here only (11).

[2] Runaway motion can also occur with two particles whose inertial masses have the same sign, e.g., in the case

$$\overset{(1)}{m}_i = \overset{(2)}{m}_i, \quad \overset{(1)}{m}_p = \overset{(2)}{m}_p, \quad \overset{(1)}{m}_a = -\overset{(2)}{m}_a$$

Eq. (4) is not satisfied, momentum is not conserved, and the third law is violated. Bondi showed that this motion is also permitted in general relativity. From the point of view adopted in this paper, this argument would lead us to deny the existence of particles having the same signs for m_i and m_p, but opposite signs for m_a.

Although the concept of a black hole is essentially one of general relativity, it is instructive in this section to consider a Newtonian interpretation that was given by McCrea [11]. Suppose we have a spherical mass m of radius r_0. Let us use units in which G and c, the speed of light, are both unity and assume the equivalence of mass and energy $E=mc^2$. Suppose that a particle of mass δm is brought slowly from infinity to the surface of m. The potential energy of δm in the field of m is $-r_0^{-1}m\,\delta m$, and the total mass of the sphere is now

$$m+\delta m-r_0^{-1}m\,\delta m \tag{13}$$

Let m and δm be positive: then when the radius r_0 is equal to m, (13) reduces to m and the additional particle confers no extra mass on the sphere. If $r_0<m$ the addition of δm actually reduces the total mass. Thus, there is a singular radius r_0 associated with the positive mass m. If, however, m and δm are both negative, δm and the potential energy both contribute (negatively) to m and by the process $|m|$ can be increased indefinitely whatever the radius of m. There is no singular radius in this case. McCrea showed how, in the case of positive mass, the argument could be adapted to general relativity, and the singular radius is then $r_0=2m$.

3. NEGATIVE MASS IN THE THEORY OF RELATIVITY

In relativity a further sort of mass arises—rest mass. The rest mass m_0 of a body need not be the same as any of m_i, m_p, m_a. Let us, however, assume that m_0 is negative along with the others. In accordance with my assumption that the laws of physics are to be unaltered, we must preserve the formula

$$E_0=m_0c^2 \tag{14}$$

so that the rest energy E_0 is negative. Indeed, since all energy has active gravitational mass, we must suppose that *all energy is negative*, except possibly in certain bodies where positive energy does occur but is outweighed by negative energy, so that the total mass is negative. An example would be a fluid sphere in which the rest density is negative but a positive pressure contributes positively to the energy density: in this case, our supposition would require the total energy to be negative. A case of this sort is described in this section.

Negative mass in special relativity has been considered at length by Terletski [12]. From the practical point of view, there would be some unfamiliar phenomena, for example, in the manifestations of the formula

(14). Theoretically, the only difficulty appears to be in the violation of causality, or equivalently, the violation of the second law of thermodynamics. However, these violations occur only if both positive and negative masses are present. From the point of view adopted in this paper, in which all masses are conceived to be negative, this difficulty would not arise. (Difficulties arising in general relativity when positive and negative matter coexist have been described by de Martins [13]).

I turn now to negative mass in general relativity. I shall use the metric signature $---+$ and field equations

$$R^{ik} - 1/2 g^{ik} R + \Lambda g^{ik} = -8\pi T^{ik} \tag{15}$$

where T^{ik} is the energy tensor and Λ the cosmological constant. In this section and Section 5, Λ will be put zero, but will be nonzero in Section 4.

We normally think of matter as being the source of gravitational fields, but there exist exact solutions of (15) with $T^{ik}=0$ and $\Lambda=0$ which contain no parameter referring to mass at all. Examples are the Kasner metric [14] and some gravitational wave metrics (e.g., plane waves [15]).

In the exact vacuum solutions which do contain parameters referring to mass, the mass concerned is, in all cases known to me, the active gravitational mass, m_a, and it enters as an arbitrary constant which can assume either sign; m_i and m_p do not occur explicitly, even in exact solutions referring to the motion of particles [16–18]. Sometimes m_i and m_p can be identified, although with considerable difficulty, in approximate solutions [19, 20].

If we take perfect fluid as the source of the field, so that, in the usual notation,

$$T^{ik} = (p+\rho)\, u^i u^k - g^{ik} p \tag{16}$$

we can expand the identities $T^{ik}{}_{;k}=0$ in the form

$$(p+\rho)\, a^i = h^{ik} p_{,k} \tag{17}$$

where $a^i = u^i{}_{;k} u^k$ is the fluid acceleration, and $h^{ik} = g^{ik} - u^i u^k$ is the tensor projecting the pressure gradient $p_{,k}$ into the 3 space orthogonal to u^i. In (17) *$p+\rho$ is clearly to be identified with the inertial mass density*. Using the principle of equivalence *we take the same expression for passive gravitational mass density*. Whittaker [21] showed that in the static case the *active gravitational mass density is $\rho+3p$*, and I shall assume this is so in what follows.

Let us now turn to the static exterior field of a spherical body of negative mass m. The Schwarzschild solution is

$$ds^2 = -\left(1+\frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \left(1+\frac{2M}{r}\right) dt^2 \tag{18}$$

where

$$-m = M > 0 \tag{19}$$

There is no horizon, and light emitted is blue-shifted. The only singularity is at $r = 0$.

Suppose that the coordinate radius of the body is r_0. Then we can fit an interior containing perfect fluid of constant proper density, just as in the case of positive mass [22]. The metric is

$$\begin{aligned} ds^2 = &-\left(1+\frac{r^2}{R^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \\ &+\left[\frac{3}{2}\left(1+\frac{r_0^2}{R^2}\right)^{1/2} - \frac{1}{2}\left(1+\frac{r^2}{R^2}\right)^{1/2}\right]^2 dt^2 \end{aligned} \tag{20}$$

and the pressure p and the density ρ_0 are given by

$$8\pi p = \frac{3}{R^2}\left[\frac{\left(1+\frac{r_0^2}{R^2}\right)^{1/2} - \left(1+\frac{r^2}{R^2}\right)^{1/2}}{3\left(1+\frac{r_0^2}{R^2}\right)^{1/2} - \left(1+\frac{r^2}{R^2}\right)^{1/2}}\right]$$

$$-8\pi\rho_0 = 3R^{-2} = 6Mr_0^{-3}$$

so that the proper density ρ_0 is negative and the constant R is real. The pressure is positive and the pressure gradient negative. (It should be noted that, since inertial mass in negative, random motions of particles produce tensions, not pressures.)

There is no lower limit on the coordinate radius r_0, in contrast to the case of positive density in which reality of the metric and the finiteness of the pressure impose lower limits on r_0.

The inertial mass density $\rho_0 + p$, and the active gravitational mass density $\rho_0 + 3p$, are both negative inside the sphere. The integral of the former over the volume of the sphere (i.e., the total inertial mass of the body), is therefore negative. Thus it is not possible, with an interior solution of constant rest density, to make a model of a sphere satisfying (12).

A different interior for a spherical body was found by Whittaker [23]. This has constant active gravitational mass density, i.e.,

$$\rho + 3p = n \text{ (const.)}$$

It is easy to check that, in the case of negative m, the field equations (15) and (16), together with continuity conditions at the boundary, require that if n is negative the inertial mass density $\rho + p$ is also negative throughout the sphere, so in this case, also, the body would not satisfy (12).

These two interior solutions constitute the support, referred to in Section 2, for ruling out (12). Of course, they do not prove that there are no interiors consistent with (12); I mention them as some justification for considering (11) rather than (12), at least in the first instance.

Let us now consider the motions of test particles in the space-time (18). These are, of course, given by the geodesics. The motions of a positive mass test particle are the same as those of a negative mass one, if they start from the same initial configuration. There are no bound orbits, as can easily be checked by studying the geodesic equations [24]. It is well-known that in Newtonian theory orbits under a central repulsive inverse square law are hyperbolas or straight lines.

Consider a test particle projected inward along a radial line. *If its velocity of projection is great enough, it can approach arbitrarily near the center, but it is always turned back before reaching* $r = 0$. A test particle projected radially outward proceeds to infinity.

It is interesting to study the corresponding situation for a ray of light. The null geodesic equations for a radial ray in space-time (17) reduce to a single equation

$$\frac{dr}{dt} = -\varepsilon\left(1 + \frac{2M}{r}\right)$$

where $\varepsilon = +1$ for an incoming and -1 for an outgoing ray. Integrating this with initial condition $r = r_1$ at $t = 0$ we find

$$\varepsilon t = r_1 - r - 2M \log[(r_1 + 2M)(r + 2M)^{-1}] \tag{21}$$

Thus an *incoming ray is not turned back* and reaches $r = 0$ in a finite coordinate time $r_1 - 2M \log(1 + r_1/2M)$.

Consider now two fixed observers 0_1 and 0_2 on the same radial line, at coordinate radii r_1 and r_2 ($r_2 > r_1$). Suppose that 0_2 sends a light ray to 0_1 who reflects it back. Then using (20) we find that the coordinate time which elapses at 0_2 during the return trip of the ray is

$$T = 2\{r_2 - r_1 - 2M \log[(r_2 + 2M)(r_1 + 2M)^{-1}]\}. \tag{22}$$

Noting that the function $f(r) \equiv r - 2M \log(r + 2M)$ increases monotonically from $r = 0$, we see that T is positive; it is also finite for finite r_2. The proper time at 0_2 corresponding to T is

$$s_2 = T(1 + 2Mr_2^{-1})^{1/2}$$

If the process is reversed so that 0_1 sends a ray to 0_2 who reflects it back, T is unchanged and the proper time at 0_1 is

$$s_1 = T(1 + 2Mr_1^{-1})^{1/2}$$

Thus, s_1 and s_2 are finite and positive. So *any two fixed radial observers can communicate with each other in finite proper time by the radar method.* (This is true even if 0_1 is at the singularity $r = 0$.) *There are no failures in communication like those due to the horizon in the Schwarzschild space-time for a positive mass.*

4. COSMOLOGY WITH NEGATIVE MASS

From purely geometrical considerations we know that the metric of a spatially homogeneous universe in comoving coordinates must be that of Robertson and Walker:

$$ds^2 = -[S(t)]^2 \{dr^2 + [f(r)]^2 (d\theta^2 + \sin^2\theta . d\phi^2)\} + dt^2 \qquad (23)$$

where $f(r) = \sin r$, r or $\sinh r$ according as the space curvature k is $+1$, 0, or -1, respectively. Supposing that the field equations are (15) and that the model is filled with perfect fluid with energy tensor (16), we can write the relevant equations as

$$8\pi\rho = S^{-2}(3\dot{S}^2 + 3k - \Lambda S^2) \qquad (24)$$

$$8\pi p = -S^{-2}(2S\ddot{S} + \dot{S}^2 + k - \Lambda S^2) \qquad (25)$$

Then for the inertial and active gravitational mass densities we have

$$8\pi(\rho + p) = 2S^{-2}(-S\ddot{S} + \dot{S}^2 + k) \leqslant 0 \qquad (26)$$

$$8\pi(\rho + 3p) = 2S^{-1}(-3\ddot{S} + \Lambda S) \leqslant 0 \qquad (27)$$

which we suppose to be nonpositive as explained in Section 3. Note that Λ does not occur in (26). $S(t)$ is taken as real and positive; I shall omit the case in which ρ and p are zero for all t.

I shall not attempt a complete classification of the possible forms of the scale function S, but content myself with describing some of the possibilities.

4.1. $k = 0, 1$

From (26) we need $\ddot{S} \geqslant 0$ so S cannot have a maximum: recollapse after expansion is not possible for the spatially flat and elliptic models. To study the possibility of a big bang, we put $S=\alpha t^n$ ($t \geqslant 0$, $n>0$, $\alpha>0$) but we find that it is not possible to satisfy (26). Thus, models with $k=0, 1$ do not admit a power-law big bang. As an example of a scale function satisfying both (26) and (27) I mention

$$S=\beta \cosh \alpha t, \beta>0, \alpha\beta>1, 3\alpha^2>\Lambda$$

where α, β are constants. The model with this form of S is free of singularity.

4.2. $k=-1$

This case admits a static universe, $S=S_0$ (const.) provided Λ is negative. In nonstatic universes, if $\Lambda \geqslant 0$ (27) again requires $\ddot{S} \geqslant 0$, so S cannot have a maximum and recollapse is not possible.

A power-law big bang is possible, but afterward the regime satisfies (26) only for a limited time.

The case with most variety is $k=-1$, $\Lambda<0$. This allows a maximum in S. It also allows small oscillations about the static universe; that is, we may take

$$S=S_0+\varepsilon f(t)$$

where ε is constant and $f(t)$ is a function oscillating about zero. Then if ε is sufficiently small, (26) and (27) are still satisfied.

4.3. $p=0$, $\Lambda=0$

This case is actually included in the foregoing, but I mention it explicitly because it corresponds to the simplest Friedmann models of traditional cosmology. It follows from (27) that $\ddot{S}$ must be nonnegative, and then from (26) that $k=-1$. Integrating (25) with $p=0$ and $k=-1$ we find

$$S=\alpha^2 \cosh^2 u$$

$$t+t_0=\alpha^2(1/2 \sinh 2u+u)$$

where α and t_0 are constants. The geometry is necessarily hyperbolic and S decreases from infinity at $t=-\infty$ to a minimum and then increases to infinity again. There is no big bang or other singularity.

To sum up, the main feature of interest in this section is *the absence of a power-law big bang leading to a permanent negative mass cosmology, and the abundance of nonsingular models.* However, as explained in Section 6, electric forces might play a prominent part in the motion of large structures in the universe. If so, the results of this section would be drastically affected.

5. THE SPACE-TIME OF A STATIC SPHERICAL CHARGE

Let us assume that three of the equations of Einstein–Maxwell theory are as usual

$$R^{ik} - 1/2g^{ik}R = -8\pi(T^{ik}_{(m)} + E^{ik}) \tag{28}$$

$$F_{ik} = A_{i;k} - A_{k;i} \tag{29}$$

$$F^{ik}{}_{;k} = 4\pi J^i \tag{30}$$

$T^{ik}_{(m)}$ being the material energy tensor, E^{ik} the electromagnetic energy tensor, A_i the four-potential for the electromagnetic field F_{ik}, and J^i the current density; a semicolon denotes covariant differentiation. To find the exterior solution for a static spherical charge we put $T^{ik}_{(m)} = 0$, and assume a static spherically symmetric metric.

An important difference from the usual derivation arises because of our assumption that inertial mass is negative. To see this, consider the energy of two charges $q_1 q_2$, held at rest in their mutual field. It is the work that must be done to bring q_2 from infinity to its actual position in the field of q_1. Since q_2 will have negative inertial mass, the work has the sign of $-q_1 q_2$, which is opposite to its sign in ordinary physics. Therefore, *in this case the energy tensor* E^{ik} *to be substituted into* (28) *must have a minus sign prefixed to the usual expression*, i.e., it must be

$$E^{ik} = F^{ia}F^{k}{}_{a} - 1/4\, g^{ik}F^{ab}F_{ab} \tag{31}$$

Let us assume that this is generally true for all forms of electromagnetic energy. Then it will ensure that, for example, incoherent radiation contributes negatively to the rest mass in (14). This is required by our supposition, for otherwise one could envisage a container of negative mass filled with sufficient radiation to give a body of positive mass, which we have ruled out.

Continuing with the spherical case, we need to substitute (31) into (28) and solve (28)–(30) for a static spherically symmetric metric and electric change q. We proceed in the usual way [26] and arrive at the solution

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1 - \frac{2m}{r} - \frac{q^2}{r^2}\right) dt^2 \tag{32}$$

$$F_{14} = qr^{-2}$$

This differs from the standard one in the negative sign before q^2, which arises because of the change in sign of E^{ik} occurring in (31).

In (32) m is negative in accordance with our fundamental assumption. The metric has a horizon where $g_{44}=0$, i.e., where

$$r_0 = -|m| + (m^2 + q^2)^{1/2}$$

Thus, a charged particle has a horizon unless its coordinate radius is greater than r_0. Therefore, although our supposition that m is negative ruled out black holes in neutral particles, *charged particles can form black holes.*

6. DISCUSSION AND CONCLUSION

In the universe we have been considering, the kinetic energy of particles, having the sign of the inertial mass, is negative. Random motions exert tensions, not pressures. The potential energy of the gravitational field is negative, as usual.

In the real universe, gravitation (although very weak compared with other forces) becomes important when its attractive character produces large masses. Repulsive gravitation, as envisaged in this paper, would tend to disperse matter; it would not form stars. Indeed, preventing the formation of large masses, gravity might not assume much importance at all. Negative inertial mass would have very far-reaching consequences, however.

Maxwell's equations are unchanged in our hypothetical universe; what is altered is the motion of charges and magnets, all endowed with negative inertial mass. The sign of the electromagnetic energy is opposite to that in ordinary theory.

The forces of electromagnetism could produce condensations. Consider two particles of equal inertial mass m_i, and charges q_1 and q_2, moving in each other's electric field with angular velocity ω in a circle of radius a about their mass center. Ignoring gravitation, we have for each particle[3]

$$-m_i a\omega^2 = q_1 q_2 (2a)^{-2}$$

Since $m_i < 0$ this requires q_1 and q_2 to have the same sign. Thus atoms would be possible and would be of two types: those of positive charge, and those of negative charge. Chemistry might be a very interesting subject in this universe. Moreover, positive atoms would attract each other, and so

[3] Coulomb's law is unaltered, in particular, in sign.

would negative atoms. In this way large condensations, each with a positive or negative charge, could form.

In fact, charge would take the place of gravitational mass in the formation of large bodies. Although there would be no neutral planets orbiting stars under the force of gravitation, there could be charged planetary systems. Intelligent life in such a universe could not be ruled out. Galaxies with charge of either sign might exist, and positively charged galaxies would repel negatively charged ones. This would have a profound effect on the purely gravitational cosmological models described in Section 4.

There would be no neutral black holes, and observers in the space-time of a neutral negative mass would have no difficulties of communication, as explained in Section 3. Charged bodies could form black holes, however, and (since the largest bodies would probably be charged) the theorists' problems with black holes would still be present.

Epistemologically, it seems that a universe of negative mass (but no positive mass) presents no logical or conceptual difficulties for general relativity or electromagnetism—although such difficulties may arise in other branches of physics. Pursuing Einstein's theological metaphor, mentioned in the Introduction, one can say that the results of this paper do not reveal why God chose matter to have positive rather that negative mass.

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