Geometrization of matter and antimatter through coadjoint action of a group on its momentum space

2: Geometrical description of Dirac's antimatter

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Abstract:

We extend the precedent group to a four-components orthochron set. This operation gives a geometrical interpretation of antimatter after Dirac.

1- Introduction

In a former paper [1] we have presented a description of elementary particles ins a ten-dimensional space, i.e. space-time (x,y,z,t) plus six additional dimensions:

(1)

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^6 \}$$

We presented a 16-dimensions group, an extension of the Poincaré orthochron subgroup, acting on:

- its 16-dimensions momentum space

- its 10-dimensional movement space.

The six additional components of the momentum have been identified to the charges of the particles:

(2)

$$\{ q , c_B , c_L , c_\mu , c_\tau , \varpi \}$$

so that the momentum becomes:

(3)

$$\mathbf{J}_{pe} = \{ q , c_B , c_L , c_\mu , c_\tau , \varpi , \mathbf{J}_p \}$$

where **J**p represent the classical moment, from the orthochron Poincaré sub-group:

(4)

$$\mathbf{J}_{po} = \{ E , \mathbf{p} , \mathbf{f} , \mathbf{l} \}$$

after J.M.Souriau [1].

We have figured the link between the species of moments and the species of movement, suggesting that:

- The movement of matter corresponds to $\{ \zeta^i > 0 \}$ sector.
- The movement of antimatter corresponds to $\{ \zeta^i < 0 \}$ sector.
- The movement of photons corresponds to $\{ \zeta^i = 0 \}$ plane.

All that must be now justified.

2- Introducing a four components group. Geometrization of Dirac's antimatter

The precedent 16-dimensional group had two components, correspondong to the two orthochron components of the Lorentz group, L_n (neutral component) and L_s , with:

(5)

$$L_o \text{ (orthochron sub-group)} = L_n \cup L_s$$

Our group was an extension of the orthochron Poincaré sub-group:

(6)

$$G_o = G_n \cup G_s$$

and we wrote it:

(7)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \phi_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & \mathbf{0} & \phi_2 \\ 0 & 0 & 1 & 0 & 0 & 0 & \mathbf{0} & \phi_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & \mathbf{0} & \phi_4 \\ 0 & 0 & 0 & 0 & 1 & 0 & \mathbf{0} & \phi_5 \\ 0 & 0 & 0 & 0 & 0 & 1 & \mathbf{0} & \phi_6 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & L_o & \mathbf{C} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ \boldsymbol{\xi} \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta_1 + \phi_1 \\ \zeta_2 + \phi_2 \\ \zeta_3 + \phi_3 \\ \zeta_4 + \phi_4 \\ \zeta_5 + \phi_5 \\ \zeta_6 + \phi_6 \\ L_o \boldsymbol{\xi} + \mathbf{C} \\ 1 \end{pmatrix}$$

The corresponding coadjoint action was:

(8)

$$\chi^{i\,\prime} = \chi^{i} \quad \text{for } i = 1 \text{ to } 6$$

$$\mathbf{P}' = \mathbf{L}\,\mathbf{P}$$

$$\mathbf{M}' = \mathbf{L}\,\mathbf{M}\,\overline{\mathbf{L}} + (\mathbf{C}\,\overline{\mathbf{P}}\,\overline{\mathbf{L}} - \mathbf{L}\,\mathbf{P}\,\overline{\mathbf{C}})$$

with:

(9)

$$\{\chi^{i}\} = \{ q , c_B , c_L , c_\mu , c_\tau , \varpi \}$$

In such a group no element transforms the movement of a matter mass-point into the movement of an antimatter mass-point, and vice versa. According to the chosen definition of antimatter, through a:

(10)

$$\zeta \text{ - Symmetry: } \{\zeta^{i}\} \text{ ----> } \{-\zeta^{i}\}$$

some element should reverse the additional dimensions. With:

(11)

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{pmatrix} \qquad \boldsymbol{\zeta} = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{pmatrix}$$

we can write the precedent group into a more compact form:

(12)

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \boldsymbol{\phi} \\ \mathbf{0} & \mathbf{L}_o & \mathbf{C} \\ 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \boldsymbol{\zeta} \\ \boldsymbol{\xi} \\ 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{\zeta} + \boldsymbol{\phi} \\ \mathbf{L}_o \boldsymbol{\xi} + \mathbf{C} \\ 1 \end{pmatrix}$$

It contains the neutral element:

(13)

$$\mathbf{1} = \begin{pmatrix} 1 & \mathbf{0} & 0 \\ \mathbf{0} & 1 & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{pmatrix}$$

The matrix that reverses the additional dimensions is be the following orthochron commuter:

(14)

$$g_{oc} = \begin{pmatrix} -1 & \mathbf{0} & 0 \\ \mathbf{0} & 1 & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{pmatrix}$$

We can duplicate the precedent group through the operation:

(15)

$$\mathbf{g}_o \times \mathbf{g}_{oc}$$

It is equivalent to write the new four component group, whose element is:

(16)

$$\begin{pmatrix} \lambda & \mathbf{0} & \lambda\phi \\ \mathbf{0} & L_o & C \\ 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta\lambda + \lambda\phi \\ L_o\xi + C \\ 1 \end{pmatrix} \quad \lambda = \pm 1$$

The corresponding coadjoint action is:

(17)

$$\chi^{i\,\prime} = \lambda\,\chi^{i} \qquad \text{for } i = 1 \text{ to } 6$$

$$P' = L\,P$$

$$M' = L\,M\,\overline{L} + (\,C\,\overline{P}\,\overline{L} - L\,P\,\overline{C}\,)$$

We see that ($\lambda = -1$) reverses the charges. In that case the inversion of the additional dimensions:

(18)

$$\zeta\text{ - Symmetry: } \{\zeta^{i}\} \longrightarrow \{-\zeta^{i}\}$$

goes with a:
(19)

C-symmetry (or charge conjugation):

$$\{ q , c_B , c_L , c_\mu , c_\tau , \varpi \} \text{ ---> } \{- q ,- c_B ,- c_L ,- c_\mu ,- c_\tau , - \varpi \}$$

which corresponds to Dirac's description of antimatter [4], so that the present paper represents a geometrization of antimatter after Dirac.

3- Coadjoint action on momentum space

In order to make the things clearer we can graphically figure it.

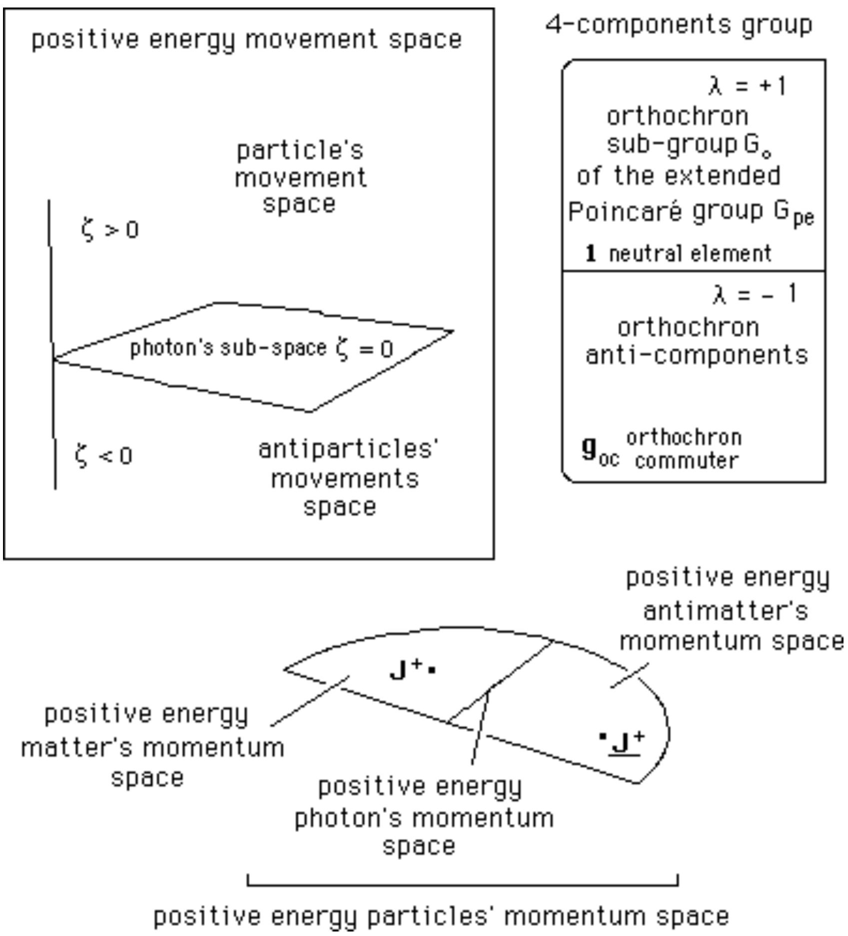

Fig.1: The four component orthochron extended group. The (λ=1) **components form a a sub-group. Below, the momentum space with its three sub-sets, figuring particles', antiparticles' and photons' worlds. Associated two-sectors movement space.**

If we choose an element picked from the ($\lambda = 1$) sub-group we refind the schemas presented in the precedent paper [1].

Examine the impact of the orthochron commuter $\mathbf{g}_{oc}$ on the moment and associated movement.

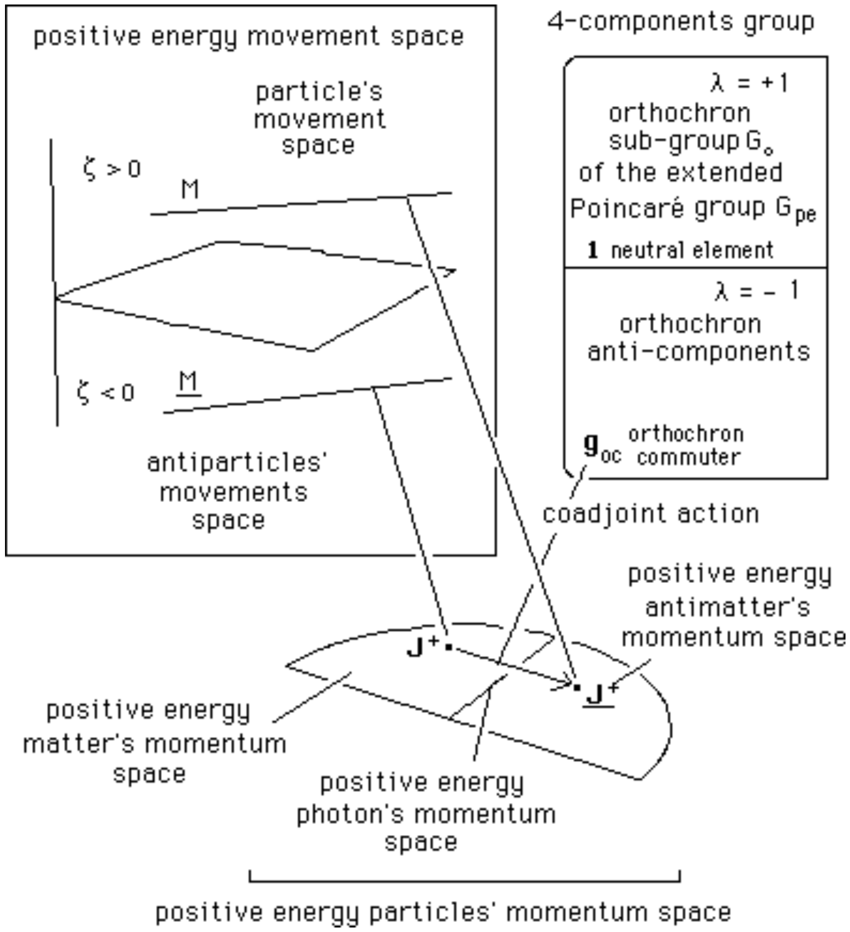

Fig.2: Coadjoint action of the orthochron commuter g_{oc}

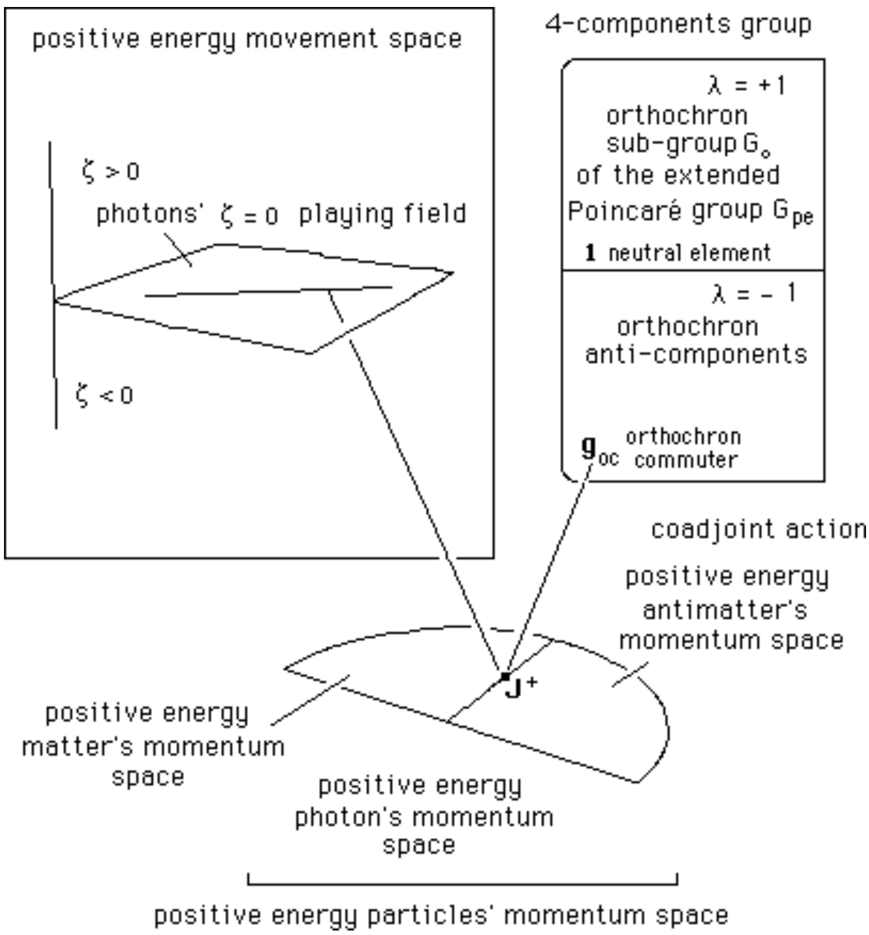

Fig.3: Coadjoint action of the orthochron commuter g_{oc} on the photon: none, for it is its own antiparticle.

Now, introduce two coupled orthochron matrixes:

(20)

$$\mathbf{g}_o \text{ and } \mathbf{g}_{oc} \times \mathbf{g}_o$$

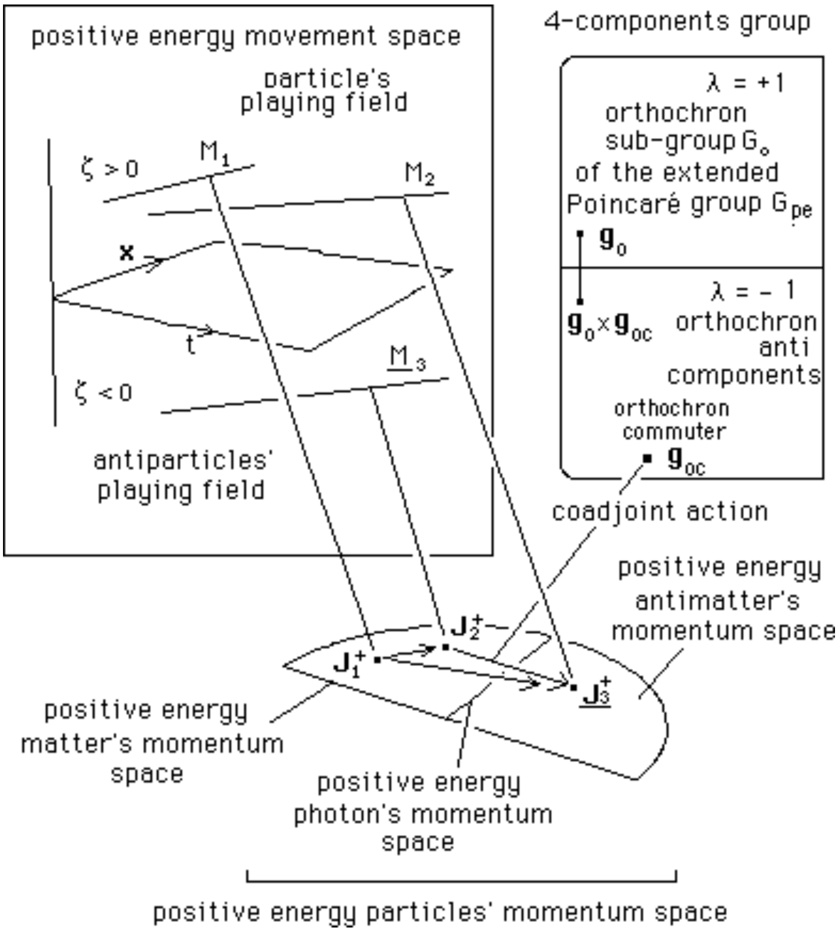

Fig.4 : **Coadjoint action of the orthochron commuter $\mathbf{g}_{oc}$ and conjugated orthochron matrixes $\mathbf{g}_o$ and $\mathbf{g}_{oc} \times \mathbf{g}_o$**

Conclusion

We start from the precedent paper [1], where we introduced a 16-dimensional group acting on its 16-dimensions momentum space and 10-dimensional movement space. As in [1] we follow the basic idea: antimatter corresponds to a ζ-Symmetry, to the inversion of the additional variables. We define a matrix, called orthochron commuter, which achieves ζ-Symmetry. Then we build a group which contains such element. We get a four components group, composed by the elements $\mathbf{g}_o$ of the ($\lambda = 1$) sub-group, and by conjugated matrixes $\mathbf{g}_{oc} \times \mathbf{g}_o$, formed through the action of the orthochron commuter $\mathbf{g}_{oc}$ on this sub-group. The antimatter becomes another movement of matter, driven by coadjoint action of the group.

References

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Acknowledgements

This work was supported by french CNRS and Brevets et Développements Dreyer company, France.
Déposé sous pli cacheté à l'Académie des Sciences de Paris, 1998.