Geometrization of matter and antimatter through coadjoint action of a group on its momentum space.

3: Geometrical description of Dirac's antimatter. A first geometrical interpretation of antimatter after Feynmann and so-called CPT-theorem.

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Abstract:

We include antichron elements in the dynamic group. Then we get movements and moments involving T-symmetry, like PT-symmetric movements and CPT-symmetric movements. The first evoke the Feynmann's vision of antimatter and the second the so calle "CPT theorem". But tim-inversion, from coadjoint action, changes the sign of the mass and energy. The PT-symmetrical of a particle of matter does not longer identify to Dirac's antiparticle, as Feynmann thaught. It is an antiparticle, but with negative mass. Same thing for CPT theorem: the CPT-symmetrcal of a particle of a particle of matter is a partcle of matter, but with negative mass.

1- Introduction

In former papers ([1] and [2]) we have given a geometrical interpretation of antimatter. Matter and antimatter are suppose to have their personal playing field $\{\zeta i > 0\}$ and $\{\zeta i < 0\}$ in a ten dimensional space:

(1)

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^6 , x , y , z , t \}$$

composed by space-time { x , y z , t } plus six additional dimensions. The playing field of photons corresponds to the $\{\zeta i > 0\}$ plane.

Our 16 dimension groups gives six additional scalars, identified to quantum charges. The basic geometric definition of antimatter we suggest corresponds to:

(2)

$$\zeta\text{-symmetry: } \{ \zeta^i \} \text{ ----> } \{ - \zeta^i \}$$

Through a four component group [2] we have shown that, in such conditions, the ζ-symmetry goes with a C-symmetry, which corresponds to Dirac's antimatter [3], [4] and [5].

Feynmann suggested an alternative description of antimatter. The argument is the following. If we consider the evolution of a particule owing a mass m and an impulsion p, its energy is:

(3)

$$E_p = \sqrt{p^2c^2 + m^2c^4}$$

Suppose that this particle, moving in the "twin fold" F*, goes from a state 1 (P*1) to a state 2 (P*2). We just keep one space marker $x = x^1$ (doing $x^2 = 0$ and $x^3 = 0$). The amplitude of such an evolution is:

(4)

$$F[2,1] = \int \frac{d^3p}{(2\pi)^3\, 2\, E_p} \exp\left(i \mid E_p (t_1 - t_2) - p\,(x_1 - x_2) \mid \right)$$

(where, conventionnaly, c = h = 1).

This path owns a conjugated image in our space-time fold F. Due to the effect of PT symmetry, the "vision" of some hypothetic observers, located in the folds F and F*, would be different. For the observer located in the fold F the particle, owing a mass m and an impulsion p moves from the state 2 to the state 1 (P and T both add a minus sign to the impulsion). This movement occurs during a time-interval $\Delta t' = t'^1 - t'^2 = t^2 - t^1$, and from a position x_2 to a position x_1.

If, for an example, a neutrino νe , with a left helicity, moves in the fold F*, from the "point of view" of the fold F its helicity will be reversed: it will be an antineutrino.

2- Shifting to complete extended Poincaré group

The idea of Feynmann (PT-symmetrical particles) implies the presence of antichron components in the group. In the group presented in reference [1] and [2] space inversion is already present, due to their presence in the basic orthochron Lorentz group. They are required to take account of disntinct helicities for photons and neutrinos.

We could extend the group, introducting a time-switch matrix:

(5)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ x \\ y \\ z \\ t \\ 1 \end{pmatrix}$$

Multiplying and the elements of the orthochron sub-group we could build the antichron components. But let us do it in a simpler way:

(6)

$$\begin{pmatrix} \boldsymbol{\lambda} & \mathbf{0} & \boldsymbol{\lambda\phi} \\ \mathbf{0} & \mu \mathbf{L}_o & \mu \mathbf{C} \\ 0 & \mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \boldsymbol{\zeta} \\ \boldsymbol{\xi} \\ 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{\zeta\lambda} + \boldsymbol{\lambda\phi} \\ \mu \mathbf{L}_o \boldsymbol{\xi} + \mu \mathbf{C} \\ 1 \end{pmatrix} \qquad \boldsymbol{\lambda} = \pm 1 \quad \mu = \pm 1$$

This group contains all the required components: orthochron and antichron, but such writing evidences the PT-symmetry ($\mu = -1$) in a convenient way. This is a eight components group (2 x 2 x 2). The group of [2] is a sub-group of (6) whence the group of [2] was a sub-group of the one of [1]. The coadjoint action is found to be:

(7)

$$\chi^{i\,'} = \lambda \mu \ \chi^i \quad \text{for } i = 1 \text{ to } 6$$

$$\mathbf{P}' = \mathbf{L}\,\mathbf{P}$$

$$\mathbf{M}' = \mathbf{L}\,\mathbf{M}\,\overline{\mathbf{L}} + (\,\mathbf{C}\,\overline{\mathbf{P}}\,\overline{\mathbf{L}} - \mathbf{L}\,\mathbf{P}\,\overline{\mathbf{C}}\,)$$

Here again, we identify the scalars χ i to the particle's charges set:

(8)

$$\{\chi^i\} = \{ q , c_B , c_L , c_\mu , c_\tau , \varpi \}$$

$\lambda = -1$ achieves:

(9)

$$\zeta \text{ - Symmetry: } \{\zeta^i\} \text{ ----> } \{-\zeta^i\}$$

Here again ζ - Symmetry is assimilated to matter-antimatter duality. With this material we can analyze the impact of the different components on the momentum. As we have antichron terms our momentum space must be extended to (E < 0) momentums sectors. See figure 1.

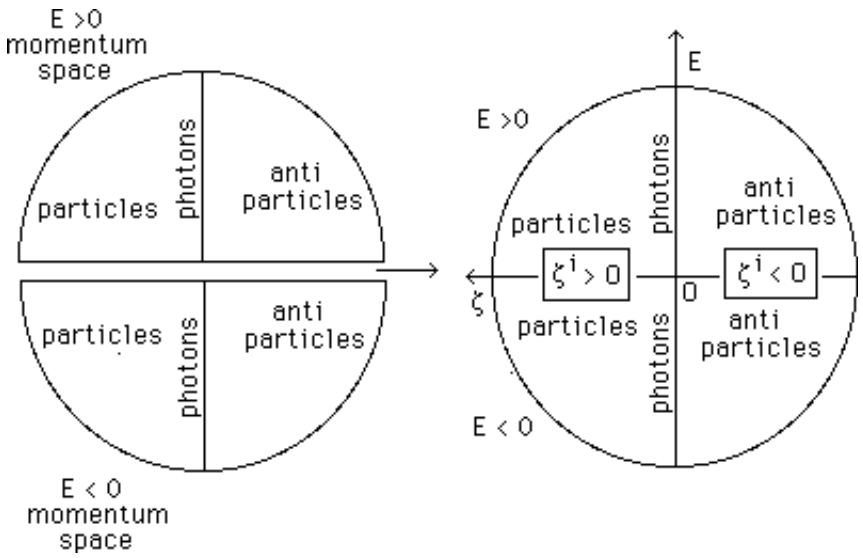

Fig.1: **Momentum space with positive and negative energies sectors.**

3- negative energies sectors

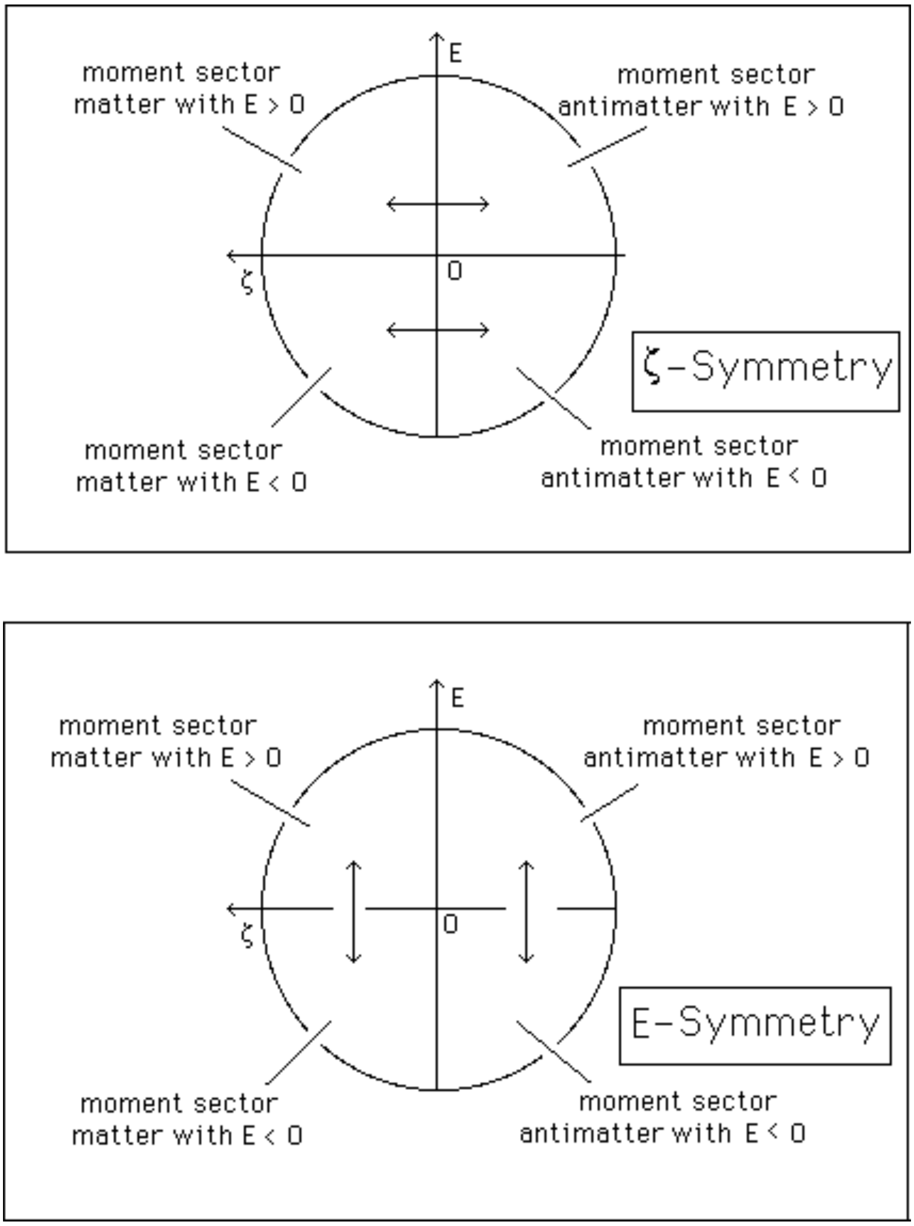

Fig.2: Subsequent symmetries

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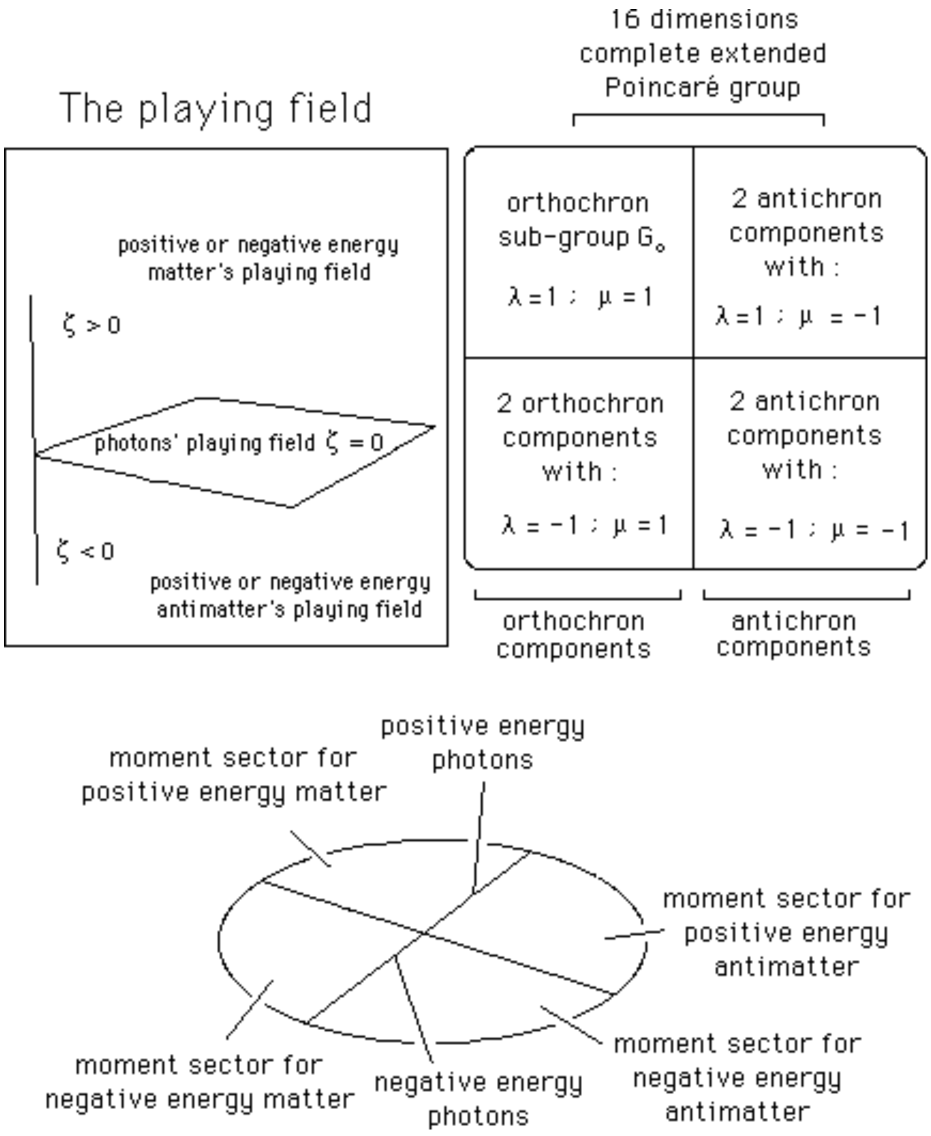

Fig.3: **The eight components group.**
its momentum and movement spaces.

It becomes easy to examine the impact of each component on momentum and movement. We shall consider a reference movement and momentum $\mathbf{J}^{+}{}_{1}$, refering to positive energy matter (the impact on positive energy photons will be analysed in a second step).

The sector of the group in which the element is chose will be grey.

Next, the movements of ordinary matter.

$$\lambda = +1 \quad \mu = +1$$

$$\lambda \mu = +1$$

The charges are unchanged. The movement M_2 refers to ($E > 0$), positive mass, orthochron matter.

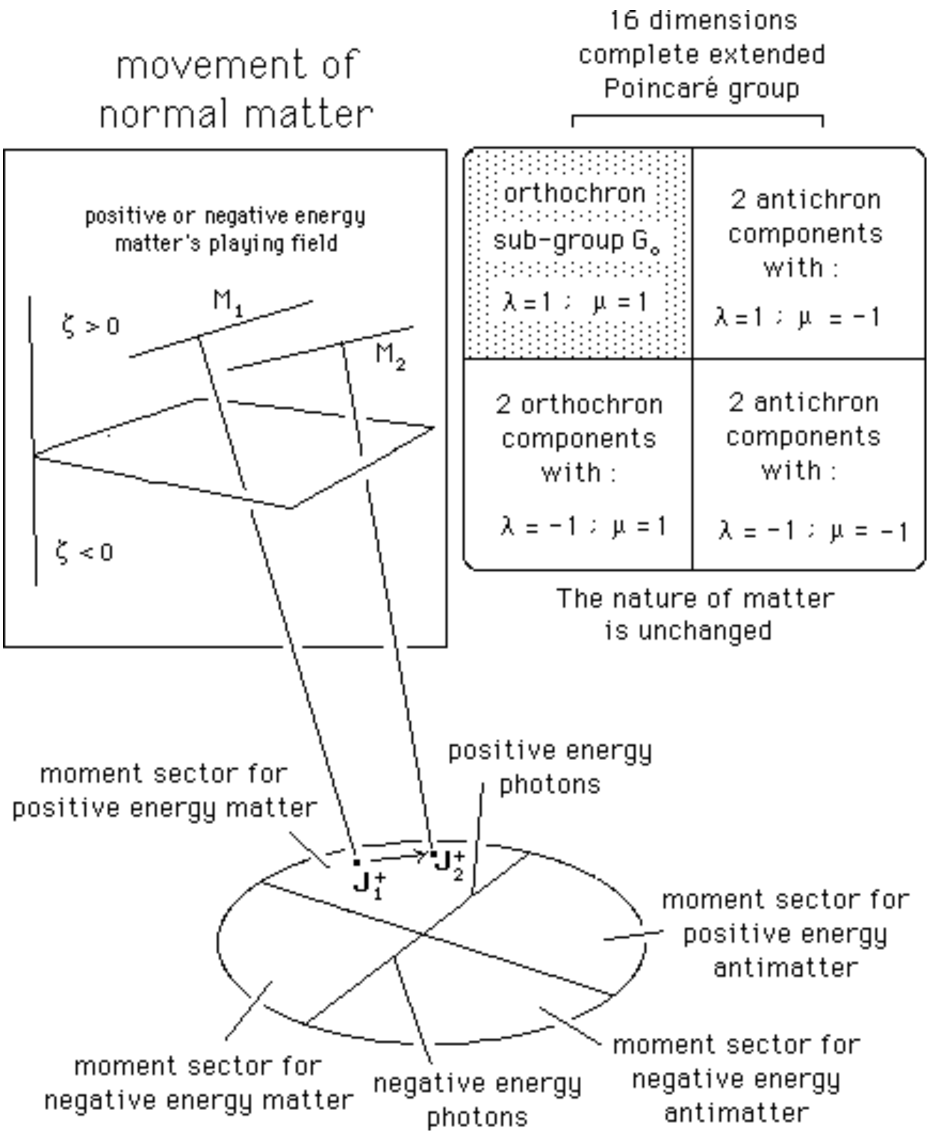

Fig.4: Movements of ordinary matter.
Action of orthochron elements of the group, with $\lambda = 1$
Charges unchanged.

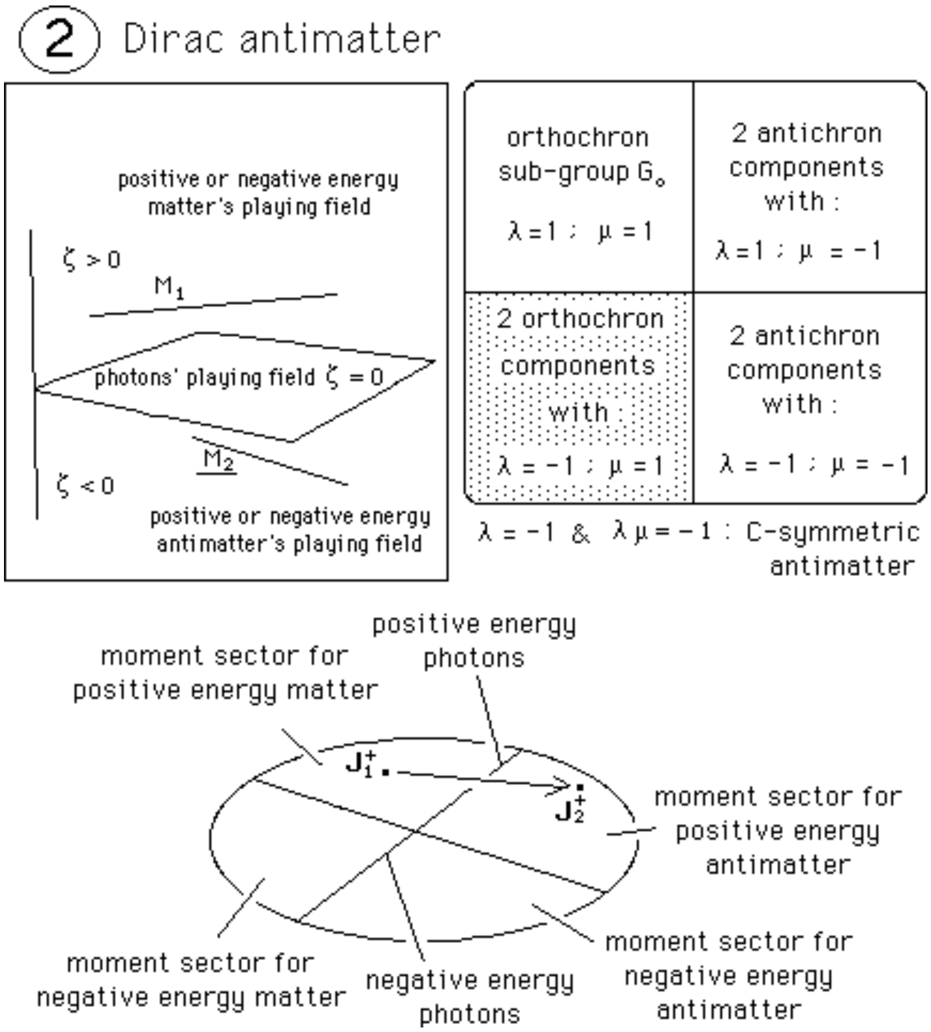

Fig. 5: Coadjoint action of a (λ = -1 ; μ = 1) element of the group on the momentum associated to the movement of normal matter: the new movement corresponds to Dirac's antimatter.

On the figure 5 the line M_1 figures the movement of normal, orthochron matter. We figures straight lines because ou group does not take account of force field, like gravitational or electromagnétic field. It only runs the behaviour of lonely particles, charged mass-points.

We choose an element in the grey area, corresponding to a (λ = -1 ; μ = 1) matrix. The (λ = - 1) value changes the signs of all the ζ^i. They become negative.

The new path is in the second sector, corresponding to antimatter. As $\lambda\ \mu$ = - 1 the charges are reversed. But as time is not reversed, the energy and the mass of the particle remains positive. This is a geometric description of (orthochron) antimatter after Dirac.

Two more sectors has to be explored. On the third we examine the impact of ($\lambda = -1$; $\mu = -1$) element on the momentum and movement.

($\lambda = -1$) reverses the $\{\zeta^i\}$. According to our geometric definition this new movement corresponds to antimatter, for it takes place in the second sector of space $\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^6 , x, y , z , t \}$.

($\mu = -1$) gives a PT-symmetry, reverses the signs of (x, y , z , t)

But ($\lambda \mu = +1$) keeps the charges unchanged. This is "PT-symmetric antimatter", so that it is a geometric description of antimatter after Feynmann.

But the group belongs to the antichron sector, so that (coadjoint action) the energy and the mass of the particle is reversed.

PT-symmetrical object does not identify completely with Dirac's antimatter, for it changes the sign of the mass. If such particles exist, they can produce full annihilation with positive mass particles.

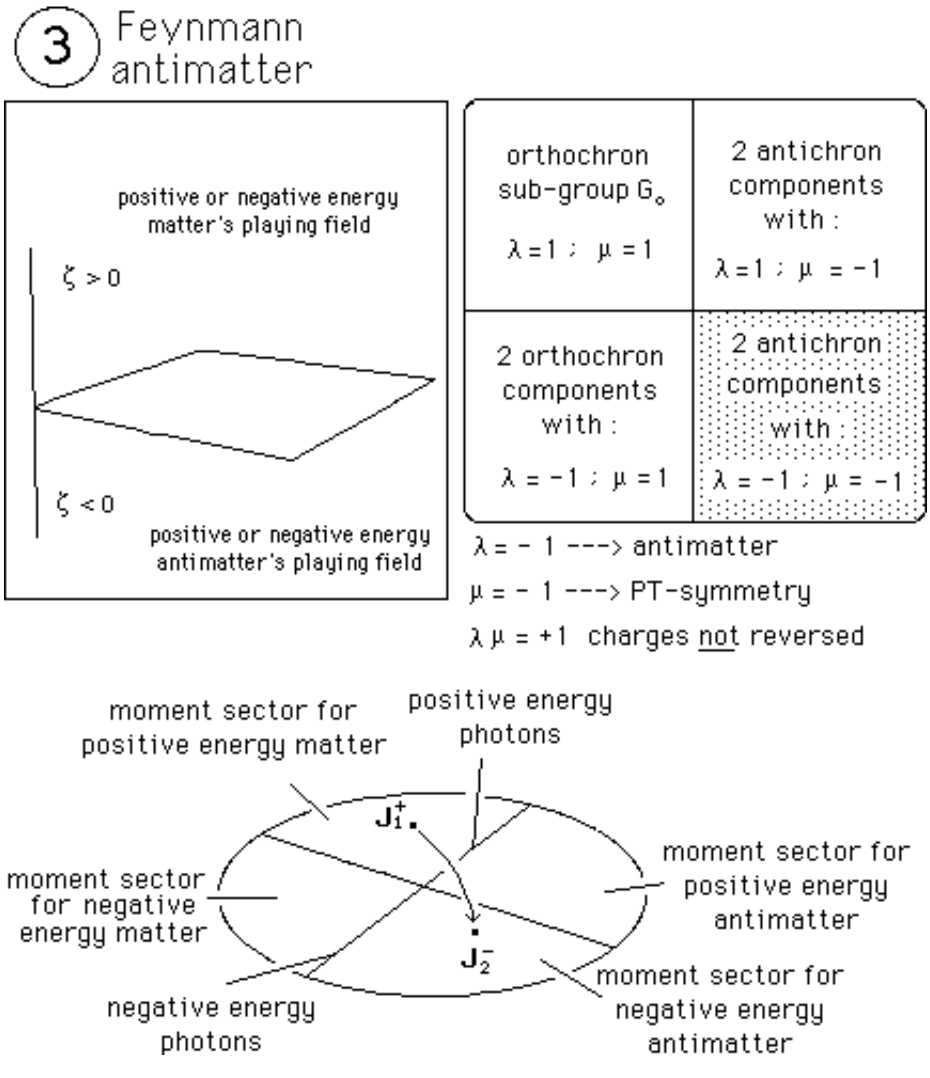

Fig. 6: ($\lambda = -1$; $\mu = -1$) elements transform movement of normal matter into movement of antimatter (ζ-symmetry) **of** PT-symmetrical **object, runing bacward in time. Geometric description of Feynmann's vision of antimatter. Does not identify completely with Dirac's one: negative mass and negative energy.**

The last elements correspond to the sector (λ= 1 ; μ = -1)

(λ = 1) --- > the movement is still in the matter's sector: no ζ-symmetry.

(μ = -1) goes with a PT-symmetry. The particule runs backward in time.

(λ = -1): C-symmetry. The charges are reversed.

This is CPT-symmetrical matter, so that it corresponds to a geometrical interpretation of the so-called "CPT theorem", which asserts that the CPT-symmetric of a particle should be identical to that particle. That's not true. This movement corresponds to an antichron movement. The particle goes backward in time, si that (caodjoint action) its mass and energy become *negative* .

If CPT-symmetrical particle do exist and if they collide normal particle, complete annihilation occurs.

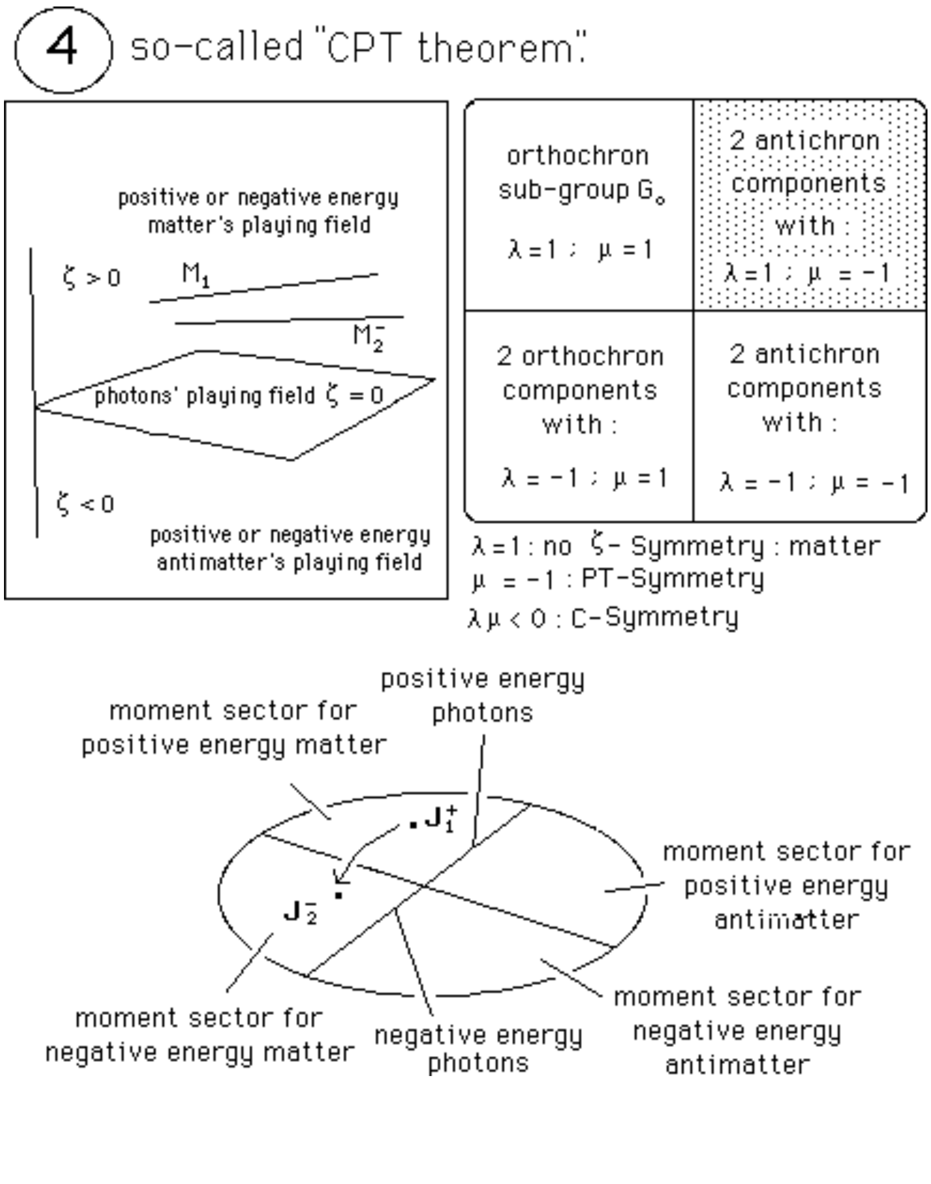

Fig.7: (λ = 1 ; μ = - 1) case. Corresponds to CPT-symmetry.
But the coadjoint action gives negative mass and energy.
The CPT-symmetric of a particle of matter is a particule of matter, but with negative mass.

Now, examine the impact on photons movement and moment. The ζ-symmetry has no impact on it: there is no "antiphoton". As all the charges of the photon are zero a does not change it. It is identical to its antiparticle.

The coadjoint action of orthochron components modifies the movement and the moment of the photon, but keep unchanged its energy. See figure 8.

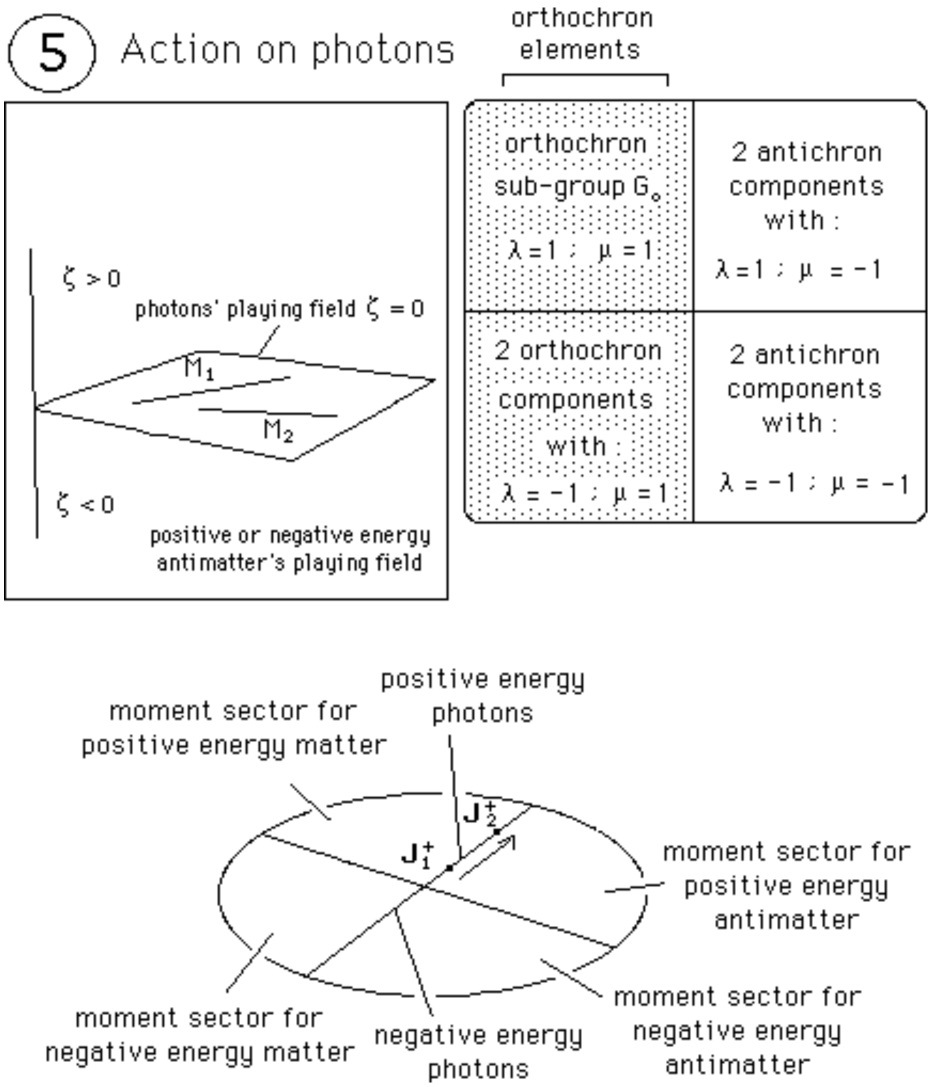

Fig. 8: **Coadjoint action of orthochron elements on photon's movement and moment.**

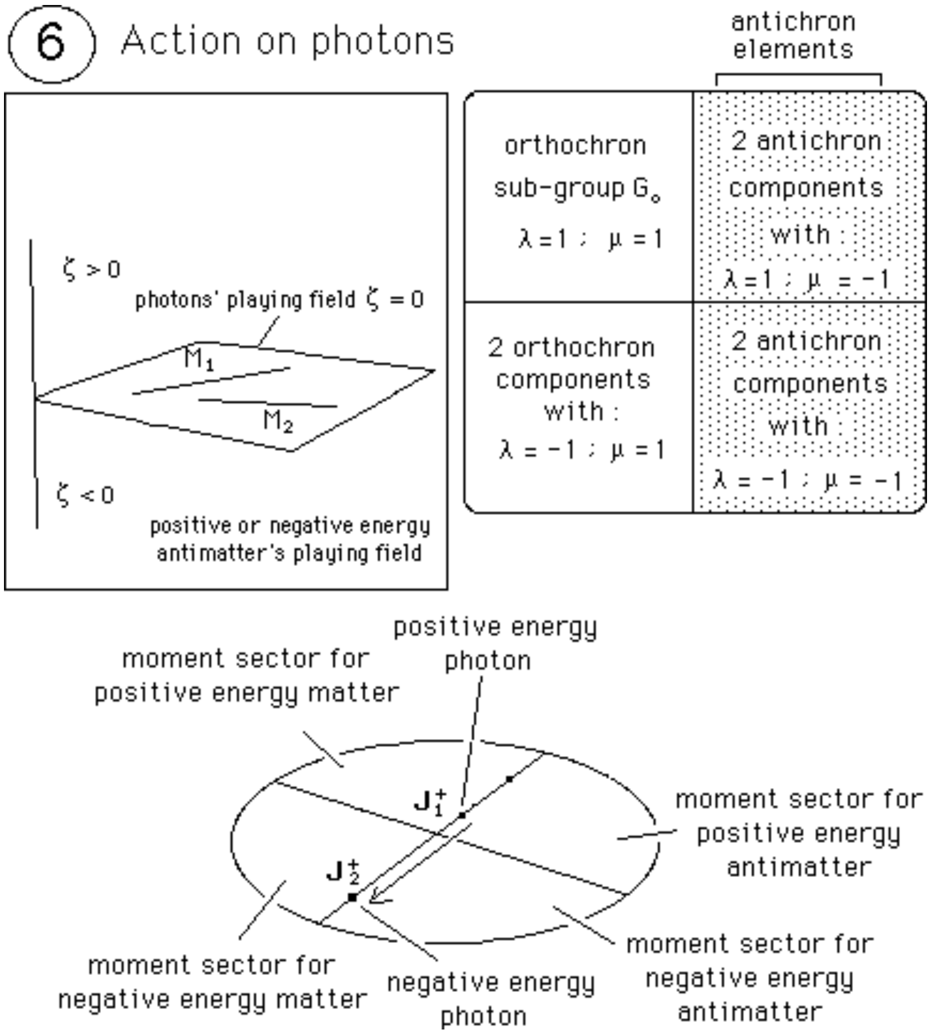

Fig.9: The coadjoint action of antichron elements on photon's movement and moment, reverses the photon energy: it travels backwards in time.

Conclusion

We have extend the group, including antichron elements. We refind the geometric description of Dirac's antimatter. But the analysis of the coadjoint action of antichron elements of the group produces PT-symmetrical and CPT-symmetrical movements.

We find that PT-symmetry goes with matter -----> antimatter transform. It joins Feynmann's idea. The PT-symmetric of a particle of matter is a particle of antimatter. But the coadjoint action of antochron elements reverses the mass and the energy. Then we cannot identify the PT-symmetrical of a particle of matter to its antiparticle, after Dirac's description.The first owns a negative mass and a negative energy.

Similarly the CPT-symmetric of a particle of matter is a partocle of matter, but with a negative mass, for it goes backwards in time.

The problem remains unsolved. As recommanded by J.M.Souriau, we could limit the dynamic group to its orthochron part, but we would'nt have PT and CPT-symmetrical object for symmetries including time-symmetry becomes forbiden.

If we keep the antichron sector we have an universe filled by positive and negative mass particles.

Charybde or Scylla?

In the next paper we shall propose another solution.

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