Geometrization of matter and antimatter through coadjoint action of a group on its momentum space.

4: The Twin group. Geometrical description of Dirac's antimatter. Geometrical interpretations of antimatter after Feynmann and so-called CPT-theorem.

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Abstract:

Starting from the work of reference [3] we modify the model, in order to avoid encounters between positive and negative mass particles. The solution is to build a two-ten-dimensional folds (F, F*) as the quotient of the group by its orthochron sub-group. Then we get two spaces with opposite arrows of time.

We study the impact of the different components of the group on momentum and movement spaces. One shows that the duality matter-antimatter occurs in boths folds, in both universes. This work gives a new insight on antimatter, through geometrical tools. For an example Dirac's antimatter is the antimatter of our own fold. The matter of the second fold is CPT-symmetrical with respect to ours. The PT-symmetrical of a matter particle that belongs to our fold is the antimatter of the other fold. Matter and antimatter particles of our universe own positive mass and energie. Matter and antimatter particles of the second fold own negative mass and energy.

1- Introduction

In a former paper [1] we have introduced a geometrical definition of antimatter, through a ζ-symmetry. Charged mass-points are supposed to move in a ten-dimensional space, with two sectors:

$$\{ \zeta^i > 0 \}\text{: and } \{ \zeta^i < 0 \}$$

The first refers to the movement of matter and the second to the movement of antimatter.

By the way, photons follows the $\{ \zeta^i = 0 \}$ surface.

It looks like Plato's cavern. The play is supposed to take place in a ten dimensional theater and, inside a four dimensional cavern called space time we observe 4d shadows, 4d movements.

In [1] we introduce a group which is an extension of the orthochron part of the Poincaré group. It makes possible to describe the charges of the particles in terms of additional components of their moment. In the paper [2] the group is duplicated, through a ζ-symmetry, which gives a geometric description of Dirac's antimatter. This last owns positive mass and energy.

Next step, paper [3], we decide to include antichron elements in the group. Then we get symmetries including T-symmetry, i.e. PT-symmetry and CPT-symmetry. We find that the PT-symmetrical of a particle of matter is an antiparticle, as suggested by Feynmann. We find that the CPT-symmetric of a particle of matter is a particle of matter too, as asserted by the so-called "CPT-theorem". But, from the coadjoint action of the group on the momentum components we find that these two own negative masses and energies. Then it is no longer possible, as suggested by Feynmann, to identify the PT-symmetry and the C-symmetry. Similarly the CPT-symmetry is different from identity, for it reverses the mass. As pointed out in [3] a solution, suggested by the mathematician J.M.Souriau [4] is to give up the antichron part of the Lorentz and Poincaré dynamical groups. But PT and CPT symmetries dissapear.

In the following we suggest another solution.

2- Building a group acting on a two folds space

From [3] the action of our 16-dimensions group on a tend-dimensional space corresponds to:

(1)

$$\begin{pmatrix} \lambda & 0 & \lambda\phi \\ 0 & \mu L_o & \mu C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta\lambda + \lambda\phi \\ \mu L_o \xi + \mu C \\ 1 \end{pmatrix} \quad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

and the corresponding coadjoint action is:

(2)

$$\chi^{i\,\prime} = \lambda\mu\ \chi^i \quad \text{for } i = 1 \text{ to } 6$$

$$P' = L\,P$$

$$M' = L\,M\,\bar{L} + (\,C\,\bar{P}\,\bar{L} - L\,P\,\bar{C}\,)$$

See computational details in the annex.We build the two-folds space as the quotient of the group by its orhochron sub-group. From (1) a point of space is defined by:

(3)

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , x , y , z , t \}$$

Introduce a fold indix f = ± 1

The a point M of the first fold, called F, is defined by:

(4)

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^5, x , y , z , t , f = +1 \}$$

and the conjugated poit M*, which belongs to the second fold F*, by:

(5)

$$\{ \zeta^1 , \zeta^2 , \zeta^3 , \zeta^4 , \zeta^5 , \zeta^5 , x , y , z , t , f = -1 \}$$

We can write the new action:

(6)

$$\begin{pmatrix} \lambda & 0 & \lambda\phi \\ 0 & \mu L_o & \mu C \\ 0 & 0 & \mu \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ f \end{pmatrix} = \begin{pmatrix} \zeta\lambda + \lambda\phi \\ \mu L_o \xi + \mu C \\ \mu f \end{pmatrix} \quad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \\ f = \pm 1 \end{matrix}$$

The coadjoint action on the momentum space is unchanged. But the interpretation of the results is different. Negative energy movements occurs in another fold. Positive and negative energy particles cannot meet, for the move in distinct ten-dimensional twin spaces.

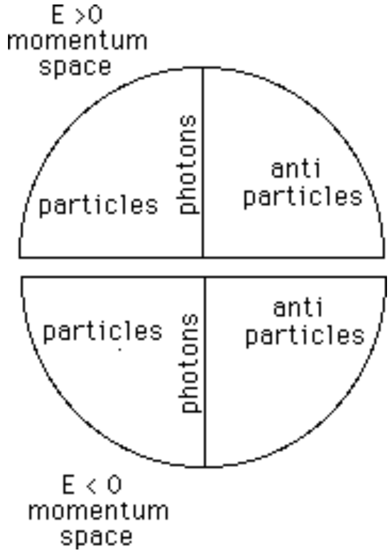

Fig.1: Two sectors momentum space.

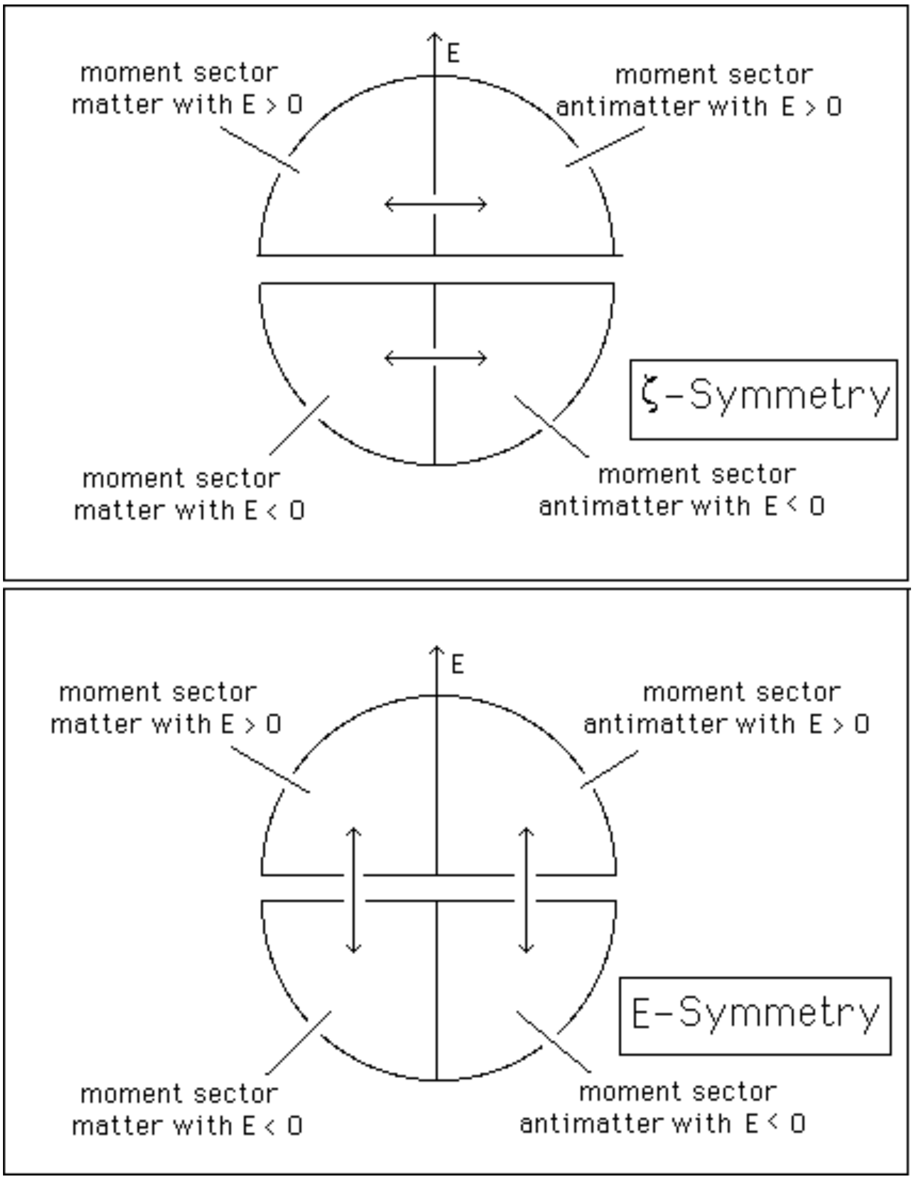

Fig.2: Associated symmetries

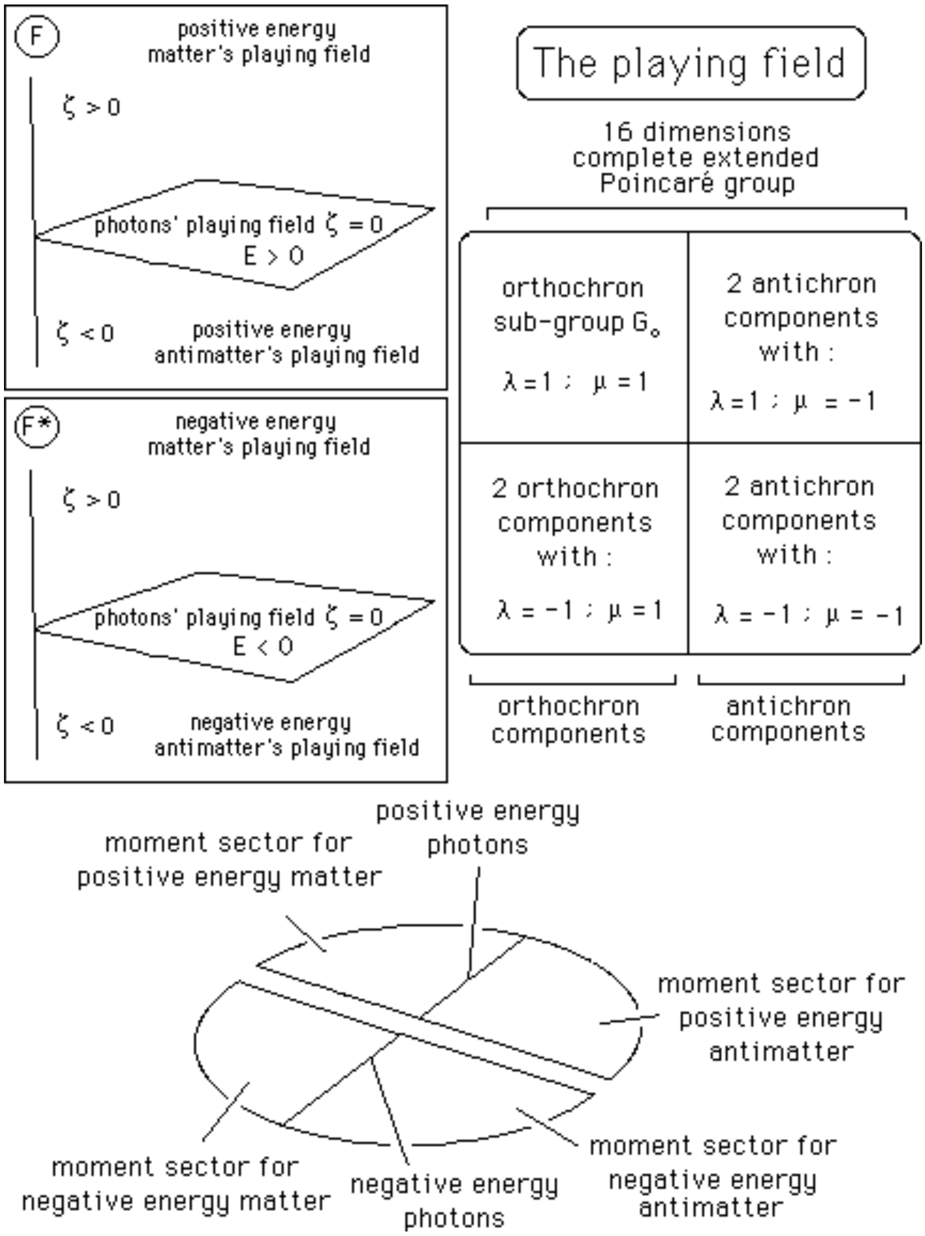

Fig.3: The playing field: a two folds (F **and** F*) **space,**
associated to a two sectors momentum space
(E > 0 **and** E < 0).

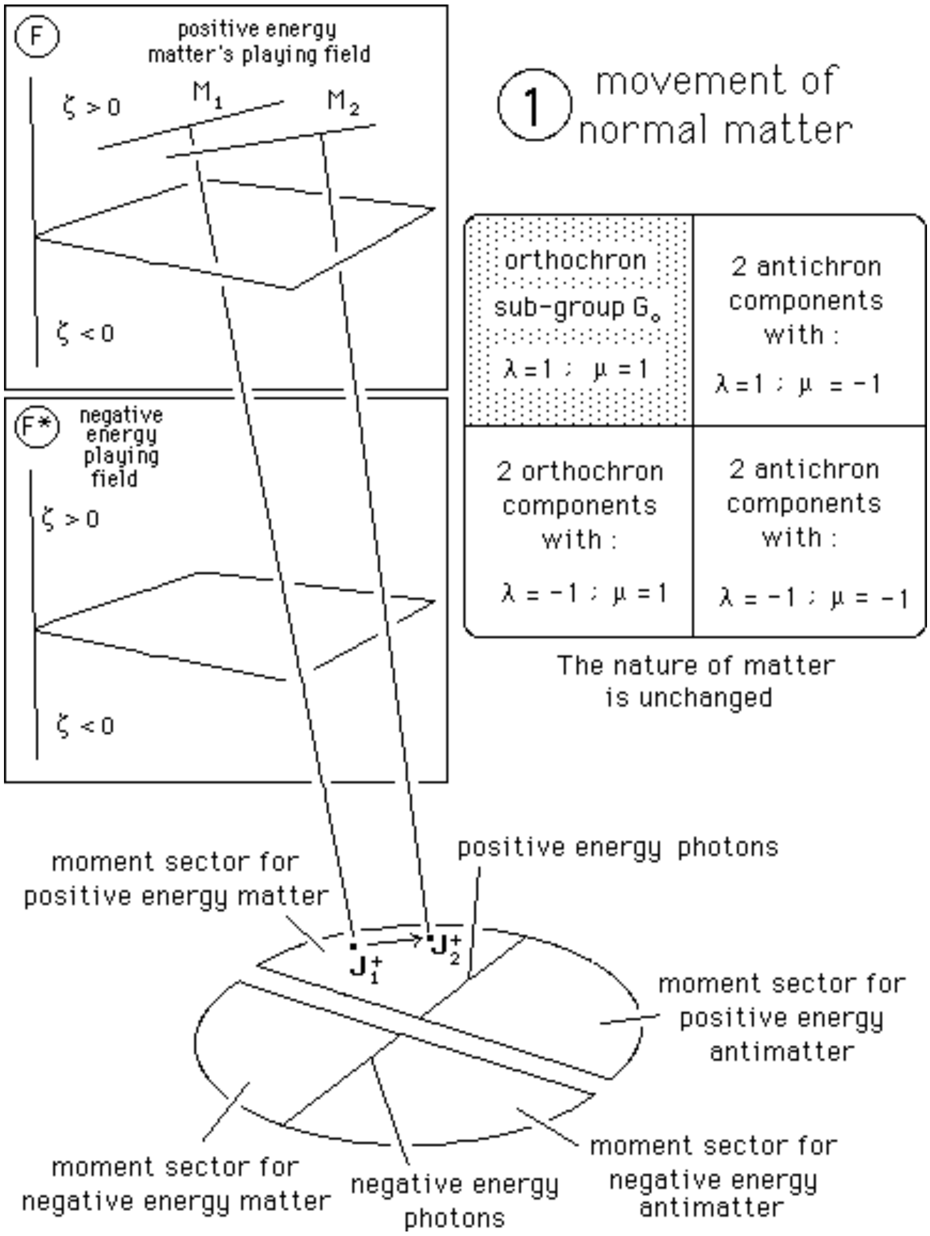

Fig.4: Movements of ordinary matter.
Action of orthochron elements of the group, with λ = 1.
Charges unchanged.

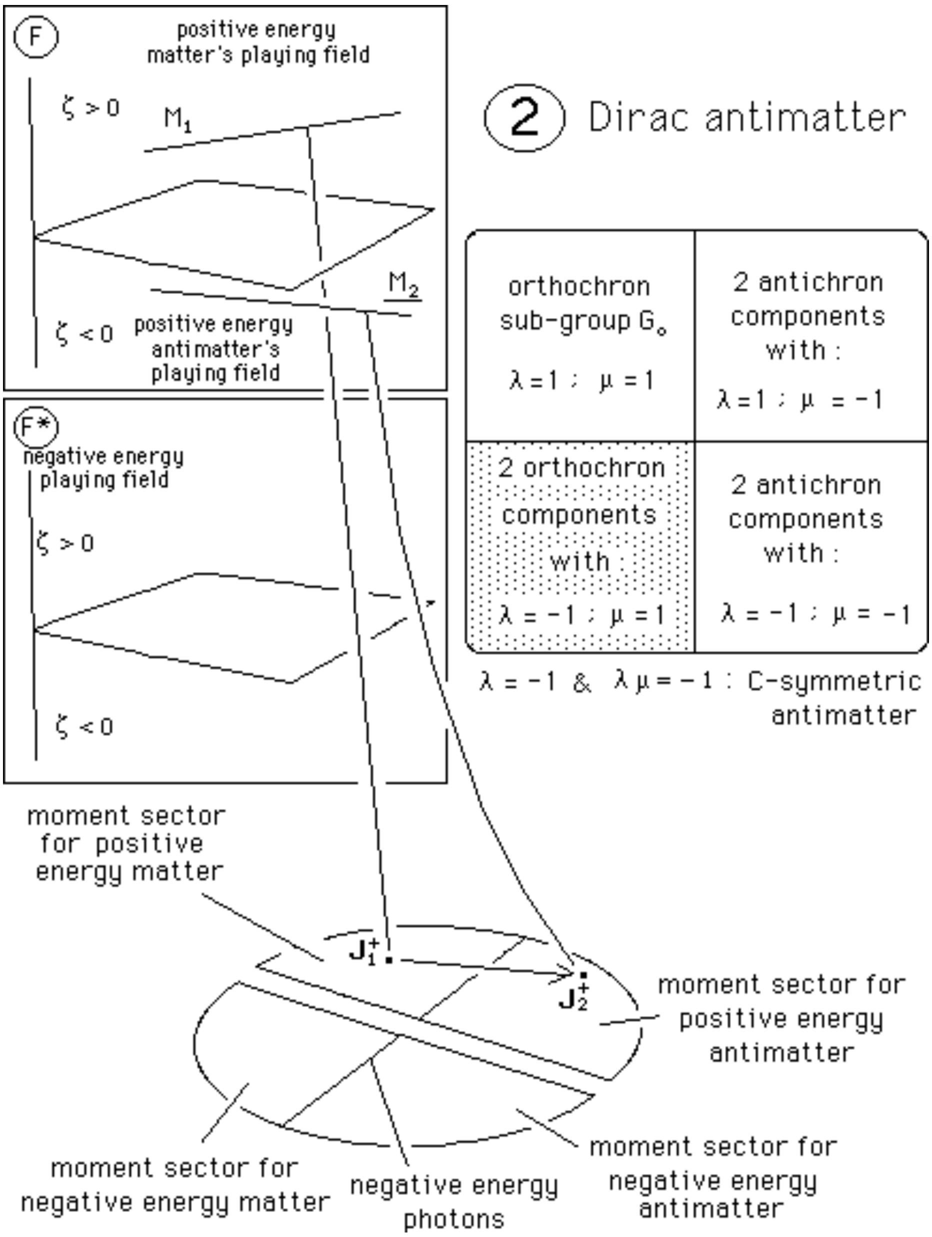

Fig. 5: Coadjoint action of a (λ = -1 ; μ = 1) element of the group on the momentum associated to the movement of normal matter: the new movement corresponds to Dirac's antimatter.

On the figure 5 the line M_1 figures the movement of normal, orthochron matter. We figures straight lines because ou group does not take account of force field, like gravitational or electromagnétic field. It only runs the behaviour of lonely particles, charged mass-points.

We choose an element in the grey area, corresponding to a ($\lambda = -1$; $\mu = 1$) matrix. The ($\lambda = -1$) value changes the signs of all the ζ^i. They become negative. The new path is in the second sector, corresponding to antimatter. As $\lambda\mu = -1$ the charges are reversed. But as time is not reversed, the energy and the mass of the particle remains positive. This is a geometric description of (orthochron) antimatter after Dirac.

Two more sectors has to be explored. On the third we examine the impact of ($\lambda = -1$; $\mu = -1$) element on the momentum and movement.

($\lambda = -1$) reverses the {ζ i}. According to our geometric definition this new movement corresponds to antimatter, for it takes place in the second sector of space { $\zeta^1, \zeta^2, \zeta^3, \zeta^4, \zeta^5, \zeta^6$, x, y , z , t }.

($\mu = -1$) gives a PT-symmetry, reverses the signs of (x, y , z , t)

But ($\lambda\mu = +1$) keeps the charges unchanged. This is "PT-symmetric antimatter", so that it is a geometric description of antimatter after Feynmann.

The movement takes place in the second space sector, in the fold F*.

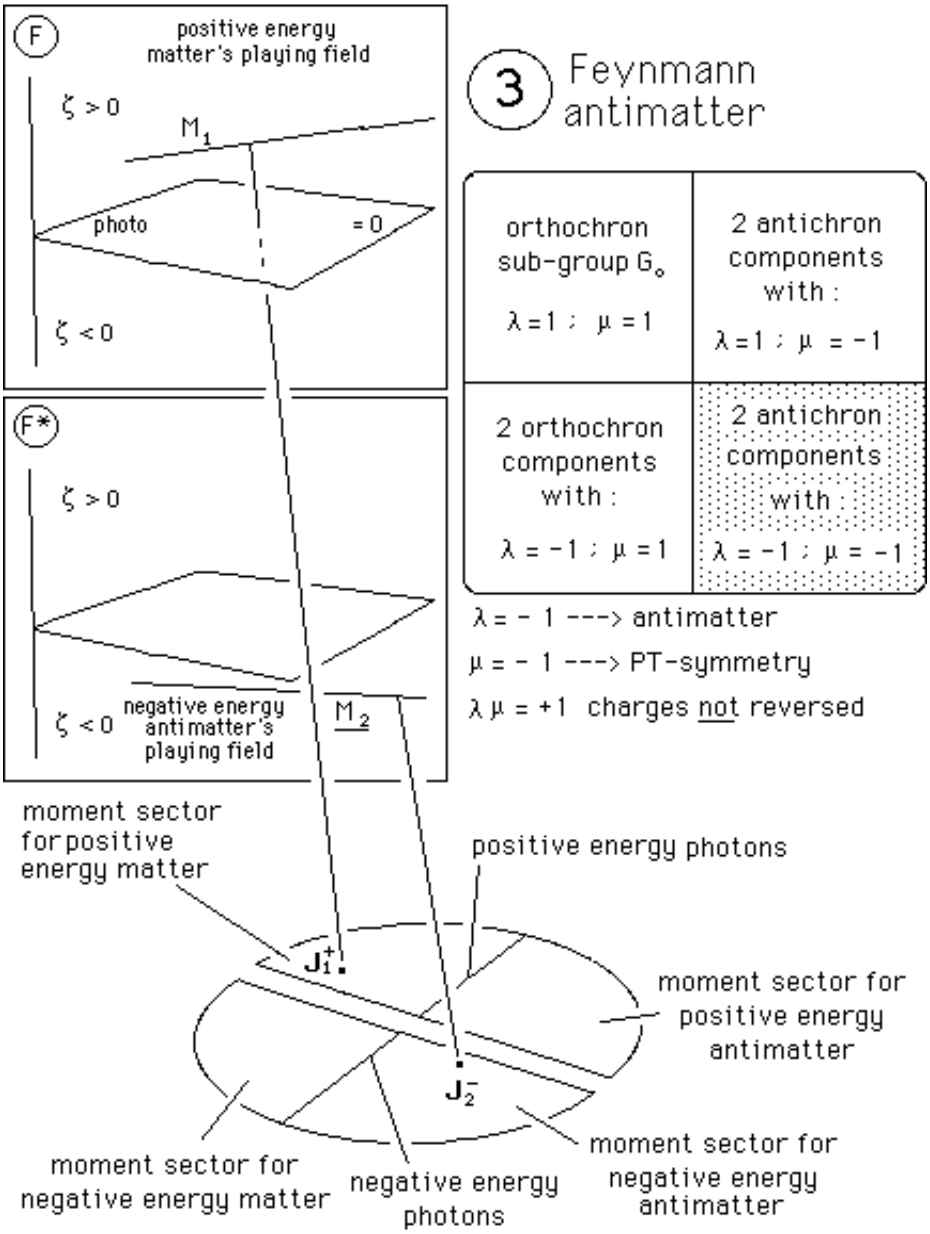

Fig.6: (λ= -1 ; μ = -1) **elements transform movement of normal matter**

into movement of antimatter (ζ-symmetry) **of** PT-symmetrical **object, runing bacward in time.**
Geometric description of Feynmann's vision of antimatter.
Does not identify vompletely with Dirac's one: negative mass and negative energy.

The last elements correspond to the sector (λ= 1 ; μ = -1)

(λ = 1) --- > the movement is still in the matter's sector: no ζ-symmetry.

(μ = -1) goes with a PT-symmetry. The particle runs backward in time.

(λ = -1): C-symmetry. The charges are reversed.

This is CPT-symmetrical matter, so that it corresponds to a geometrical interpretation of the so-called "CPT theorem", which asserts that the CPT-symmetric of a particle should be identical to that particle. That's not true. This movement corresponds to an antichron movement. The particle goes backward in time, si that (coadjoint action) its mass and energy become *negative*.

The movement of a particle which is the CPT-symmetrical of a normal particle takes place in the fold F*.

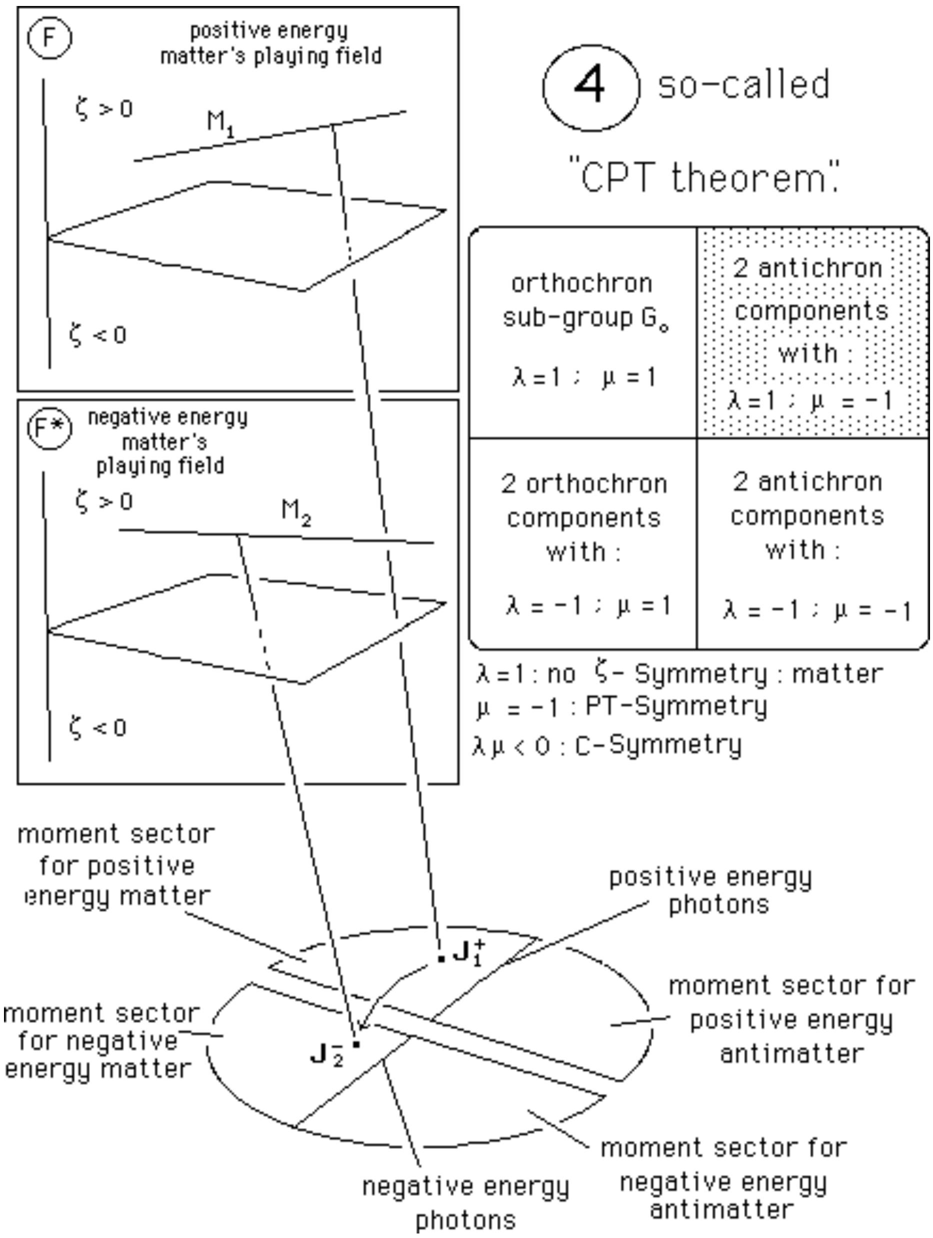

Fig.7: (λ = 1 ; μ = - 1) case. Corresponds to CPT-symmetry.

But the coadjoint action gives negative mass and energy.
The CPT-symmetric of a particle of matter is a particle of matter, but with negative mass.

Now, examine the impact on photons movement and moment. The ζ-symmetry has no impact on it: there is no "antiphoton". As all the charges of the photon are zero a does not change it. It is identical to its antiparticle.

The coadjoint action of orthochron components modifies the movement and the moment of the photon, but keep unchanged its energy. See figure 8.

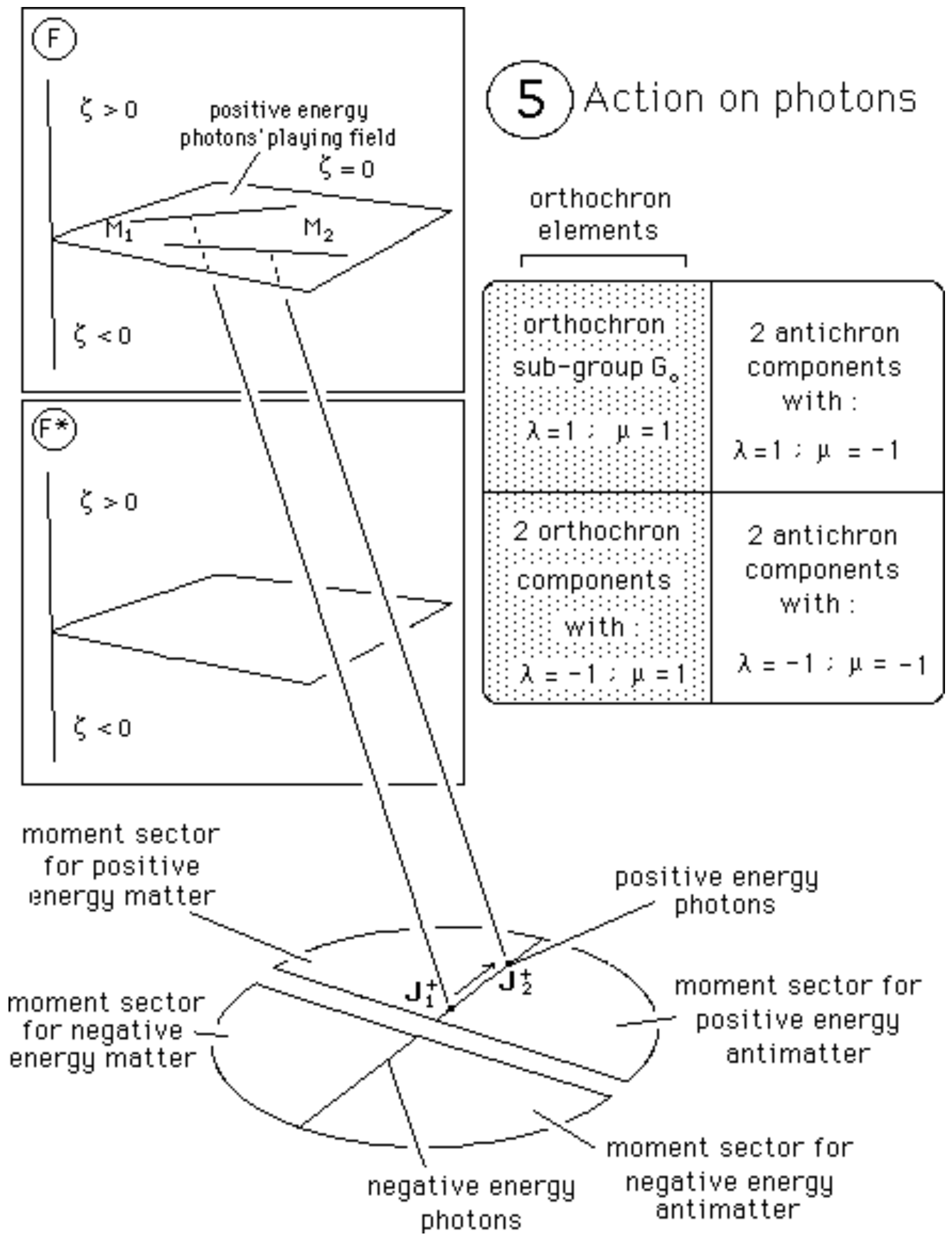

**Fig. 8: Coadjoint action of orthochron elements
on photon's movement and moment.**

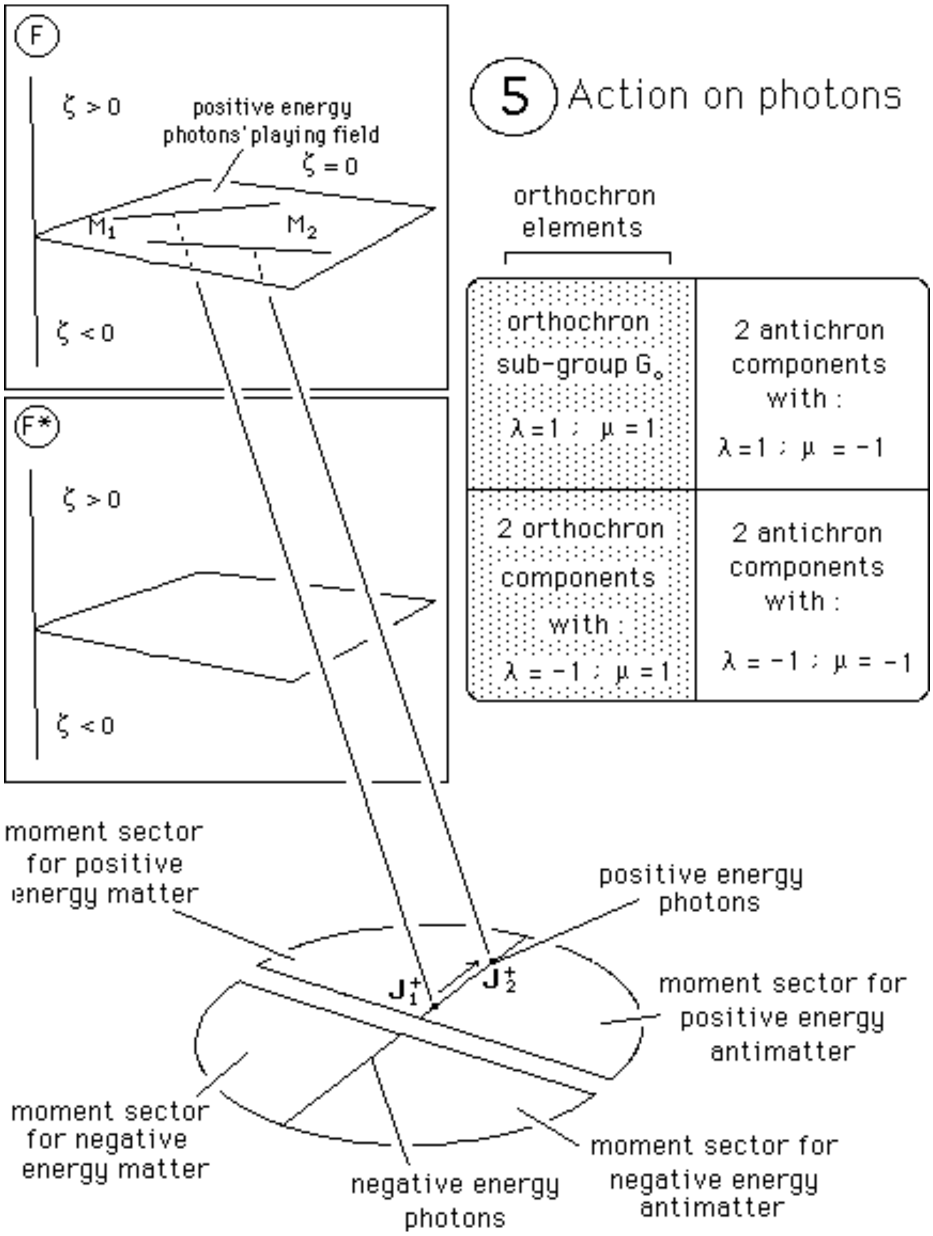

Fig.9: The coadjoint action of antichron elements on photon's movement and moment, reverses the photon energy: it travels backwards in time.

The antichron elements of the group acts on the momentum through coadjoint action. Such elements transform the normal photon, travelling forward in time un the $\zeta = 0$ plan of the fold F into a movement in the corresponding plane of the fold F*. This photon travels backwards in time. Its energy is negative

.3- About the contents of the two folds

They can be summarized in the following tables.

	proton	electron	neutron	photon	neutrinos		
					ν_e	ν_μ	ν_τ
electric charge	1	-1	0	0	0	0	0
baryonic charge	1	0	1	0	0	0	0
leptonic charge	0	1	0	0	1	0	0
muonic charge	0	0	0	0	0	1	0
tauonic charge	0	0	0	0	0	0	1
magnetic gyrofactor	ϖ_p	ϖ_e	ϖ_n	0	?	?	?
mass	m_p	m_e	m_n	0	0	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$		

particules

	anti proton	anti electron	anti neutron	photon	anti neutrinos		
					$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$
electric charge	-1	1	0	0	0	0	0
baryonic charge	-1	0	-1	0	0	0	0
leptonic charge	0	-1	0	0	-1	0	0
muonic charge	0	0	0	0	0	-1	0
tauonic charge	0	0	0	0	0	0	-1
magnetic gyrofactor	$-\varpi_p$	$-\varpi_e$	$-\varpi_n$	0	-?	-?	-?
mass	m_p	m_e	m_n	0	0	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$		

antiparticules

Fig.10: Classical particles-antiparticles zoo.

	ghost proton	ghost electron	ghost neutron	ghost photon	ghost neutrinos ν_e	ν_μ	ν_τ
electric charge	-1	1	0	0	0	0	0
baryonic charge	-1	0	1	0	0	0	0
leptonic charge	0	-1	0	0	-1	0	0
muonic charge	0	0	0	0	0	-1	0
tauonic charge	0	0	0	0	0	0	-1
magnetic gyrofactor	$-\varpi_p$	$-\varpi_e$	$-\varpi_n$	0			
mass	$-m_p$	$-m_e$	$-m_n$	0	0	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$		

particules

	ghost anti proton	ghost anti electron	ghost anti neutron	ghost photon	ghost anti neutrinos $\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$
electric charge	1	-1	0	0	0	0	0
baryonic charge	1	0	-1	0	0	0	0
leptonic charge	0	1	0	0	1	0	0
muonic charge	0	0	0	0	0	1	0
tauonic charge	0	0	0	0	0	0	1
magnetic gyrofactor	ϖ_p	ϖ_e	ϖ_n	0			
mass	$-m_p$	$-m_e$	$-m_n$	0	0	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$		

antiparticules

Fig.11: Ghost particles-ghost-antiparticles zoo.

$$\mathbf{G} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This last matrix is linked to the metric:

(7)

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = \overline{d\boldsymbol{\xi}}\ \mathbf{G}\ d\boldsymbol{\xi}$$

with

(8)

$$\mathbf{G} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This last matrix is linked to the metric :

(9)

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = \overline{d\boldsymbol{\xi}}\ \mathbf{G}\ d\boldsymbol{\xi}$$

So that the two folds have same signature. If they are described as Minkiwski space times, their metrics are identical. But their arrows of time are opposite. But their arrows of time are opposite.

If one wants to describe the two folds, the two universes, one have to choose his own arrow of time and space orientation.

It is clear that the duality matter-antimatter occurs in both folds. If we call the second fold "twin fols" (Sakharov) or "shadow fold" (Green, Schwarz and Salam) or "ghost fold" (the author's choice) the arrow of time in this second fold is opposite (T-symmetry), as predicted by A. Sakharov, and space structures are enantiomorphic (P-symmetry).

In the second fold the matter is CPT-symmetric with respect to ours. Whence, in that fold, a proton owns a negative charge and an electron a positive charge. Conversely, an anti-electron of that fold, PT-symmetric with respect to ours, owns a negative charge, whence an antiproton of the second fold has a positive charge. To sum up, the second fold is CPT symmetric with respect to ours. As suggested by Andrei Sakharov, we can expect that the violation of the parity principle could be reversed in that fold. If the absence of antimatter, in our fold, is a direct consequence of the violation of the parity principle, it is possible that such dissymmetry would be reversed in the other fold.

4- Interacting folds

All our work in astrophysics and cosmology comes from a system of two coupled field equations:

(10)

$$\mathbf{S} = \chi\ (\mathbf{T} - \mathbf{T}^*)$$

(11)

$$\mathbf{S}^* = \chi\ (\mathbf{T}^* - \mathbf{T})$$

The two minus signs were introduced as an a priori hypothesis. At the end of this work, based on group theory, the explanation arises. The two folds *must* have opposite arrows of time and *must* be enantiomorphic in order to fit constrainsts coming from the group structure.

So that the other matter, located in the other fold, for an orbserver located in the first, bahaves as if it own a negative mass, which comes from the coadjoint action and the T-symmetry.

Conclusion

Starting from the work of reference [3] we have modified the model, in order to avoir encounters between positive and negative mass particles. The solution was to build a two-ten-dimensional folds (F,F*) as the quotient of the group by its orthochron sub-group.

Then we get two spaces with opposite arrows of time.

We study the impact of the different components of the group on momentum and movement spaces. One shows that the duality matter-antimatter occurs in boths folds, in both universes. This work gives a new insight on antimatter, through geometrical tools.

For an example Dirac's antimatter is the antimatter of our own fold.
The matter of the second fold is CPT-symmetrical with respect to ours.
The PT-symmetrical of a matter particle that belongs to our fold is the antimatter of the other fold.

Matter and antimatter particles of our universe own positive mass and energy.
Matter and antimatter particles of the second fold own negative mass and energy.

A N N E X

Extension of the group

Consider a group Γ composed by matrixes:
(1)

$$\mathbf{\Gamma} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mu \end{pmatrix}$$

A is a square matrix. **B** is a column matric and **O** a ligne matrix, composed by null terms. Consider the extension:
(2)

$$\mathbf{g}_3 = \begin{pmatrix} \lambda & \lambda\boldsymbol{\vartheta} \\ \mathbf{0} & \mathbf{\Gamma} \end{pmatrix}$$

where $\boldsymbol{\vartheta}$ is the following ligne sub-matrix:
(3)

$$\boldsymbol{\vartheta} = \begin{pmatrix} \mathbf{0} & \vartheta \end{pmatrix}$$

ϑ being a scalar. Check that (2) is a group:
(4)

$$\begin{pmatrix} \lambda_1 & \lambda_1\boldsymbol{\vartheta}_1 \\ \mathbf{0} & \mathbf{\Gamma}_1 \end{pmatrix} \times \begin{pmatrix} \lambda_2 & \lambda_2\boldsymbol{\vartheta}_2 \\ \mathbf{0} & \mathbf{\Gamma}_2 \end{pmatrix} = \begin{pmatrix} \lambda_1\lambda_2 & \begin{matrix} \lambda_1\lambda_2\boldsymbol{\vartheta}_2 \\ + \lambda_1\lambda_2\boldsymbol{\vartheta}_1\mathbf{\Gamma}_2 \end{matrix} \\ \mathbf{0} & \mathbf{\Gamma}_1\mathbf{\Gamma}_2 \end{pmatrix} = \begin{pmatrix} \nu_3 & \begin{matrix} \lambda_3\boldsymbol{\vartheta}_2 \\ + \lambda_3\boldsymbol{\vartheta}_1\mathbf{\Gamma}_2 \end{matrix} \\ \mathbf{0} & \mathbf{\Gamma}_3 \end{pmatrix}$$

(5)

$$\boldsymbol{\vartheta}_1\mathbf{\Gamma}_2 = \begin{pmatrix} \mathbf{0} & \vartheta_1 \end{pmatrix} \times \begin{pmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{0} & \mu \end{pmatrix} = \mu \begin{pmatrix} \mathbf{0} & \vartheta_1 \end{pmatrix} = \mu\,\boldsymbol{\vartheta}_1$$

(6)

$$\lambda_3\boldsymbol{\vartheta}_2 + \lambda_3\boldsymbol{\vartheta}_1\mathbf{\Gamma}_2 = \lambda_3(\boldsymbol{\vartheta}_2 + \mu\boldsymbol{\vartheta}_1) = \lambda_3\boldsymbol{\vartheta}_3$$

Then:
(7)

$$\begin{pmatrix} \lambda_1 & \lambda_1\boldsymbol{\vartheta}_1 \\ \mathbf{0} & \mathbf{\Gamma}_1 \end{pmatrix} \times \begin{pmatrix} \lambda_2 & \lambda_2\boldsymbol{\vartheta}_2 \\ \mathbf{0} & \mathbf{\Gamma}_2 \end{pmatrix} = \begin{pmatrix} \lambda_3 & \lambda_3\boldsymbol{\vartheta}_3 \\ \mathbf{0} & \mathbf{\Gamma}_3 \end{pmatrix}$$

The inverse matrix is:
(8)

$$\mathbf{g}_3^{-1} = \begin{pmatrix} \lambda & -\lambda\mu\boldsymbol{\vartheta}\boldsymbol{\Gamma}^{-1} \\ \mathbf{0} & \boldsymbol{\Gamma}^{-1} \end{pmatrix}$$

The element of the Lie algebra is:
(9)

$$\delta\mathbf{g}_3 = \begin{pmatrix} 0 & \lambda\,\delta\boldsymbol{\vartheta} \\ \mathbf{0} & \delta\boldsymbol{\Gamma} \end{pmatrix} = \begin{pmatrix} 0 & \lambda\,\delta\boldsymbol{\vartheta} \\ \mathbf{0} & \boldsymbol{\gamma} \end{pmatrix}$$

Calculate the action of **g**3-1 on the element of the Lie algebra element δ**g3**
(10)

$$\mathbf{g}_3^{-1} \times \delta\mathbf{g}_3 \times \mathbf{g}_3 =$$

$$\begin{pmatrix} \lambda & -\lambda\mu\boldsymbol{\vartheta}\boldsymbol{\Gamma}^{-1} \\ \mathbf{0} & \boldsymbol{\Gamma}^{-1} \end{pmatrix} \times \begin{pmatrix} 0 & \lambda\,\delta\boldsymbol{\vartheta} \\ \mathbf{0} & \boldsymbol{\gamma} \end{pmatrix} \times \begin{pmatrix} \lambda & \lambda\boldsymbol{\vartheta} \\ \mathbf{0} & \boldsymbol{\Gamma} \end{pmatrix} = \begin{pmatrix} 0 & \begin{matrix}\delta\boldsymbol{\vartheta}\boldsymbol{\Gamma} \\ -\lambda\mu\boldsymbol{\vartheta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}\boldsymbol{\Gamma}\end{matrix} \\ \mathbf{0} & \boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}\boldsymbol{\Gamma} \end{pmatrix}$$

(11)

$$\delta\boldsymbol{\vartheta}\boldsymbol{\Gamma} = \mu\,\delta\boldsymbol{\vartheta} \qquad\qquad \boldsymbol{\vartheta}\boldsymbol{\Gamma}^{-1} = \mu\,\boldsymbol{\vartheta}$$

$\boldsymbol{\gamma}$ is a matrix :
(12)

$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\alpha} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

so that:
(13)

$$-\lambda\mu\boldsymbol{\vartheta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}\boldsymbol{\Gamma} = \mathbf{0}$$

The identification:
(14)

$$\delta\mathbf{g}_3' = \begin{pmatrix} 0 & \lambda\,\delta\boldsymbol{\vartheta}' \\ \mathbf{0} & \boldsymbol{\gamma}' \end{pmatrix} = \mathbf{g}_3^{-1} \times \delta\mathbf{g}_3 \times \mathbf{g}_3 = \begin{pmatrix} 0 & \mu\,\delta\boldsymbol{\vartheta} \\ \mathbf{0} & \boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}\boldsymbol{\Gamma} \end{pmatrix}$$

gives:
(15)

$$\delta\vartheta' = \lambda\,\mu\,\vartheta$$

(16)

$$\mathbf{\gamma}' = \mathbf{\Gamma}^{-1}\mathbf{\gamma}\mathbf{\Gamma}$$

The equation (16) is the action on the Lie algebra element, corresponding to the group Γ. The coadjoint action is the dual of this action and is based on the invariance of a scalar. Call S this scalar from which one computes the coadjoint action of the group Γ on its momentum. We compute the coadjoint action of the group $\mathbf{g}_3$ from the scalar:
(17)

$$\chi\,\delta\vartheta + S$$

Then the coadjoint action of the group $\mathbf{g}_3$ on its momentum is:
(18)

$$\chi' = \lambda\mu\,\chi$$

$$\left[\begin{array}{l}\text{coadjoint action}\\ \text{of the group } \mathbf{\Gamma} \text{ on}\\ \text{its momentum}\end{array}\right.$$

The moment of the group $\mathbf{g}_3$ is:
(19)

$$\mathbf{J} = \{\ \chi\ ,\ \text{momentum of the group } \Gamma\ \}$$

The extension of the group adds a component χ to the moment, which obeys (20). In particular, if $\mathbf{\Gamma} = \mathbf{g}_2$, i.e:
(20)

$$\mathbf{g}_3 \times \begin{pmatrix} \zeta \\ \boldsymbol{\xi} \\ \sigma \end{pmatrix} = \begin{pmatrix} \lambda & \mathbf{0} & \lambda\phi \\ \mathbf{0} & \mu\mathbf{L} & \mu\mathbf{C} \\ 0 & \mathbf{0} & \mu \end{pmatrix} \times \begin{pmatrix} \zeta \\ \boldsymbol{\xi} \\ \sigma \end{pmatrix} \qquad \begin{array}{l} \mathbf{L} \text{ is is the neutral component} \\ \text{of Lorentz group} \\ \lambda = \pm\ 1 \\ \mu = \pm\ 1 \\ \sigma = \pm\ 1 \end{array}$$

its coadjoint action is:
(21)

$$\chi' = \lambda\,\mu\,\chi$$

(22)

$$\mathbf{M}' = \mathbf{L}\,\mathbf{M}\,\overline{\mathbf{L}} + (\ \mathbf{C}\,\overline{\mathbf{P}}\,\overline{\mathbf{L}} - \mathbf{L}\,\mathbf{P}\,\overline{\mathbf{C}}\ )$$

(23)

$$\mathbf{P}' = \mathbf{L}\,\mathbf{P}$$

The equations (22) + (23) identifies to the coadjoint action of the Poincaré group when **L** is the neutral component of the Lorentz group.

We know that we can put the momentum $\mathbf{J}_p$ of the Poincaré group $\mathbf{g}_p$ into an antisymmetric matrix :
(24)

$$\mathbf{J}_p = \begin{pmatrix} \mathbf{M} & -\mathbf{P} \\ \bar{\mathbf{P}} & 1 \end{pmatrix}$$

Then its action on this momentum is:
(25)

$$\mathbf{J}'_p = \mathbf{g}_p \, \mathbf{J}_p \, \bar{\mathbf{g}}_p$$

Then we can write:
(26)

$$\mathbf{J} = \{ \chi , \mathbf{J}_p \}$$

and:
(27)

$$\chi' = \lambda \mu \chi$$

$$\mathbf{J}'_p = \mathbf{g}_p \, \mathbf{J}_p \, \bar{\mathbf{g}}_p$$

The Dimension of the Poincaré group is ten. The dimension of this extended group is eleven, due to adding the new variable ϕ . ($\lambda = \pm 1$) and ($\mu = \pm 1$) are not new dimensions of the group.

This method can be extended as many times as one wants. Consider the following matrix:
(28)

$$\begin{pmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \lambda_1\phi_1 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 & \mathbf{0} & \lambda_2\phi_2 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 & \mathbf{0} & \lambda_3\phi_3 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 & \mathbf{0} & \lambda_4\phi_4 \\ 0 & 0 & 0 & 0 & \lambda_5 & 0 & \mathbf{0} & \lambda_5\phi_5 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6 & \mathbf{0} & \lambda_6\phi_6 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mu\mathbf{L} & \mu\mathbf{C} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & \mu \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \\ \boldsymbol{\xi} \\ \sigma \end{pmatrix} = \begin{pmatrix} \lambda_1\zeta_1 + \sigma\lambda_1\phi_1 \\ \lambda_2\zeta_2 + \sigma\lambda_2\phi_2 \\ \lambda_3\zeta_3 + \sigma\lambda_3\phi_3 \\ \lambda_4\zeta_4 + \quad \lambda_4\phi_4 \\ \lambda_5\zeta_5 + \sigma\lambda_5\phi_5 \\ \lambda_6\zeta_6 + \sigma\lambda_6\phi_6 \\ \mu\mathbf{L}\,\boldsymbol{\xi} + \sigma\mu\mathbf{C} \\ \mu\sigma \end{pmatrix}$$

The Poincaré group depends owns ten dimensions. The set

$$(\phi^1 , \phi^2 , \phi^3 , \phi^4 , \phi^5 , \phi^5)$$

adds si more dimensions. The scalar ($\lambda_1 , \lambda_2 , \lambda_3 , \lambda_4 , \lambda_5 , \lambda_5$) are fixed and do not correspond to new dimensions.

The coadjoint action of the group on its momentum
(29)

$$\mathbf{J} = \{ \chi^1 , \chi^2 , \chi^3 , \chi^4 , \chi^5 , \chi^6 , \mathbf{J}_p \}$$

is:
(30)

$$\chi'^i = \lambda_i \, \mu \, \chi^i \text{ with } i = \{ 1 , 2 , 3 , 4 , 5 , 6 \}$$

$$\mathbf{J}'_p = \mathbf{g}_p \, \mathbf{J}_p \, \bar{\mathbf{g}}_p$$

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