# Cosmological bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe 

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#### Abstract

An extension of a previously published model of a bimetric Universe is presented, where the speeds of light associated to positive and negative mass species are different. As shown earlier, the asymmetry of the model explains the acceleration of the positive species, while the negative one slows down. Asymmetry affects scale factors linked to lengths, times and speeds of light; so that if a mass inversion of a craft can be achieved, then interstellar travels would become non-impossible at a velocity less than the speed of light of the negative sector, and possibly much higher than that of the positive sector.


Keywords: Janus cosmological model; dark energy; bimetric model; negative mass; interstellar travel.

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## 1. Introduction

Only a few decades ago there were many scientists who doubted that intelligent life could exist in the Universe elsewhere than on Earth. Some of them doubted about the possibility of life being present in other planets on the sole grounds of probabilities. This position can be summed up by the famous Drake formula which is expressed as a product of multiple probabilities. And skeptics say: "Suppose that one of those factors is zero, and then the probability would fall to nothing". Even further, there were scientists who wondered if a planetary system around a star could ever be a common thing in the Universe.

Observations in the last decade have completely upset this view. But in fact we are witnessing a rearguard action aimed at challenging the existence of planets with intelligent life with the same level of organization than that found on Earth.

When the first exoplanet was discovered in 1995 by Mayor and Queloz, 40 lightyears away from Earth, it was evidently a giant planet unsuitable for hosting life. Skeptics asked whether planetary systems with telluric planets could ever exist. They thought that even if telluric planets were to exist in extrasolar systems, it was unlikely they might bear surface conditions conducive to the emergence of life. In the line of arguments against this eventuality was the requirement of the existence of a satellite around the planet, allowing the latter to stabilize its rotation movement. In fact, proponents of the theory of deterministic chaos had shown, using computer simulations, that the axis of rotation of a lonely planet would be unstable and topple completely, making the emergence of life very problematic. But these simulations do consider the Earth as a rigid sphere, which she is far from being at the scale of the phenomena considered.

Returning to the so now called exoplanets, their number has rapidly grown, and it will still grow with increasing acceleration, as it happens for any existing object in the cosmos, as a function of the progress of detection systems. Telluric planets, including those lying "within the band of water", were soon discovered next to the stars they orbit.

All the prejudices on which one could rely to defy the existence of life on a planet collapsed one after another. And it became not as evident as before the fact that not only Sun-like stars may be conducive to the emergence of life in its associated planetary system.

Nowadays the view of a scientist like Drake has been totally reversed, and it is likely that his formula will soon be forgotten. According to Ref. 41, the NASA, based on the fact that 1500 planets were detected within a radius of 50 light-years from the Earth, used this figure to estimate 100 billions of the minimum number of planets in the Milky Way. This number should be weighted by a coefficient of habitability on which we should still come to an agreement. But that makes us to think that the probability of the Earth not being the only inhabited planet in our galaxy cannot be zero. An estimation of this number being made, if it is multiplied by the number of galaxies in the observable Universe, a considerable figure is anyway obtained.

One can sum this up in one sentence: "The probability that there does not exist, not only in the Universe but in our own galaxy, a planet other than ours, which is carrier of organized and intelligent life, must be considered as zero". So it was compelling to recognize that the belief of the skeptics was nothing more than the manifestation of a ridiculous geocentrism.

You can still find however scientists that doubt that the level of organization of life, such as it exists on Earth, can be found elsewhere in other planets. Some of them say: "Life elsewhere, yes, but only in bacterial form".

Humans are indeed facing an unprecedented awareness in Earth's history, with the obvious impacts on all their religious beliefs. Some precursors, such as Andrei Sakharov, have expressed a very advanced view on this, which is summarized in the last lines of his acceptance speech of the Nobel Peace Prize in 1975. Given
the stature of this scientist and the outstanding quality of his contributions in various scientific fields, ranging from hot plasma physics (he was the designer of the first Soviet hydrogen bomb) to magnetohydrodynamics (where he was, during the '50s, one of the pioneers) and cosmology, it seemed important to us to quote his own words:

> Thousands of years ago tribes of human beings suffered great privations in the struggle to survive. In this struggle, it was important not only to be able to handle a club, but also to possess the ability to think reasonably, to take care of the knowledge and experience garnered by the tribe, and to develop the links that would provide cooperation with other tribes. Today the entire human race is faced with a similar test. In infinite space many civilizations are bound to exist, among them civilizations that are also wiser and more "successful" than ours. I support the cosmological hypothesis which states that the development of the Universe is repeated in its basic features an infinite number of times. In accordance with this, other civilizations, including more "successful" ones, should exist an infinite number of times on the "preceding" and the "following" pages of the Book of the Universe. Yet this should not minimize our sacred endeavors in this world of ours, where, like faint glimmers of light in the dark, we have emerged for a moment from the nothingness of dark unconsciousness of material existence. We must make good the demands of reason and create a life worthy of ourselves and of the goals we only dimly perceive.

In this text is explicitly mentioned the fact that earthlings might not be the most advanced intelligent species in terms of science and technology. It is the logical view every reasonable scientist should assume. But it leads immediately to one question: More advanced civilizations that our own, have they been able to visit us, including ancient times of our history? If such visits were technically and scientifically possible, do they currently continue?

Faced with this question, skeptics exhibits two arguments. First, as once did Enrico Fermi, they are amazed that there has not been any contact at all. Second, they evoke the impossibility of interstellar travel due to the severe limitation imposed on the speed of a craft by the laws, repeatedly checked at all scales, of special relativity.

On the first argument, let us say that any contact between civilizations with very different technical and scientific levels of development is unavoidably accompanied by a shock of cultures. Different scenarios are then possible. In one of them, the low-level scientific and technological civilization quite simply collapses. In a second one the most advanced civilization does swallows up the one which it comes in contact with. It can assimilate it, or enslave it (colonization). In a third scenario, a powerful scientific and technological transfer occurs, which is immediately converted with priority, as in any fundamental progress of technology, into new weapons.

Any progress in science is inevitably accompanied by a subsequent progress in destructive capacity. So that if some advanced sophisticated technoscience arose which made interstellar travel possible, the corollary would be the emergence of huge means of destruction. In comparison, such new weapons would make our thermonuclear warheads look like match sticks.

I do not think I would be contradicted by saying that wisdom is far from Earth where, despite past dramas, we wonder if humanity would not soon switch into a third world war. Thus, any technology transfer would not do other but accentuate the deadly imbalance which we suffer and that is not going to get better with time.

This paper focuses on the second argument, the feasibility of interstellar travel.

## 2. A Bimetric Model

The cosmological model of General Relativity represents an extension of the revolution represented by the appearance of Special Relativity. It is worth to think for a moment about the essence of these profound changes of paradigm. The transition from Newtonian physics to relativistic physics is based on a modification of the geometric paradigm. Special Relativity can be summed up in the sentence: "We live in a spacetime which is an $\mathrm{M}_{4}$ manifold with a hyperbolic Lorentz metric of signature ( +--- )". General Relativity can be stated accordingly: "The Universe is an $\mathrm{M}_{4}$ manifold with a Riemannian metric of signature $(+---)$ " which is a solution of the Einstein equation:

$$
\begin{equation*}
R_{\mu \nu}-\frac{1}{2} R g_{\mu \nu}+\Lambda g_{\mu \nu}=\chi T_{\mu \nu} \tag{1}
\end{equation*}
$$

Every travel in this context can proceed only by moving along the geodesics associated with the metric $g_{\mu \nu}$ that is solution of the equation. These geodesics include two subsets: those of nonzero length followed by particles that own a mass, and those of zero length, followed by photons. This statement implies an unavoidable limit for all velocities, that is the speed of light $c$. In the domain of relativistic velocities, an enormous energy must be involved, and the limitation to a speed $c$ implies the fact that, according to this vision of the Universe, an almost infinite energy is necessary to reach it.

Everything else follows and all those who have tried to model the interstellar travel are led to travel times that are incompatible with the duration of human life. The conclusion is therefore the following: if interstellar travel is scientifically and technically feasible, that is, compatible with the duration of human life, then a deep change in the current cosmological paradigm, which prohibits such an eventuality must be operated.

One could say that the current situation somehow calls for such a change of paradigm. Theoretical astrophysics virtually stopped in the early '70s when theorists, disregarding the approach initiated by Chandrasekhar in $1941,{ }^{1}$ abandoned any attempt to build a self-consistent model of galaxies and their spiral structure, whenever they possess one, in favor of numerical computer simulations on which
high hopes were deposited and which have strictly produced nothing in half a century. The improvement of observational data on the velocity curves of rotation of gas in galaxies, and the discovery of gravitational arcs, soon attributed to gravitational lens effects and incompatible with the amount of matter in galaxies and clusters of galaxies from photometric observations, gave birth to a deus ex machina: the dark matter. But all attempts to identify this new component, being it MACHOs or astroparticles, have failed. And it comes that since ten years ago it is confirmed, against all odds, that the Universe, rather than decelerate as a result of the attractive forces of visible matter and dark matter, accelerates instead (see Refs. 2-10). Faced with this new dilemma, astrophysicists have not done anything but to create a new word: dark energy. And have done so without having the slightest idea of the nature of this new component, which would represent $70 \%$ of the cosmic content. Many attempts were made in order to explain the nature of such phenomenon. Among them, let us cite Refs. 11 to 14.

In 1967, Sakharov proposed a twin description of the Universe with the aid of two entities linked by a Big Bang singularity, and with antiparallel arrows of time. ${ }^{15-17}$ If we exclude our own work, which was initiated in $1977^{18,19}$ using the tools of non-relativistic Newtonian cosmology, and ignore the existence of his own, this approach had no continuators. In 1994, ${ }^{20}$ we clarified the approach by offering a bimetric description of the Universe. Though it has nothing to do with the bimetric models of Refs. 21 and 22 where the second metric refers to gravitons with nonzero mass. Strictly speaking, these models have not produced anything.

In our model, the Universe is an $\mathrm{M}_{4}$ manifold associated not to one single metric, but to two: $g_{\mu \nu}^{(+)}$and $g_{\mu \nu}^{(-)}$, the former linked to species of positive mass and energy, the latter to species of negative mass and energy. From these metrics, one can build the associated Ricci tensors, $R_{\mu \nu}^{(+)}$and $R_{\mu \nu}^{(-)}$. A system of two coupled field equations was then proposed: ${ }^{20}$

$$
\begin{gather*}
R_{\mu \nu}^{(+)}-\frac{1}{2} R^{(+)} g_{\mu \nu}^{(+)}=\chi\left(T_{\mu \nu}^{(+)}+T_{\mu \nu}^{(-)}\right),  \tag{2a}\\
R_{\mu \nu}^{(-)}-\frac{1}{2} R^{(-)} g_{\mu \nu}^{(-)}=-\chi\left(T_{\mu \nu}^{(+)}+T_{\mu \nu}^{(-)}\right), \tag{2b}
\end{gather*}
$$

where the tensors $T_{\mu \nu}^{(+)}$and $T_{\mu \nu}^{(+)}$represent positive and negative energy contents (and positive and negative mass contents as well). Previously, in 1957, Bondi ${ }^{23}$ did study the possibility of introducing negative masses into the Einsteinian model with a single metric. As a result, positive masses attracted everything and negative masses repelled everything. It then led to a phenomenon that was called "runaway": when a mass $+m$ met a mass $-m$, the positive mass fled, pursued by the negative mass, both undergoing a continuous acceleration, with conservation of energy, since one of the energies was positive and the other negative.

This idea of cohabitation of masses of opposed signs was therefore abandoned during 57 years.

Quantum Field Theory (QFT) rejects the existence of negative energy states on the basis that no state could exist with energy less than that of the vacuum. This argument is developed by Weinberg in his book (Sec. 2.6 of Ref. 24). But it is more a hypothesis than a demonstration in the sense that everything depends on the choice of the $T$ operator, that is, time inversion. If it is to be considered as unitary and linear, then QFT enables the existence of negative energies, time inversion being synonymous of energy inversion (which is the case in the Theory of Dynamical Groups, build with real coefficients ${ }^{25}$ ). In order to avoid this "disastrous" situation, QFT imposes the choice of an anti-unitary and anti-linear T operator, which does not invert energy.

The paper of $1994^{20}$ was completed the following year with another publication ${ }^{26}$ and with a communication in an international workshop. ${ }^{27}$ This theoretical description was also recently revisited. ${ }^{28}$ The main lines of this model are as follows:

- Particles of positive mass follow non-null geodesics of the $g_{\mu \nu}^{(+)}$metric.
- Photons of positive energy follow null geodesics of the $g_{\mu \nu}^{(+)}$metric.
- Particles of negative mass follow non-null geodesics of the $g_{\mu \nu}^{(-)}$metric.
- Photons of negative energy follow null geodesics of the $g_{\mu \nu}^{(-)}$metric.
- The two families of geodesics, one followed by particles of positive energy and the other followed by particles of negative energy, are a priori disjoint.
- Particles of positive mass emit positive energy photons that can in turn be captured by observational devices constituted also by positive masses.
- Particles of negative mass emit negative energy photons that cannot be captured by our observational devices.

References 26 and 27 contain results of 2D numerical simulations carried out in the Daisy Laboratory in Germany with the computing capabilities of that time. They give an idea of the fertility of such approach when it comes to explain the large-scale structure of the Universe, galaxy confinement and durable spiral structures. The initial results could not be pursued because of the lack of access to adequate means of calculation. Beyond that, the goal is obviously to build a complete cosmological model, with a radiation dominated era and a matter dominated era. The schema includes therefore a radiative phase "with constant variables" and free of redshift. We published the first paper referring to a cosmological model with variable light speed in this same journal in $1988,,^{29,30}$ and then in 1995. ${ }^{26}$ Subsequently, we modified the model ${ }^{28}$ on the assumption that the secular drift of the constants ceased when the radiation was no longer the dominant component. Laws of evolution which are not semi-empirical, but are derived from a universal gauge relationship assuring the invariance of the equations of physics (field equation, Maxwell equations, quantum physics equations) were provided for the different constants. If $a$ is the scale factor associated with length, the evolution of $c$ is: ${ }^{26,29}$

$$
\begin{equation*}
c \sim \frac{1}{\sqrt{a}} . \tag{3}
\end{equation*}
$$

It leads to a cosmological horizon that varies with $a$, which ensures cosmic homogeneity and makes the appeal to inflation theory unnecessary, what was already stated in the conclusions of Ref. 30. Moreover, this evolution was "Lorentz invariant". But this is simply a remark. The formulation of the cosmic evolution in its radiative phase and in the regime of "constant variables" will be the subject of a future paper. It is at the end of this phase that the two cosmic sets, one of positive energy, the other of negative energy, are provided with constant but very different $c^{(+)}$and $c^{(-)}$light speeds.

We mention here the work of Couannier, ${ }^{31}$ Hossenfelder ${ }^{32}$ and Milgrom ${ }^{33}$ which also aim at the construction of a bimetric description of the Universe.

## 3. A Model with Two Different Light Speeds

This section deals with a regime with two different light speeds. Some previous work was presented in Ref. 28. We take the system of two coupled field equations:

$$
\begin{align*}
& R_{\mu \nu}^{(+)}-\frac{1}{2} R^{(+)} g_{\mu \nu}^{(+)}=\chi^{(+)}\left(T_{\mu \nu}^{(+)}+\varphi T_{\mu \nu}^{(-)}\right)  \tag{4a}\\
& R_{\mu \nu}^{(-)}-\frac{1}{2} R^{(-)} g_{\mu \nu}^{(-)}=-\chi^{(-)}\left(\phi T_{\mu \nu}^{(+)}+T_{\mu \nu}^{(-)}\right) \tag{4b}
\end{align*}
$$

For the sake of brevity, we will sometimes use the notation: $f \in\{+,-\}$. In mixed form, we will write the tensors:

$$
T_{\nu}^{(f) \mu}\left(\begin{array}{cccc}
\rho^{(f)}\left(c^{(f)}\right)^{2} & 0 & 0 & 0  \tag{5}\\
0 & -p^{(f)} & 0 & 0 \\
0 & 0 & -p^{(f)} & 0 \\
0 & 0 & 0 & -p^{(f)}
\end{array}\right) \text { with } \rho^{(f)}>0, p^{(f)}>0 \text { for } f="+"
$$

$\varphi$ and $\phi$ are functions determined from conservation energy requirements. Positive energy photons cruise along null geodesics of the metric $g_{\mu \nu}^{(+)}$and negative energy photons along null geodesics of the metric $g_{\mu \nu}^{(-)}$. Note that the speeds $c^{(+)}$and $c^{(-)}$ on one hand, and the constants $\chi^{(+)}$and $\chi^{(-)}$on another hand, may be different.

We assume this bimetric Universe is homogeneous and isotropic, so that the Riemannian metric becomes:

$$
\begin{equation*}
\left(d s^{(f)}\right)^{2}=\left(c^{(f)}\right)^{2} d t^{2}-\left(a^{(f)}\right)^{2} \frac{d u^{2}+u^{2}\left(d \theta^{2}+\sin ^{2} \theta d \varphi^{2}\right)}{\left(1+k k^{(f)} \frac{u^{2}}{4}\right)^{2}} . \tag{6}
\end{equation*}
$$

Introducing these metrics in the system (3a)+(3b), we get classical systems:

$$
\begin{gather*}
\frac{3}{\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{2}}\left(\frac{d a^{(+)}}{d t}\right)^{2}+\frac{3 k^{(+)}}{\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{2}} \\
\quad=-\chi^{(+)}\left[\rho^{(+)}\left(c^{(+)}\right)^{2}+\varphi \rho^{(-)}\left(c^{(-)}\right)^{2}\right] \tag{7a}
\end{gather*}
$$

$$
\begin{gather*}
\frac{2}{\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{2}} \frac{d^{2} a^{(+)}}{d t^{2}}+\frac{1}{\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{2}}\left(\frac{d a^{(+)}}{d t}\right)^{2}+\frac{k^{(+)}}{\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{2}}=0,  \tag{8a}\\
\frac{3}{\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{2}}\left(\frac{d a^{(-)}}{d t}\right)^{2}+\frac{3 k^{(-)}}{\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{2}} \\
=+\chi^{(-)}\left[\phi \rho^{(+)}\left(c^{(+)}\right)^{2}+\rho^{(-)}\left(c^{(-)}\right)^{2}\right],  \tag{7b}\\
\frac{2}{\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{2}} \frac{d^{2} a^{(-)}}{d t^{2}}+\frac{1}{\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{2}}\left(\frac{d a^{(-)}}{d t}\right)^{2}+\frac{k^{(-)}}{\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{2}}=0 . \tag{8b}
\end{gather*}
$$

Applying classical mathematical methods ${ }^{34}$ the compatibility conditions of sets $(7 \mathrm{a})+(7 \mathrm{~b})$, and $(8 \mathrm{a})+(8 \mathrm{~b})$ give:

$$
\begin{align*}
& 3 \frac{d a^{(+)}}{a^{(+)}}+\frac{d\left[\rho^{(+)}\left(c^{(+)}\right)^{2}+\varphi \rho^{(-)}\left(c^{(-)}\right)^{2}\right]}{\left[\rho^{(+)}\left(c^{(+)}\right)^{2}+\varphi \rho^{(-)}\left(c^{(-)}\right)^{2}\right]}=0  \tag{9a}\\
& 3 \frac{d a^{(-)}}{a^{(-)}}+\frac{d\left[\phi \rho^{(+)}\left(c^{(+)}\right)^{2}+\rho^{(-)}\left(c^{(-)}\right)^{2}\right]}{\left[\phi \rho^{(+)}\left(c^{(+)}\right)^{2}+\rho^{(-)}\left(c^{(-)}\right)^{2}\right]}=0 \tag{9b}
\end{align*}
$$

The conservation of energy:

$$
\begin{equation*}
E=\rho^{(+)}\left(c^{(+)}\right)^{2}\left(a^{(+)}\right)^{3}+\rho^{(-)}\left(c^{(-)}\right)^{2}\left(a^{(-)}\right)^{3} \tag{10}
\end{equation*}
$$

is ensured if:

$$
\begin{equation*}
\varphi=\left(\frac{a^{(-)}}{a^{(+)}}\right)^{3} \quad \phi=\left(\frac{a^{(+)}}{a^{(-)}}\right)^{3} \quad \phi=\varphi^{-1} . \tag{11}
\end{equation*}
$$

So that our coupled field equation system becomes:

$$
\begin{gather*}
R_{\mu \nu}^{(+)}-\frac{1}{2} R^{(+)} g_{\mu \nu}^{(+)}=\chi^{(+)}\left[T_{\mu \nu}^{(+)}+\left(\frac{a^{(-)}}{a^{(+)}}\right)^{3} T_{\mu \nu}^{(-)}\right],  \tag{12a}\\
R_{\mu \nu}^{(-)}-\frac{1}{2} R^{(-)} g_{\mu \nu}^{(-)}=-\chi^{(-)}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^{3} T_{\mu \nu}^{(+)}+T_{\mu \nu}^{(-)}\right] \tag{12b}
\end{gather*}
$$

as previously established in Ref. 28. Let:

$$
\begin{gather*}
\left(a^{(+)}\right)^{2} \frac{d^{2} a^{(+)}}{d t^{2}}=\frac{\chi^{(+)}}{2} E,  \tag{13a}\\
\left(a^{(-)}\right)^{2} \frac{d^{2} a^{(-)}}{d t^{2}}=-\frac{\chi^{(-)}}{2} E,  \tag{13b}\\
\chi^{(+)}=-\frac{8 \pi G}{c^{4}} \text { with } c^{(+)}=c \quad \text { and } \quad G^{(+)}=G . \tag{14}
\end{gather*}
$$

Let us assume $E<0$. Then $a^{(+) \prime}>0$ and $a^{(-) \prime \prime}<0$. If we assume further that our visible part of the Universe corresponds to positive mass, then it accelerates, while
the negative species decelerates. As in Ref. 28, we find that the evolution equation (13b) is identical to Friedmann's equation, while (13a) is identical to Bonnor's solution: ${ }^{35}$

$$
\begin{align*}
& a^{(+)}(\mu)=\alpha^{2} \operatorname{ch}^{2} u \\
& t^{(+)}(\mu)=\alpha^{2}\left(1+\frac{\operatorname{sh} 2 u}{2}+u\right) . \tag{15}
\end{align*}
$$

Let us now develop the Newtonian approximation. The Universe owns a global nonzero curvature. In steady-state or quasi steady-state condition, the Universe can be considered as a large 3D hypersurface where very small regions contain matter surrounded by large empty spaces where the metric corresponds to the Lorentz solution of the $R_{\mu \nu}=0$ equation. Consider a portion of empty space with a finite extension. If we consider phenomena occurring over a characteristic time that is small compared to the one corresponding to the evolution of the Universe, the description by time-independent metric holds. Introduce a finite space extension perturbation, corresponding to the terms $\gamma_{\mu \nu}^{(+)}$and $\gamma_{\mu \nu}^{(-)}$.

$$
\begin{equation*}
g^{(+)}=\eta^{(+)}+\varepsilon \gamma^{(+)}, \quad g^{(-)}=\eta^{(-)}+\varepsilon \gamma^{(-)} . \tag{16}
\end{equation*}
$$

More explicitly:

$$
\begin{align*}
& \left(d s^{(+)}\right)^{2}=\left(c^{(+)}\right)^{2} d t^{2}-\left(a^{(+)}\right)^{2}\left[\left(d \xi^{1}\right)^{2}+\left(d \xi^{2}\right)^{2}+\left(d \xi^{3}\right)^{2}\right]  \tag{17a}\\
& \left(d s^{(-)}\right)^{2}=\left(c^{(-)}\right)^{2} d t^{2}-\left(a^{(-)}\right)^{2}\left[\left(d \xi^{1}\right)^{2}+\left(d \xi^{2}\right)^{2}+\left(d \xi^{3}\right)^{2}\right] \tag{17b}
\end{align*}
$$

In quasi steady-state conditions, the two scale factors are considered as constants. Then, the field equations are expanded into a series. Neglecting second-order terms in the expansion, we find:

$$
\begin{align*}
& \varepsilon \gamma_{00|\beta| \beta}^{(+)}=-\chi^{(+)}\left[\delta \rho^{(+)}\left(c^{(+)}\right)^{2}+\left(\frac{a^{(-)}}{a^{(+)}}\right)^{3} \delta \rho^{(-)}\left(c^{(-)}\right)^{2}\right],  \tag{18a}\\
& \varepsilon \gamma_{00|\beta| \beta}^{(-)}=\chi^{(-)}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^{3} \delta \rho^{(+)}\left(c^{(+)}\right)^{2}+\delta \rho^{(-)}\left(c^{(-)}\right)^{2}\right] . \tag{18b}
\end{align*}
$$

Defining the potentials:

$$
\begin{equation*}
\varphi^{(+)}=\varepsilon \gamma_{00}^{(+)} \frac{\left(c^{(+)}\right)^{2}}{2}, \quad \varphi^{(-)}=\varepsilon \gamma_{00}^{(-)} \frac{\left(c^{(-)}\right)^{2}}{2} \tag{19}
\end{equation*}
$$

we get:

$$
\begin{align*}
& \sum_{\alpha=1}^{3} \frac{\partial^{2} \varphi^{(+)}}{\partial \xi^{\alpha} \partial \xi_{a}}=-\frac{\chi^{(+)}\left(a^{(+)}\right)^{2}}{2}\left[\delta \rho^{(+)}\left(c^{(+)}\right)^{2}+\left(\frac{a^{(-)}}{a^{(+)}}\right)^{3} \delta \rho^{(-)}\left(c^{(-)}\right)^{2}\right]  \tag{20a}\\
& \sum_{\alpha=1}^{3} \frac{\partial^{2} \varphi^{(-)}}{\partial \xi^{\alpha} \partial \xi_{\alpha}}=\frac{\chi^{(-)}\left(a^{(-)}\right)^{2}}{2}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^{3} \delta \rho^{(+)}\left(c^{(+)}\right)^{2}+\delta \rho^{(-)}\left(c^{(-)}\right)^{2}\right] \tag{20b}
\end{align*}
$$

which are Poisson-like equations that represent an extension of Ref. 28 when the two light speeds are different. Using the two forms of geodesic equations we get for the positive mass particles:

$$
\begin{equation*}
\frac{d^{2} \xi^{\alpha}}{d t^{2}}=-\frac{1}{\left(a^{(+)}\right)^{2}} \frac{\partial \varphi^{(+)}}{\partial \xi_{\alpha}} . \tag{21a}
\end{equation*}
$$

And for the negative mass particles:

$$
\begin{equation*}
\frac{d^{2} \xi^{\alpha}}{d t^{2}}=-\frac{1}{\left(a^{(-)}\right)^{2}} \frac{\partial \varphi^{(-)}}{\partial \xi_{\alpha}} \tag{21b}
\end{equation*}
$$

Building the dynamics from this set of equations would require a more refined solution, involving time scale factors $T^{(+)}$and $T^{(-)}$. This will be developed in a future paper. Anyway, it is possible to derive the laws in the simplified case $a^{(+)}=a^{(-)}, c^{(+)}=c^{(-)}$. Then:

$$
\begin{gather*}
\Delta \varphi^{(+)}=-\Delta \varphi^{(+)}=4 \pi G\left(\delta \rho^{(+)}+\delta \rho^{(-)}\right),  \tag{22}\\
\frac{d^{2} x^{\alpha}}{d t^{2}}=-\frac{\partial \varphi^{(+)}}{\partial x_{\alpha}}, \quad \frac{d^{2} x^{(-) \alpha}}{d t^{2}}=-\frac{\partial \varphi^{(-)}}{\partial x_{\alpha}}=\frac{\partial \varphi^{(+)}}{\partial x_{\alpha}} . \tag{23}
\end{gather*}
$$

Referring to the previous works of Bondi ${ }^{23}$ and Bonnor, ${ }^{35}$ the puzzling problem of runaway phenomenon is eliminated. This model fits classical verification associated with General Relativity. As the two species repel each other, where positive mass is present, negative mass is almost absent and the system of coupled field equations reduces to:

$$
\begin{align*}
& R_{\mu \nu}^{(+)}-\frac{1}{2} R^{(+)} g_{\mu \nu}^{(+)} \approx \chi^{(+)} T_{\mu \nu}^{(+)}=-\frac{8 \pi G}{c^{4}} T_{\mu \nu}^{(+)},  \tag{24a}\\
& R_{\mu \nu}^{(-)}-\frac{1}{2} R^{(-)} g_{\mu \nu}^{(-)} \approx-\chi^{(-)}\left(\frac{a^{(+)}}{a^{(-)}}\right)^{3} T_{\mu \nu}^{(+)} . \tag{24b}
\end{align*}
$$

On the other hand, considering portions of space containing a sphere filled with constant density matter (positive or negative) surrounded by vacuum, makes "internal" and "external" Schwarzschild solutions features clearer, as first presented in Ref. 26 introducing a negative gravitational lensing effect.

To give the relative size of the parameters involved, we need to develop the non-steady solution applying to the radiative era with variable speeds of light. This will also be done in a future paper.

The basic features of the bimetric model, interaction laws and negative lensing effect, were developed in Refs. 20, 26 and 27. In Ref. 28, from dynamical group theory, ${ }^{25}$ the particles of the second sector are CPT-symmetrical with respect to positive energy particles (protons, neutrons, electrons, photons and so on), which gives them negative energy and mass. So the matter-antimatter duality holds in the two sectors.

Given that masses of opposite signs repel each other, the component responsible for the strong effects of gravitational lensing around galaxies and clusters of galaxies
is in an ultra-rarified state in the galaxies and also in the solar system (where it could be responsible for the deceleration of the Pioneer space probes, a subject to be examinated). If the proposed model is correct, then attempts to find evidence for the existence of astroparticles would be doomed to failure.

Reference 28 was focused on the explanation of the acceleration of the Universe, caused by a negative pressure associated to the negative energy component. The present paper has shown that a bimetric model can be associated with two different light speeds.

## 4. Back to the Problem of Interstellar Travel

The present work needs completion. A study of the radiation dominated era with "variable constants" must be carried out in order to justify the difference between $c^{(+)}$and $c^{(-)}$. As evoked above, time scale factors should be added, which we have not done in the present paper.

Species with positive and negative mass behave differently, the whole being fully asymmetric. Sakharov ${ }^{15-17}$ was the first to imagine an asymmetry in the characteristic times of production of baryons from quarks, and of antibaryons from antiquarks in our Universe. He suggested that the two twin Universes owned opposite arrows of time (notice that according to dynamical group theory ${ }^{25}$ time inversion goes with mass inversion). He suggested that different rates of production of baryons and antibaryons could explain the absence of cosmological antibaryons in our orthochron Universe of positive masses and energies, while "primeval baryons" would be absent in the twin Universe. This would go with a remnant of free antiquarks in our fold and a remnant of quarks with negative energy in the twin.

The absence of primeval antimatter is therefore the first smoking gun of such asymmetry.

The second one is the Very Large Structure of the Universe, which according to that model looks like joint soap bubbles (see Refs. 36-39). Negative mass would form a set of clumps located at the center of each bubble.

As evoked in Ref. 28 a new cosmic cartography could be built on the basis of a recent idea published by a Japanese group, ${ }^{40}$ which would make it possible to size the ratio between averaged densities of the two species $\rho^{(+)}$and $\rho^{(-)}$. Further data could come from the analysis of magnitude of distant galaxies with strong redshifts, considered classically as dwarfs. We believe they are "normal" galaxies whose signal is weakened by negative gravitational lensing effects. Notice that those galaxies emit positive energy photons which can cross negative mass clumps.

As shown in former 2D simulations, ${ }^{26}$ macroscopic structures are different in positive and negative worlds. Negative mass forms spheroidal clumps, similar to huge proto-stars whose cooling time is so great with respect to the age of the Universe that fusion cannot occur. In the negative world there are no stars, no galaxies, no heavy elements, no planets. Life is absent.

Scale factors are also different: $a^{(+)}>a^{(-)}$, so that in order to go from a
point $A$ to a point $B$, two ways are possible along two geodesic paths whose lengths are different depending on the mass of the object, positive or negative.

The evolution of the bimetric Universe in a radiation dominated era with variable constants and $a^{(-)}<a^{(+)}$implies $c^{(-)}>c^{(+)}$:

$$
\begin{equation*}
c^{(+)} \sim \frac{1}{\sqrt{a^{(+)}}}, \quad c^{(-)} \sim \frac{1}{\sqrt{a^{(-)}}} \tag{25}
\end{equation*}
$$

that is:

$$
\begin{equation*}
\frac{c^{(-)}}{c^{(+)}} \sim \sqrt{\frac{a^{(+)}}{a^{(-)}}}>1 \tag{26}
\end{equation*}
$$

If mass inversion could be achieved, then the feasibility of interstellar travel should be reconsidered. In effect, a transferred vehicle would cruise along geodesics of the metric $g^{(-)}$, with several gains:

- The speed limit $\left(c^{(-)}>c^{(+)}\right)$would be higher.
- Distances would be shorter: $a^{(-)}<a^{(+)}$.
- The addition of a different time gauge could reduce travel duration (to be developed in a future work).

Two questions remain:

- Is it possible to achieve mass inversion of a craft and its passengers?
- How to give to the craft a relativistic velocity with respect to the new value $c^{(-)}$?

Mass inversion is accompanied by a local modification of the geometry, which implies a deconnection of the geodesics of one system and a reconnection with the geodesics of the other system. We believe that such process occurs in nature and will give a theoretical description of it in a future paper, showing subsequent observations. As we will show, if a mass is inverted, it does not imply that it goes backwards in time and could emerge at a point of its own past. This is so because the proper time is not modified by the process.

The theoretical physicist Michio Kaku suggests a classification of civilizations of "types I, II, III, etc.". He imagines that the last of them would be capable of using energies defying imagination, which would make interstellar travel possible. Would it be necessary, for example, in order to ensure mass inversion of a given mass $M$, to use an energy comparable to $M c^{2}$ ? Should we create a "wormhole" and jump in? We believe that the energy we can handle today through nuclear technology and, in the future, matter-antimatter annihilation, could be enough. We believe also that our present science and technology has reached the required threshold if used in a new geometric context.

If a given technology makes a craft's mass inversion possible, it would seem to dematerialize if observed by a witness. Negative mass particles, atoms and molecules no longer interact with the positive molecules of the surrounding air except for gravitation (in fact, anti-gravitation). We conjecture that the mass inversion process would be symmetrical. If a positive mass is inverted in a given volume, this would
affect the few negative masses present in the volume, whose mass would be inverted too, and become positive. As a consequence, a highly rarefied medium would appear in the volume, composed by few positive mass hydrogen atoms. For a positive mass observer, the volume occupied by the craft would look empty. The air would fill it immediately, the process producing a hydrodynamical perturbation. If a plane is flying too close to a large craft that is inverting its own mass, the subsequent gas perturbation could break it.

We will elaborate below, on a very conjectural basis, on a possible modification of the craft kinetic parameters after the hypothetic mass inversion process. Anyway, an inverted mass would be repelled by the Earth, so that "it would fall upward". An appropriate cyclic mass inversion could cancel weight. In effect, values of the craft weight would be alternatively $P=M g$ and $P=-M g$, that is zero on average.

A witness would say that the force of gravity is cancelled. Combination with appropriate modifications of the kinetic parameters would ensure the crafts movement.

Particles with mass $m$ own a characteristic length, the Compton's length $\lambda_{c}$ :

$$
\begin{equation*}
\lambda_{c}=\frac{h}{m c} . \tag{27}
\end{equation*}
$$

If mass inversion follows gauge laws as defined in Ref. 26, then:

$$
\begin{equation*}
c \sim \frac{1}{\sqrt{a}}, \quad h \sim a^{\frac{3}{2}}, \quad m \sim a . \tag{28}
\end{equation*}
$$

It is an "apparent mass", concept that will be developed in a future paper. This leads to:

$$
\begin{equation*}
\lambda_{c} \sim a . \tag{29}
\end{equation*}
$$

Mass inversion would be accompanied with some kind of "Gulliver effect". Transferred in the new frame of reference, the inverted particles would have a Compton's length greater than that of their sisters of negative mass because $a^{(-)}<a^{(+)}$. The subsequent energy is:

$$
\begin{equation*}
E=\frac{h c}{\lambda} . \tag{30}
\end{equation*}
$$

Such transfer would occur with loss of energy. If we assume that mass transfer implies energy conservation during the process of mass sign inversion, the transferred particles must have Compton's lengths equal to that of their "sisters" owing negative mass. This would be possible if, after mass inversion, particles gain relativistic velocity so that their Compton's length shortening is ensured by the Lorentz contraction. But the process should orient velocities in the same direction. If not, the craft and the passengers would be splitted. Would the process be possible, interstellar crafts would not need any propeller at all. The words "acceleration" and "deceleration" would lose meaning. Thus, if a huge amount of energy is necessary to accelerate a positive mass at a relativistic velocity, once its mass is inverted, a comparable amount of energy is required to bring its speed to zero when the craft cruises in the new negative frame of reference. After mass inversion, a craft would
go so fast that it could not slow down. But, arriving at its destination, a new mass inversion would give it again its former kinetic parameters.

Even in a very primitive way, these travel techniques have nothing to do with rocket propulsion. All relies on mass manipulation, together with the manipulation of other parameters as the spin (quite easy to handle with a magnetic field). We suggest that energy conservation could be accompanied by a modification of the dynamical parameters that defies imagination. After mass inversion, a new modification of parameters, when the craft owns a negative mass, could give it in the positive realm a velocity vector oriented in another direction, while the length of the vector and the kinetic momentum would be conserved. If such thing is possible, a positive mass observer would see the craft performing turns at right angles, or a total inversion of its velocity, both incompatible with the conventional laws of physics.

If, in a distant future, technical progress makes it possible for a craft to travel between stars in times comparable with human life duration, what would the travelers see through the windows when they go over the negative world? They could no longer see the Universe of positive mass. They could not see planets, stars or galaxies. Instead, being able to capture images mediated by photons of negative energy, it would be possible for them to see the distant fuzzy spheroidal clusters of negative mass emitting weakly, like proto-stars, reddish and infrared light.

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