A Symplectic Cosmological Model

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Abstract

In this paper, we use the Lie algebra of the dual Poincaré dynamical group which when acted upon by its coadjoint , displays energy, momentum and spin as pure geometrical quantities. When extended to the full group, one obtains negative mass species in accordance with our *Janus Cosmologial Model* and the twin universe model conjectured by A.Sakharov.

Within a *5D Kaluza Space*, the theory leads to a new matter/antimatter duality, implying negative energy photons emitted on the negative domain of the twin Universe. This account for the dark matter and dark energy which are thereof impossible to detect in our domain.

Finally, we show that shifting to a Hermitean space-time with an associated dynamic group yields imaginary energy, imaginary mass and imaginary charges all imbeded in a symplectic (complex) framework which remains open to wide inverstigations.

Notations

Space time : indices 0,1,2,3

Space-time signature -2

Introduction

Symplectic geometry relies on symplectic manifolds. They are said symplectic when they are endowed with a symplectic form that allows the measurement of sizes of 2-dimensional objects. In Riemanian geometry, the metric tensor probes lengths and angles, whereas the sympectif form measures areas.

The term sympletic was first coined by H.Weyl in 1939 as a substitute to rather confusing (line) complex groups and/or Abelian linear groups.

The relativistic Symplectic Mechanics [1] was primarly developped by the french mathematician J.M.Souriau from dynamic groups theory.

It provides a new definition of energy, momentum and spin only in terms of pure geometrical quantities. This arises from two objects : *n*-dimensional space and its isometry group.

1 – **The Janus Cosmological Model**

The main mathematical tool is the so-called « *momentum map* » which is inferred from the coadjoint action of the group on its *Lie algebra* (*the coadjoint of the Lie representation is the dual of the adjoint representation*).

Applying the technique of this coadjoint action leads to the appearence of generalized linear and angular momenta : $\left\{ Energy\ E\ ,\ 3-momentum\ p\ ,\ spin\ s \right\}$

The action corresponds to :

$$M' = L\ M\ {}^tL + N\ {}^tP\ {}^tL - L\ P\ {}^tN \qquad (1)$$

$$P' = L\ P \qquad (2)$$

Where *P* is the energy-impulsion 4-vector :

$$P = \begin{pmatrix} E \\ p_x \\ p_Y \\ p_z \end{pmatrix} \qquad (3)$$

L is the element of the Lorentz group and *N* the boost 4-vector .

In the classical treatment, one merely considers *the restricted Poincaré group* which is formed with the orthochronous components L_o . Hence the full Poincaré Group can be written as

$$\begin{pmatrix} \lambda L_o & N \\ 0 & 1 \end{pmatrix} \qquad (4)$$

with $\lambda = \pm 1$

We then obtain two kinds of matters and two kinds of photons which each an opposite mass and energy. This copes with the Janus Cosmological Model (JCM) we developped earlier ([2], [3], [4]).

Such a model involves particles ith opposite masses and energy.

However as shown by H.Bondi in 1957 [5], the field equation cannot sustain this duality due to the subsequent and unmanageable « runaway effect ». In short General Relativity deals with positive masss that are attractive, while negative masses woud exhibit repelling forces.

The issue can be evaded by cinsidering a bi-metric (our JCModel) with a single manifold M4 equipped with two metric tensors $g^{(+)}_{\mu\nu}$ and $g^{(-)}_{\mu\nu}$, which define two field equations : (5)

$$R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} = \chi \left[T^{(+)}_{\mu\nu} + \sqrt{\frac{g^{(-)}}{g^{(+)}}}\, T^{(-)}_{\mu\nu} \right] \quad (5)$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} = -\chi \left[\sqrt{\frac{g^{(+)}}{g^{(-)}}}\, T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right] \quad (6)$$

whose time-independent and time-dependent solutions fit observational data.

3. Extension to a wider geometrical framework

We now turn to a possible extension of the group to a five dimensional scheme so as to obtain an *isometry group* which acts on a *Kaluza space-time* :

$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & N \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

with $\lambda = \pm 1$ and $\mu = \pm 1$

By extension to the fifth dimension, the *Noether theorem* indices an additional conserved scalar quantity which can be identified to the electric charge *q*.

$\mu = -1$ implies the inversion of the fifth dimension, which is just the geometrical expression of the matter-antimatter duality, as firstly presented by J.M.Souriau in 1964[6] .

Therefore the physics ruled by the dynamical group (7) exhibits straightforwardly the matter-antimatter symmetry in the two sectors, with opposite mass and energy.

If we now add *p*-Kaluza like dimensions, we obtain the metric under the form :

$$ds^2 = dx_o{}^2 - dx_1{}^2 - dx_2{}^2 - dx_3{}^2 - d\xi_1{}^2 - d\xi_2{}^2 \ldots - d\xi_p{}^2 \quad (8)$$

This can be coupled to an isometry group :

$$\begin{pmatrix} \lambda\mu & 0 & \ldots & 0 & 0 & \phi_1 \\ 0 & \lambda\mu & \ldots & 0 & 0 & \phi_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \lambda\mu & 0 & \phi_p \\ 0 & 0 & \ldots & 0 & \lambda L_o & N \\ 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

with $\lambda = \pm 1$ and $\mu = \pm 1$

The electric charge is one of the quantum charges. Here again $\mu = -1$ reflects the C-symmetry : they account for the classical matter-antimatter representation.

$\left(\lambda = -1 \;;\; \mu = -1 \right)$ correspond to the PT symmetry, classically associated to the so-called « Feynman Antimatter », which is no longer identified to the « C-symmetrical antimatter », that we could call the « Dirac-antimatter ». This is due to the presence of the (real) time reversal operator T inducing both time, mass and energy inversions.

In other words the group representation (9), which is the basis of the JCModel, provides two distinct types of antimatters :

- The C-symmetrical type, corresponding to Dirac's antimatter

- The PT-symmetrical type, corresponding to Feynman's antimatter

4. **Remark about Andrei Sakharov schema** [7]

In classical cosmology a severe problem remains, due to the absence of observation od primeval antimatter. In 1967 A.Sakharov suggested the Universe comprises two domains : the actual Universe and its twin Universe, each connected through a singularity ([8] , [9], [10]), the two being CPT-symmetrical. Both are CPT-symmetrical.

Since the mass inversion goes with T-symmetry , our JC-Model . ([3] ,[4]) corresponds to such CPT-symmetry. The so-called *twin matter* becomes nothing but a copy of ordinary particles, with opposite mass and energy.

If, as suggested by A.Sakharov, poitive passes are synthetized by positive energy quarks faster the the synthesis of negative mass matter ftom positive energy antiquarks, then, in the positive sector we find :

- Remnant positive mass matter
- The equivalent (ration 3/1) of positive energy antiquarks
- Positive energy photons.

In analogy to Sakharov's idea the *negative energy domain* would be composed by :

- Remnant negative mass antimatter
- The equivalent (ration 3/1) of negative energy quarks
- Negative energy photons.

As shown in ([3],[4]) the negative material suitably replaces both dark matter and so-called dark energy. Accordingly, emitting negative energy photons the remnant negative mass antimatter is geniusly invisible.

5. **Remark about the Quantum Theory of Fields** [7]

In QFT the time reversal operator is a complex operator which can be *linear* and *unitary*, as well as *antilinear* and *anti-unitary*. If chosen as linear and unitary this operator implies the existence of negative energy states, which is *a priori* banned by QFT. In Weinberg [7] we quote : « *In order to avoid this disastrous conclusion we are forced to conclude that T is antilinear and anti-unitary* ».

On page 104 Weinberg also writes that « *No examples are known of particles that furnish unconventional representation of inversions, so these possibilitys will not be pursed further here* ».

Actually, thi was true until the discovery of the acceleration of the expanding Universe which implies the action of a negative pressure.

Aa a pressure is linkened to an energy density, this new phenomenon implies the existence of negative energy states and as a result it questions QFT by itself. In the same manner, it also raises some questions as to the validity of the so-called CPT theorem and the vacuum instability. Classically one considers that a particle may loose energy by the emission of a photon, so that such phenomenon would lead to negative energy states. But if we consider that a negative mass particle emits negative energy photons, that leads to stable zero energy state.

6. **Extension of the method to a complex field.**

In one replaces the Minkowski coordinates $\left\{ x_o , x_1 , x_2 , x_3 \right\}$ with complex coordinates we may form the *Hermitean Riemann metric* :

$$ds^2 = dx_o * dx_o - dx_1 * dx_1 - dx_2 * dx_2 - dx_3 * dx_3 \qquad (10)$$

The metric is defined on a *Hermitean manifold*

Let us now consider the real matrix G :

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (11)$$

an the complex Lorentz group defined as :

$$*L\, G\, L = G \qquad (12)$$

$*L$ stands for the adjoint of L .

One can easily show that *the complex Poincaré group*

$$\begin{pmatrix} L & N \\ 0 & 1 \end{pmatrix} \qquad (13)$$

is a isometry group of such a Hermitean space and can be considered as a dynamical group. Suprizingly, all classical (matrix) calculations can be extended to such complex framework, by simply substititing the matrices $*A$ to the transposed matrices ^{t}A .

As a result, the complex momentum obeys the law :

$$M' = L\, M^* L + N^* P^* L - L\, P^* N \tag{14}$$

$$P' = L\, P \tag{15}$$

where P is a *complex energy-momentum 4-vector with complex energy* and *complex momentum.*

This extended physics grants the mass a possibly complex nature implying the possible existence of purely real masses $\pm m$ and purely imaginary masses $\pm\ (-1)^{\frac{1}{2}} m$.

At the same time, such mases can exchange imaginary photons, whose energies are $\pm\ (-1)^{\frac{1}{2}} E$.

Conclusion

J.M.Souriau gave the first purely geometrical interpretation of all classical physics features, namely energy, momenta and spin.

When extended to higher dimensions it provides a geometrical interpretation of the matter-antimatter duality.

In addition, one can notice that the complex approach of space definition yields complex physical quantities.

The physical meaning of such complex quantities should demand further scrutiny and as such remains a new field of investigantions.

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