Modern Physics Letters A
Vol. 34 (2019) 1950084 (6 pages)

DOI: 10.1142/S0217732319500846

Negative mass scenario and Schwarzschild spacetime in general relativity

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Received 3 October 2018
Revised 27 December 2018
Accepted 8 January 2019
Published 19 March 2019

In this note, we discuss the hypothesis of a presence of the negative mass matter in the Schwarzschild's spacetime and demonstrate, that the discrete PT transformation of the coordinates and mass inversion in the Schwarzschild's metric solution is equivalent to the inversion of the Kruskal–Szekeres coordinates [M. D. Kruskal, *Phys. Rev.* **119**, 1743 (1960); G. Szekeres, *Publ. Mat. Debrecen* **7**, 285 (1960)], in the full region of the solution's spacetime. As the consequence of the result, it is argued that the whole Schwarzschild spacetime can be described in terms of regions of positive and negative masses interconnected by the discrete transforms.

Keywords: General relativity; Schwarzschild spacetime; negative mass.

PACS No.: 04.20.-q

1. Introduction

The motivation of the negative mass gravitation scenario in the different cosmological models is very clear. In any scenario, see Refs. 2 and 3 for example, the presence of some kind of repulsive gravitation forces in our Universe helps with explanation of the existence of dark energy and dark matter in the models, see Refs. 4–8 and references therein. There are few possibilities to introduce the gravitational repulsive force to the scene. In the scenario of Ref. 2, it is argued that the repulsive gravitation is arising between matter and antimatter (in the sense of charge) particles without an introduction of any negative masses. It is also possible to consider a model where our Universe antimatter has a negative mass, see, for example Ref. 3, where the negative mass extension of Kerr's solution is populated by our world antiparticles which possess not positive but negative masses. Additionally, we can consider the negative mass particles as the antiparticles of the regular ones with the mass sign of them changed from positive to negative in comparison to the mass sign of the particles and antiparticles of our Universe. In this case, the annihilation between the particles of two types of the matter will happen for the particles of any charge. Important, that the gravitation properties of this matter is also described by Einstein equations after the discrete symmetry transformations applied, see Refs. 2 or 3 and 9 for the discrete symmetries applications in the case of the quantum and classical systems.

The presence of this type of negative mass antimatter as well explains the existence of dark energy in our Universe and in this note we discuss the possible window for the presence of this type of the matter in the spacetime solution given by Schwarzschild's metric. The consideration is based on the following main statements.

(1) An assumption about the full symmetry between isolated behavior of the positive and negative mass matter. Namely, in the absence of our, "normal", matter, the negative mass matter behaves as the usual one, i.e. it has the positive inertial mass and mutual attraction sign in Newton's gravitation law in the weak field approximation, see Ref. 11. As the consequence of this request, in the given framework, we have to reproduce the usual Schwarzschild metric solution in the regions with the negative mass present.
(2) The anti-gravity assumption about the interaction of positive and negative masses which means that the interaction of the matter and antimatter is due to the repulsion force, i.e. with repulsion in the weak field approximation.
(3) The unified description assumption. Both solutions for different masses must be encoded in the same Einstein's equations, similarly, for example, to the particle–antiparticle interpretation of the solutions of QFT equations. This statement does not mean only the invariance of the equation under the CPT transformation and mass inversion. It postulates that both solutions of positive and negative masses are present in the whole spacetime which is defined in terms of some suitable coordinate system, Kruskal–Szekeres for example. For the Schwarzschild spacetime it means that both solutions are subject to the same solution of the general relativity equations. Therefore, any transforms of the interests do not pull out the solutions from the spacetime defined by this way.

The last request clarifies that we do not consider any bi-metric theories of the gravity as in Refs. 4 and 10 but mainly try to follow the ideas of Refs. 2 and 3 with, nevertheless, a different interpretation of what provides the possible repulsive gravitational forces in our Universe.

The first two properties of the matter and antimatter can be explained through the application of the CPT symmetry transformation to the Einstein equations, see Refs. 2 and 3. It can be understood as a consequence of the physics law invariance under the transform. Namely, introducing gravitational source and probe masses in the problem, the equations of motion of the source masses of different signs will be the same whereas the relative sign in the Newton's gravitation law will be different when it is applied for the probe and source masses of the different signs. Correspondingly, the third statement above can be considered as a condition of the self-consistency of the negative mass hypothesis. So far, there are no any experimental facts about a non-correctness of the general relativity results or predictions. Therefore, it looks natural to check the possibility of the negative masses presence in any solution of Einstein equations, beginning from Schwarzschild's metric solution. If no contradictions in the construction of the negative mass scenario in the present framework are found, then can be considered as some verification of the possibility of its existence, which, of course, does not prove the reality of the negative mass. In any case, only experimental checks of the hypothesis can prove or disprove it. Therefore, the main goal of the note is a check of the possibility to reveal the negative mass solution in the spacetime determined by the Schwarzschild solution. In Sec. 2, we discuss Schwarzschild's metric solution in its transformation in the proposed framework and Sec. 3 concludes the paper with the representation of the discussion of possible consequences of the hypothesis.

2. Schwarzschild Metric and Negative Mass Spacetime

The easiest way to understand the negative mass structure of the Schwarzschild metric is to consider the metric in the Eddington–Finkelstein coordinates. For the ongoing Eddington–Finkelstein coordinates (v, r, t), which describe the metric structure in I–II regions (see the definitions in Ref. 12 for example), we have as usual

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2\, dv\, dr + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

with

$$v = t + r^* = t + r + 2M \ln\left|\frac{r}{2M} - 1\right|. \tag{2}$$

Correspondingly, the outgoing Eddington–Finkelstein coordinates (u, r, t) describe the solution in III–IV regions, the metric there has the following form:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) du^2 - 2\, du\, dr + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{3}$$

where

$$u = t - r^*, \tag{4}$$

see Ref. 12. Now, we would like to continue the metric in the region of negative masses performing a substitution $M \to -\tilde{M}, \tilde{M} > 0$. Nevertheless, this naive continuation is not correct, the particles of negative masses must preserve the same dynamics as the "normal" ones. Therefore, the time and radial coordinate must be reversed as well in order to preserve the horizon properties of Schwarzschild's black hole solution: $t \to -\tilde{t}, \tilde{t} > 0$ and $r \to -\tilde{r}, \tilde{r} > 0$, see Refs. 2 and 3. The new variables cover the same R^1 region as the "normal" ones, and now we can construct the full spherically symmetrical vacuum spacetime in Kruskal–Szekeres coordinates accurately performing the discrete transforms. Consider first the $U < 0, V > 0$ region (the region I with $r > 2M$) of the (U, V) coordinate plane. We have there

$$U = -e^{-u/4M}, \quad V = e^{v/4M}. \tag{5}$$

Now, in order to continue this quadrant of the space in the region of negative mass, we perform the discrete transforms. For the incoming coordinate, we obtain

$$v = t + r^* \to \tilde{v} = -\tilde{t} - \tilde{r} + 2M\left(\ln\left(\frac{\tilde{r}}{2M} + 1\right) + \imath\pi\right), \tag{6}$$

after the subsequent t and r reverse and

$$\begin{aligned}\tilde{v} &= -\tilde{t} - \tilde{r} - 2\tilde{M}\left(\ln\left(\frac{\tilde{r}}{2\tilde{M}} - 1\right) + 2\imath\pi\right)\\ &= -\tilde{t} - \tilde{r}^* - 4\imath\pi\tilde{M} = -v - 4\imath\pi\tilde{M},\end{aligned} \tag{7}$$

after the final reversion of the mass. For the outgoing coordinate, we will have similarly

$$u = t - r^* \to \tilde{u} = -\left(\tilde{t} - \tilde{r}^*\right) + 4\imath\pi\tilde{M} = -u + 4\imath\pi\tilde{M}. \tag{8}$$

Therefore, the continuation of Eq. (5) coordinates in the region of negative mass will have the following form:

$$U = -e^{-u/4M} \to \tilde{U} = e^{-\tilde{u}/4\tilde{M}} = -U, \tag{9}$$

$$V = e^{v/4M} \to \tilde{V} = -e^{-\tilde{v}/4\tilde{M}} = -V. \tag{10}$$

This inversion of the signs of the (U, V) coordinate axes will hold correspondingly in all regions of (U, V) plane after analytical continuation, Eqs. (7) and (8). Formally, from the point of view of the discrete transform performed in (U, V) plane, the transformations Eq. (9) are equivalent to the full reversion of the Kruskal–Szekeres "time"

$$T = \frac{1}{2}(V + U) \to -T \tag{11}$$

and "coordinate"

$$R = \frac{1}{2}(V - U) \to -R, \tag{12}$$

of the complete Schwarzschild spacetime. The spacetime with $(\tilde{U}, \tilde{V})$ variables, obtained after the applied discrete transforms, must be understood as spacetime with these "watching point" in the regions of the negative mass. Therefore, the region I in the $(\tilde{U}, \tilde{V})$ plane corresponds to the region III in (U, V) plane, the "white hole" region, IV in the space of normal mass, will transform to the black hole region, II, in the $(\tilde{U}, \tilde{V})$ coordinate plane and vice versa. Importantly, both spaces of normal and negative masses are encoded in the same solution of the Einstein equation in correspondence to our third proposition.

3. Conclusions

In this short note, we discuss the possibility of the presence of the negative mass matter in the world as described by Schwarzschild spacetime. Basing on the three statements above, see Sec. 1, we obtained that regions III–IV of the Schwarzschild's metric solution in Kruskal–Szekeres coordinates, disconnected from our "normal" world, can be interpreted as the regions populated by the matter of the negative mass. This result is clarified by the demonstration that the application of the inversion of the mass, radial coordinate and time reversing in the Schwarzschild's metric solution is equivalent to the full inverse of the Kruskal–Szekeres coordinates in whole spacetime determined by Schwarzschild's metric, which is the most important result of this note. In our future research, we plan to investigate whether this result can be justified in the case of more complex symmetrical solutions of Einstein equations as well, see Refs. 12 and 13.

The first important consequence of the obtained result is that the regions of the negative mass are disconnected from the spacetime of "normal" mass, see, for example Ref. 14. Namely, with the Schwarzschild's metric as the solution of the Einstein equations, there is no direct connection between the regions populated by the negative and "normal" masses. Nevertheless, in this scenario, the white holes in our part of the spacetime are inverted into the black holes which consist of the mass of another type, this is the only sign of the negative masses in our part of the full spacetime for the case of Schwarzschild's solution, see also Ref. 13. The influence of these "negative" black holes on the "normal" matter is in the repulsion of the matter around, no other signals can be detected from the objects. Taking into account that the negative mass scenario is proposed to explain the dark energy presence in our universe, we conclude that in the given framework the dark energy can correspond to the mass of the "negative" black holes in the negative mass region of the spacetime. Another interesting observation is a possibility of modifications and extensions of some non-canonical theories of cosmology in the spacetime with negative mass present, see Refs. 15 and 16.

In conclusion, we underline that any proposed scenario of the negative mass existing has to explain the features of our region of spacetime and any cosmological properties of the observed universe without redundant modification of existing theories and it must be verified in both theoretical and experimental ways. In our opinion, further work in this direction is a very intriguing and interesting subject of research.

Acknowledgments

The author is grateful to M. Zubkov, M. Schiffer and A. Kashi for the numerous and useful discussions.

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