Bimetric models

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Abstract

Following a brief review of existing bimetric models, we show that Janus Cosmological Model brings an alternative solution to the ΛCDM model by introducing negative masses that preserve action-reaction principle (eliminating the runaway effect). By limiting ourselves to Newtonian approximation solutions, an action leads to coupled field equations satisfying Bianchi identities. This model provides an explanation to the recently discovered Great Repeller effect.

Keywords : Cosmology, General Relativity, Bimetric models

1 Introduction

Nowadays the central question in Cosmology and Astrophysics can be summarized to the following:

> With no evidence so far of dark matter existence and a clear dark energy model, can we continue to be contented with the mainstream ΛCDM, or are we ready to reconsider completely our theoretical tools, including General Relativity?

Subsequent to the recent failures in attempts to reveal the existence of dark matter, whether under deep layers of rock or in space from the international space station, the second option should be seriously considered. Pushing through the quest of dark matter detection, assuming each failure implies weaker and weaker interaction with conventional matter, brings experiments to boundaries where the detection become practically impossible, the signal being scrambled by cosmic neutrinos flow noise.

The MOND (Modified Newtonian Dynamics) that consist in proposing a modification of the Newton law in order to account for some observations like the flatness of rotation curves in galaxies. But the adaptation of the law is ad-hoc and it is difficult to find ontological base that would serve as a starting point of this cosmologic model revision.

For an alternative to general relativity to be able to compete with the mainstream ΛCDM model, it has to describe the nature of those two unidentified components that are dark matter and black energy. Several attempts have been made in this direction, via cosmological modes including some negative masses. Among them some keep the geometrical vision of the General Relativity, being variations the Einstein equation, others propose a complete change of paradigm by introducing a biometric description of the universe.

Let us start with the first models. There is the approach of the French researchers Gabriel Chardin et Benoit-Lévy [1] that suggest turning to the 1933 Dirac Milne model [2]. This model is based on the hypothesis of matter-antimatter symmetric cosmology in which antimatter is supposed to present a negative active gravitational mass. In addition the global mass is assumed to be zero in average so that the model gives a linear evolution of the scale factor in time, in contradiction with the acceleration of the cosmic expansion ([3],[4],[5]) while still bringing a solution to the problem of the cosmological horizon. This model furthermore assumes the existence of a mechanism insuring the separation of this mix of matter/anti-matter. The interest in such a model, although missing theoretical justifications of its assumptions, is to bring an insight about the abundance of light elements in the hypothesis of a dynamic close to an expansion linear in time. Antimatter classical models confer to it a priori a positive mass. The authors of this model have high expectations form weighting experiments of the antimatter that are ongoing that the CERN (alpha et Gbar experiments).

We must then quote the article of the English researcher J. Farnes [6]. He proposes to unify dark matter and dark energy in a single entity of negative mass. In order to fit with the mainstream ΛCDM model, in which the equivalent density in constant over time, and to mimic the cosmological constant Λ, the author is led to invoke an hypothetical mechanism of negative mass continuous creation hence bringing more questions than answers. Nevertheless, some numerical simulations are given based on the assumption that galaxies would fit in gaps inside some negative masses distribution, this later one confining the galaxies. The author recovers the flatness of rotation curves by using ideas introduced earlier in [7]. For a detailed analysis of that publication see the references [8], [9] and [10].

In those two approaches the authors keep the geometric context of General Relativity, meaning that their model is derived from Einstein equations.

2 The runaway effect

This effect, exhibited in 1957 by Hermann Bondi [11] comes from the fact that the behavior of test masses, put in a gravitational field, is deduced from a single metric solution. Therefore, put in a gravitational field created by positive masses, the positive or negative test masses are subject to an attraction force. Put in a gravitational field created by negative masses, the positive or negative test masses are subject to a repulsion force. Hence the schema of positive/negative masses interaction, according to the Einstein model, correspond to

- Positive masses attract each other according to the Newton law
- Negative masses repel each other according to an (anti-)Newton law
- When masses of opposite signs are put in the presence of each other, the positive mass run away, followed by the negative mass

If those masses are equal in absolute value, then while undergoing a uniformly accelerated motion, the distance between them stay constant. At the extreme, this paradoxical effect (called runaway effect) is done at constant energy as the kinetic energy of the negative mass is itself negative. Let us note also that this effect is in contradiction with the action-reaction principle.

In [6] J.Farnes simply says that this runaway phenomenon could be at the origin of the huge energy of the so-called cosmic rays in spite his introduction of negative masses in the geometrical context of general relativity implies the abandonment of the action-reaction principle. Can we envisage physics with such a choice?

3 The choice of a bimetric geometry

If we plan to maintain the action-reaction principle, it is necessary to envision a major paradigm shift in which the 1917 Einstein model is a step in building a more elaborated bimetric cosmological model.

Historically, a first proposal has been made in the 1994 article [12]. In this first article, several aspects are covered, like the fact that galaxies could be trapped in gaps in the negative mass field exercising a confinement. If the model presented raises the explanation of those phenomena with the help of coupled field equations, it lacks mathematical foundation. The year after, the author introduces the negative gravitational lensing effect to explain the strong lensing effect observed that always suggest introducing some missing mass.

The first paper to introduce a bimetric model based on more solid mathematical foundation is the one of T.Damour and I.Kogan, in 2002 ([13], [14]). The authors invoke two left and right *branes*, that interact via some graviton having a mass spectrum. Later several interaction models including this kind of vectors are considered. Noticing that taking into account Kaluza type models lead to the conclusion that a gap would exist between light and heavy gravitons, the authors decide to focus on light gravitons. The two *right* and *left* populations behavior is described by two metrics $g^R_{\mu\nu}$ and $g^L_{\mu\nu}$ from which they derive two Ricci tensor fields $R_{\mu\nu}(g^R_{\mu\nu})$ and $R_{\mu\nu}(g^L_{\mu\nu})$. Then an action integral is proposed :

$$S = \int d^4x\sqrt{-g_L}(M_L^2 R(g_L) - \Lambda_L) + \int d^4x\sqrt{-g_L}L(\phi_L, g_L)$$
$$+ \int d^4x\sqrt{-g_R}(M_R^2 R(g_R) - \Lambda_R) + \int d^4x\sqrt{-g_R}L(\phi_R, g_R) - \mu^4 \int d^4x(g_R g_L)^{\frac{1}{4}} V(g_L, g_R) \quad (1)$$

in that expression we find the Lagrangian densities

$$\sqrt{-g_L}(M_L^2 R(g_L) + L(\phi_L, g_L) - \Lambda_L)$$

and

$$\sqrt{-g_R}(M_R^2 R(g_R) + L(\phi_R, g_R) - \Lambda_R)$$

that reveal a generalization of the Lagrangian derivation technic whih is at the base of the Einstein equation construction. The quantities $d^4x\sqrt{-g_L}$ and $d^4x\sqrt{-g_R}$ represent two elementary hypervolumes defined in the two branes. In order to introduce an interaction term, they suggest to use kind of an average hypervolume $d^4x\sqrt{g_L g_R}$.

Variation calculus lead them to propose the following system of two coupled equations :

$$2M_L^2(R_{\mu\nu}(g^L) - \frac{1}{2}g^L_{\mu\nu}R(g^L)) + \Lambda_L g^L_{\mu\nu} = t^L_{\mu\nu} + T^L_{\mu\nu}$$
$$2M_R^2(R_{\mu\nu}(g^R) - \frac{1}{2}g^R_{\mu\nu}R(g^R)) + \Lambda_R g^R_{\mu\nu} = t^R_{\mu\nu} + T^R_{\mu\nu} \quad (2)$$

The quantities $R(g_L), R(g_R)$ are two Ricci scalars. The terms $T^L_{\mu\nu}, T^R_{\mu\nu}$ are the left and right matter tensor fields. The tensors $t^L_{\mu\nu}, t^R_{\mu\nu}$ are supposed to be terms describing the interaction between the two branes. From those two metrics we build the covariant derivation operators $\nabla^L_\nu, \nabla^R_\nu$ from which Bianchi identities are supposed to be expressed. The article does not describe the construction of the tensor fields $t^L_{\mu\nu}, t^R_{\mu\nu}$ and their theoretical progress stops there.

A more elaborated work is the one of the researcher Hossenfelder ([15], [16], [17]). The two populations are associated to metrics designated by the letters g and h. Le matter tensor fields are designated by the letters ϕ and ψ . She proposes the following action :

$$S = \int d^4x\sqrt{-g}({}^{(g)}\frac{R}{8\pi G} + L(\psi)) + \int d^4x\sqrt{-h}P_{\underline{h}}(\underline{L}(\underline{\phi}))$$
$$+ \int d^4x\sqrt{-\underline{h}}({}^{(\underline{h})}\frac{R}{8\pi G} + \underline{L}(\underline{\phi})) + \int d^4x\sqrt{-\underline{g}}P_g(L(\psi)) \quad (3)$$

There are four Lagrangian densities in the integral.
The terms $\sqrt{-g}({}^{(g)}\frac{R}{8\pi G}+L(\psi))$ and $\sqrt{-\underline{h}}({}^{(\underline{h})}\frac{R}{8\pi G}+\underline{L}(\underline{\phi}))$ are the generalization of the General Relativity Lagrangian densities. The terms $\sqrt{-h}P_{\underline{h}}(\underline{L}(\underline{\phi}))$ and $\sqrt{-\underline{g}}P_g(L(\psi))$ characterize the interaction of the the two entities, one on the other. Those terms are built from the quadridimensional hypervolumes $d^4x\sqrt{-h}$ and $d^4x\sqrt{-\underline{g}}$.
From this action the author produces a system of two equations:

$${}^{(g)}R_{\kappa\nu} - \frac{1}{2}g^{(g)}_{\kappa\nu}R = T_{\kappa\nu} - (\underline{V}a^\nu_\nu)\sqrt{\frac{h}{g}}T_{\nu\kappa}$$
$${}^{(h)}R_{\nu\kappa} - \frac{1}{2}h^{(h)}_{\nu\kappa}R = -[(Wa^\kappa_\kappa a^\nu_\nu)\sqrt{\frac{g}{\underline{h}}}T_{\kappa\nu} - \underline{T}_{\nu\kappa}] \quad (4)$$

The underline indicates that two different coordinate systems are $\{x^0, x^1, x^2, x^3\}$ and $\{\underline{x}^0, \underline{x}^1, \underline{x}^2, \underline{x}^3\}$ the line elements can be written :

$$ds^{(g)2} = g_{\kappa\nu}dx^\kappa dx^\nu \ ; \ ds^{(h)2} = g_{\underline{\kappa\nu}}d\underline{x}^\kappa d\underline{x}^\nu$$
$$g_{\kappa\nu} = g_{\kappa\nu}(x^\kappa, x^\nu) \ ; \ \underline{h}_{\kappa\nu} = h_{\kappa\nu}(\underline{x}^\kappa, \underline{x}^\nu) \quad (5)$$

The map a , introduces a link between the two metrics that the author define by:

$$g_{\tau\lambda} = a^{\nu}_{\tau} a^{\kappa}_{\lambda} h_{\nu\kappa} \ ; \ g_{\underline{\tau\lambda}} = a^{\nu}_{\underline{\tau}} a^{\underline{\kappa}}_{\underline{\lambda}} h_{\underline{\nu}\kappa} \tag{6}$$

depending whether the application is defined in one coordinate system or the other. The search of a solution implies to define that application. In the article [15] the author assumes, *for sake of symmetry*, that one of the two population is a copy of the other, which hence make it physically less sound. The author focuses on giving an interpretation to the significance of the cosmological constant, the fact that its value is small. She mainly considers that the interaction between the two classes of matter is of relative importance, *due to the weakness of the force of gravity.*

4 Differential geometry in a dimensionless context

Let us consider the Einstein equation without its cosmological constant:

$$R^{\nu}_{\mu} - \frac{1}{2} R \delta^{\nu}_{\mu} = \chi T^{\nu}_{\mu} \tag{7}$$

In which the Einstein constant takes the form:

$$\chi = -\frac{8\pi G}{c^2} \tag{8}$$

The dimension of the the gravitation constant is $G \sim \frac{L^3 T^{-2}}{M}$, the one of the tensor T^{ν}_{μ} in this form is ML^{-3} and the dimension of the Ricci tensor is $R \sim L^{-2}$.
We write the metric

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{9}$$

where gaussian coordinates $\{x^0, x^1, x^2, x^3\}$ are lengths.
$\{x^1, x^2, x^3\}$ are then space coordinates and x^0 is a time-marker classically identified to ct.
The s variable is also a length. Divided by the speed of light c it becomes the proper time. Therefore $g_{\mu\nu}$ numbers do not have any dimension.
We usually write the FLRW in the form:

$$ds^2 = dx^{0^2} - a^2 [\frac{du^2}{1 - ku^2} + u^2 d\theta^2 + u^2 sin^2\theta \ d\phi^2] \tag{10}$$

Let us divide the terms by a characteristic length L by setting:

$$\sigma = \frac{s}{L}, \xi^0 = \frac{x^0}{L}, \alpha = \frac{a}{L}$$

the FLRW metric is then written, in a dimensionless form:

$$d\sigma^2 = d{\xi^0}^2 - \alpha^2[\frac{du^2}{1-ku^2} + u^2 d\theta^2 + u^2 sin^2\theta \; d\phi^2] \tag{11}$$

The choice in the dimension conferred to s and to other variables determines the dimension of the metric coefficients. With our choice, those are numbers. In order to indicate that our metric coefficients are in a dimensionless form, we will use greek letters and replace $g_{\mu\nu}$ by $\bar{\omega}_{\mu\nu}$. The metric can then be written:

$$\bar{\omega}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{\alpha^2}{1-ku^2} & 0 & 0 \\ 0 & 0 & -\alpha^2 u^2 & 0 \\ 0 & 0 & 0 & -\alpha^2 u^2 sin^2\theta \end{pmatrix} \tag{12}$$

where all coefficients are dimensionless numbers . We can therefore build dimensionless Ricci tensor Γ_μ^ν and Ricci scalar Γ . We will also write the matter tensor in a dimensionless way, here in a mixed form:

$$\Xi_\mu^\nu = \begin{pmatrix} \omega & 0 & 0 & 0 \\ 0 & -\pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & -\pi \end{pmatrix} \tag{13}$$

Hence the Einstein equation would write in a mixed dimensionless form:

$$\Gamma_\mu^\nu - \frac{1}{2}\Gamma\delta_\mu^\nu = 8\pi\Xi_\mu^\nu \tag{14}$$

5 The dimensionless bimetric approach

We shall start from a manifold M4 equipped with a dimensionless coordinate system (simples numbers designated by Greek letters) :

$$\{\xi^0, \xi^1, \xi^2, \xi^3\} \tag{15}$$

or, in polar form

$$\{\xi^0, u, \theta, \phi\} \tag{16}$$

and two Riemanian metrics $\bar{\omega}^{(+)}_{\mu\nu}$, $\bar{\omega}^{(-)}_{\mu\nu}$ with signature (+ - - -).
We now have to build an action $S(\bar{\omega}^{(+)}_{\mu\nu}, \bar{\omega}^{(-)}_{\mu\nu})$, knowing that its variation will be built from the variations of both metrics and that those variations will be linked

$$\delta\bar{\omega}^{(+)}_{\mu\nu}, \delta\bar{\omega}^{(+)}_{\mu\nu} \to \delta S \tag{17}$$

The two metrics belong to the functional space of Riemanian metrics with signature (+ - - -). We link those variations by the following simple relation:

$$\delta\bar{\omega}^{(+)}_{\mu\nu} = -\delta\bar{\omega}^{(-)}_{\mu\nu} \tag{18}$$

This will be motivated later, on the ground of linearized solutions coming from coupled field equation resulting from the lagrangian derivation under those symmetry constraints.
Let us recall that we want to build a system of coupled field equations satisfying to equation identities coming from Bianchi identities and allowing to produce solutions that can be written in the form of a couple of metrics $(\bar{\omega}^{(+)}_{\mu\nu}, \bar{\omega}^{(-)}_{\mu\nu})$, linearized and corresponding to the following symmetries :

- Stationary linearized solutions with spherical symmetry of the homogeneous system (in vacuum)
- Stationary linearized solutions with spherical symmetry of the inhomogeneous system (in the presences of masses)
- Stationary linearized solutions with axial symmetry
- Non-stationary linearized solutions with isotropy and homogeneity

Relation (18) will be justified a-posteriori.
We are writing the action:

$$\Sigma = \int_{D4} [(\Gamma^{(+)} - 2\lambda^{(+)})\sqrt{-\bar{\omega}^{(+)}} + 2\hat{\lambda}^{(-)}\sqrt{-\bar{\omega}^{(-)}} + (\Gamma^{(-)} + 2\lambda^{(-)})\sqrt{-\bar{\omega}^{(-)}} - 2\hat{\lambda}^{(+)}\sqrt{-\bar{\omega}^{(+)}}] d^4\xi \tag{19}$$

We have

$$\delta \int_{D4} [\Gamma^{(+)}\sqrt{-\bar{\omega}^{(+)}}] d^4\xi = \int_{D4} [\Gamma^{(+)}_{\mu\nu} - \frac{1}{2}\Gamma^{(+)}\bar{\omega}^{(+)}_{\mu\nu}]\sqrt{-\bar{\omega}^{(+)}}\delta\bar{\omega}^{(+)\mu\nu} d^4\xi \tag{20}$$

$$\delta\int_{D4}[\Gamma^{(-)}\sqrt{-\bar\omega^{(-)}}]d^4\xi=\int_{D4}[\Gamma^{(-)}_{\mu\nu}-\frac{1}{2}\Gamma^{(-)}\bar\omega^{(-)}_{\mu\nu}]\sqrt{-\bar\omega^{(-)}}\delta\bar\omega^{(-)\mu\nu}d^4\xi \tag{21}$$

$$\delta\int_{D4}[\lambda^{(+)}\sqrt{-\bar\omega^{(+)}}]d^4\xi=\int_{D4}\frac{1}{2}\Xi^{(+)}_{\mu\nu}\sqrt{-\bar\omega^{(+)}}\delta\bar\omega^{(+)\mu\nu}d^4\xi \tag{22}$$

$$\delta\int_{D4}[\lambda^{(-)}\sqrt{-\bar\omega^{(-)}}]d^4\xi=\int_{D4}\frac{1}{2}\Xi^{(-)}_{\mu\nu}\sqrt{-\bar\omega^{(-)}}\delta\bar\omega^{(-)\mu\nu}d^4\xi \tag{23}$$

Taking (18) into account we can write:

$$\delta\int_{D4}[\hat\lambda^{(+)}\sqrt{-\bar\omega^{(+)}}]d^4\xi=-\int_{D4}\frac{1}{2}\hat\Xi^{(+)}_{\mu\nu}\sqrt{\frac{\bar\omega^{(+)}}{\bar\omega^{(-)}}}\sqrt{-\bar\omega^{(-)}}\delta\bar\omega^{(-)\mu\nu}d^4\xi \tag{24}$$

$$\delta\int_{D4}[\hat\lambda^{(-)}\sqrt{-\bar\omega^{(-)}}]d^4\xi=-\int_{D4}\frac{1}{2}\hat\Xi^{(-)}_{\mu\nu}\sqrt{\frac{\bar\omega^{(-)}}{\bar\omega^{(+)}}}\sqrt{-\bar\omega^{(+)}}\delta\bar\omega^{(+)\mu\nu}d^4\xi \tag{25}$$

We get the coupled field equations:

$$\Gamma^{(+)}_{\mu\nu}-\frac{1}{2}\Gamma^{(+)}\bar\omega^{(+)}_{\mu\nu}=-[\Xi^{(+)}_{\mu\nu}+\sqrt{\frac{\bar\omega^{(-)}}{\bar\omega^{(+)}}}\hat\Xi^{(-)}_{\mu\nu}] \tag{26}$$

$$\Gamma^{(-)}_{\mu\nu}-\frac{1}{2}\Gamma^{(-)}\bar\omega^{(-)}_{\mu\nu}=[\Xi^{(-)}_{\mu\nu}+\sqrt{\frac{\bar\omega^{(+)}}{\bar\omega^{(-)}}}\hat\Xi^{(+)}_{\mu\nu}] \tag{27}$$

which are the Janus equations, written in dimensionless form. We can then look for FLRW solutions, written in dimensionless form:

$$d\sigma^{(+)^2}=d\xi^{(0)^2}-\alpha^{(+)^2}[\frac{du^2}{1-k^{(+)}u^2}+u^2d\theta^2+u^2sin^2\theta\ d\phi^2] \tag{28}$$

$$d\sigma^{(-)^2}=d\xi^{(0)^2}-\alpha^{(-)^2}[\frac{du^2}{1-k^{(-)}u^2}+u^2d\theta^2+u^2sin^2\theta\ d\phi^2] \tag{29}$$

whose determinants are

$$\bar\omega^{(+)}=-\frac{\alpha^{(+)^6}u^6}{1-k^{(+)}u^2}\ ,\ \bar\omega^{(-)}=\frac{\alpha^{(-)^6}u^6}{1-k^{(-)}u^2} \tag{30}$$

By limiting ourself (this will be justified in the sequel) to hyperbolic solutions:

$$k^{(+)}=k^{(-)}=-1 \tag{31}$$

Hence

$$\sqrt{\frac{\bar\omega^{(-)}}{\bar\omega^{(+)}}}=(\frac{\alpha^{(-)}}{\alpha^{(+)}})^3 \tag{32}$$

and we obtain

$$\Gamma^{(+)}_{\mu\nu} - \frac{1}{2}\Gamma^{(+)}\bar{\omega}^{(+)}_{\mu\nu} = -[\Xi^{(+)}_{\mu\nu} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\hat{\Xi}^{(-)}_{\mu\nu}] \tag{33}$$

$$\Gamma^{(-)}_{\mu\nu} - \frac{1}{2}\Gamma^{(-)}\bar{\omega}^{(-)}_{\mu\nu} = [\Xi^{(-)}_{\mu\nu} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\hat{\Xi}^{(+)}_{\mu\nu}] \tag{34}$$

those are the equations used in 2014's article [18], written in dimensionless form, that allowed to bring out the acceleration of the cosmic expansion. In that article the form of the second members have been determined to ensure the existence of the solution (in virtue of mass conservation) of the system of differential equations determining the solutions $\alpha^{(+)}(\xi^0)$ and $\alpha^{(-)}(\xi^0)$.

6 Induced Geometry

The geometry of the entities are described by the two metrics $\bar{\omega}^{(+)}_{\mu\nu}$and $\bar{\omega}^{(-)}_{\mu\nu}$. In the right hand side of the two equations (33) and (34) are some terms $\hat{\Xi}^{(+)}_{\mu\nu}$ and $\hat{\Xi}^{(-)}_{\mu\nu}$ that can be considered as the sources of induced geometries, by the matter of one species on the geometry of the other. As show in 2019's article [19], it is the satisfaction of the Bianchi identities that determines their form. By considering the (constant) Lorentz matrix

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{35}$$

We have

$$\hat{\Xi}^{(+)}_{\mu\nu} = \eta_{\mu\nu}\Xi^{(+)}_{\mu\nu} \; ; \; \hat{\Xi}^{(-)}_{\mu\nu} = \eta_{\mu\nu}\Xi^{(-)}_{\mu\nu} \tag{36}$$

or, more explicitely

$$\Xi^{\nu\,(+)}_{\mu} = \begin{pmatrix} \omega^{(+)} & 0 & 0 & 0 \\ 0 & -\pi^{(+)} & 0 & 0 \\ 0 & 0 & -\pi^{(+)} & 0 \\ 0 & 0 & 0 & -\pi^{(+)} \end{pmatrix} \; ; \; \Xi^{\nu\,(-)}_{\mu} = \begin{pmatrix} \omega^{(-)} & 0 & 0 & 0 \\ 0 & -\pi^{(-)} & 0 & 0 \\ 0 & 0 & -\pi^{(-)} & 0 \\ 0 & 0 & 0 & -\pi^{(-)} \end{pmatrix} \tag{37}$$

$$\hat{\Xi}_\mu^{\nu(+)} = \begin{pmatrix} \omega^{(+)} & 0 & 0 & 0 \\ 0 & \pi^{(+)} & 0 & 0 \\ 0 & 0 & \pi^{(+)} & 0 \\ 0 & 0 & 0 & \pi^{(+)} \end{pmatrix} \; ; \; \hat{\Xi}_\mu^{\nu(-)} = \begin{pmatrix} \omega^{(-)} & 0 & 0 & 0 \\ 0 & \pi^{(-)} & 0 & 0 \\ 0 & 0 & \pi^{(-)} & 0 \\ 0 & 0 & 0 & \pi^{(-)} \end{pmatrix} \tag{38}$$

Let us review the linearized solutions corresponding to the different symmetries considered earlier. In order to do that let us write the field equations in their mixed form:

$$\Gamma_\mu^{(+)\nu} - \frac{1}{2}\Gamma^{(+)}\delta_\mu^\nu = -[\Xi_\mu^{(+)\nu} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\hat{\Xi}_\mu^{(+)\nu}] \tag{39}$$

$$\Gamma_\mu^{(-)\nu} - \frac{1}{2}\Gamma^{(-)}\delta_\mu^\nu = [\hat{\Xi}_\mu^{(-)\nu} + (\frac{\alpha^{(+)}}{\alpha^{(-)}})^3\Xi_\mu^{(-)\nu}] \tag{40}$$

7 Solution in the vacuum with spherical symmetry

The equations are

$$\Gamma_\mu^{(+)\nu} - \frac{1}{2}\Gamma^{(+)}\delta_\mu^\nu = 0 \tag{41}$$

$$\Gamma_\mu^{(-)\nu} - \frac{1}{2}\Gamma^{(-)}\delta_\mu^\nu = 0 \tag{42}$$

whose solution is the classical exterior Schwarzschild solution:

$$d\sigma^{(+)^2} = (1 - \frac{2\mu}{u})d\xi^{0^2} - \frac{du^2}{1 + \frac{2\mu}{u}} - u^2(d\theta^2 + sin^2\theta \; d\phi^2) \tag{43}$$

$$d\sigma^{(-)^2} = (1 + \frac{2\mu}{u})d\xi^{0^2} - \frac{du^2}{1 - \frac{2\mu}{u}} - u^2(d\theta^2 + sin^2\theta \; d\phi^2) \tag{44}$$

We consider them in their linearized form:

$$d\sigma^{(+)^2} = (1 - \frac{2\mu}{u})d\xi^{0^2} - (1 + \frac{2\mu}{u})du^2 - u^2(d\theta^2 + sin^2\theta \; d\phi^2) \tag{45}$$

$$d\sigma^{(-)^2} = (1 + \frac{2\mu}{u})d\xi^{0^2} - (1 - \frac{2\mu}{u})du^2 - u^2(d\theta^2 + sin^2\theta \; d\phi^2) \tag{46}$$

Those metrics belong to the sub space of spherically symmetric Riemanian metrics of signature (+ - - -), solution of the system of the two field equations without right hand side. Those solutions depend in a single parameter and the satisfy relation (18) :

$$\delta\bar{\omega}^{(+)}_{\mu\nu} = \delta\mu \begin{pmatrix} -\frac{2}{u} & 0 & 0 & 0 \\ 0 & -\frac{2}{u} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \; ; \; \delta\bar{\omega}^{(-)}_{\mu\nu} = \delta\mu \begin{pmatrix} \frac{2}{u} & 0 & 0 & 0 \\ 0 & \frac{2}{u} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (47)$$

We might argue that the relation (18) is not *covariant*, i.e. not independent of the choice of coordinates. Let us remind here that in 2017 this *covariance principle* has been questioned subsequently through the presentation of a large scale map of the universe [20] in a huge cube of 1.5 billion light years extension with our galaxy in the center. The kinetic map has been obtained by subtracting the Hubble field hence bringing out the proper motion of the galaxies with respect to the space itself.
Moreover, the observers determined the dynamic parameters of an observing system that would annihilate the anisotropy of the CMB. It turned out that an observer moving at a speed of 651 km/s relative to our galaxy, pointing towards the geometric centre of the Great Repeller, would capture a totally isotropic image of the CMB. We can therefore consider that this observer would be *motionless in relation to the universe itself*, animated at zero speed. The flow of time would therefore be maximal for this observer. This corresponds to a resurgence of Mach's principle, linking space to matter, identifying container and content.
According to this principle, by considering a concentration of matter centered on a point, this point can then claim to play the role of the origin of a radial coordinate u.

8 Spherically symmetrical solution in material of constant density

We find the solution of the system of two equations with a second member, which is associated with so-called *inner Schwarzschild metrics.* These solutions depend only on one parameter, the density ω (expressed in dimensionless form).

In a non-adimensional form, inside a sphere of radius r_s :

$$ds^{(+)^2} = [\frac{3}{2}\sqrt{1-\frac{r_s^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{\hat{R}^2}}]^2 c^2 dt^2 - \frac{dr^2}{1-\frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 sin^2\theta \, d\phi^2 \quad (48)$$

$$ds^{(-)^2} = [\frac{3}{2}\sqrt{1+\frac{r_s^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1+\frac{r^2}{\hat{R}^2}}]^2 c^2 dt^2 - \frac{dr^2}{1+\frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 sin^2\theta \, d\phi^2 \quad (49)$$

with

$$\hat{R}^2 = \frac{8\pi G|\rho|}{3c^2} \quad (50)$$

But in dimensionless form with a dimensionless density ω we get :

$$d\sigma^{(+)^2} = [\frac{3}{2}\sqrt{1-\omega u_s^2} - \frac{1}{2}\sqrt{1-\omega u^2}]^2 d\xi^{(0)^2} - \frac{du^2}{1-\omega u^2} - u^2 d\theta^2 - u^2 sin^2\theta \, d\phi^2 \quad (51)$$

$$d\sigma^{(-)^2} = [\frac{3}{2}\sqrt{1+\omega u_s^2} - \frac{1}{2}\sqrt{1+\omega u^2}]^2 d\xi^{(0)^2} - \frac{du^2}{1+\omega u^2} - u^2 d\theta^2 - u^2 sin^2\theta \, d\phi^2 \quad (52)$$

These metrics depend only on the single parameter ω . By linearizing and differentiating these metrics also satisfy the relationship (26).

9 Linearized metric coupled stationary axisymmetric solutions

(after Lense and Thiring)

$$ds^{(+)^2} = (1-\frac{2m}{r})c^2dt^2 - (1+\frac{2m}{r})dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) + \frac{4GJ}{c^2 r} sin^2\theta \, d\phi \, dt \quad (53)$$

$$ds^{(-)^2} = (1+\frac{2m}{r})c^2dt^2 - (1-\frac{2m}{r})dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) - \frac{4GJ}{c^2 r} sin^2\theta \, d\phi \, dt \quad (54)$$

The couple of metrics depends on two parameters, m and J (angular momentum). Switching from one to the other is done by changing both m into - m and J into - J.

In their dimensionless form

$$d\sigma^{(+)^2} = (1-\frac{\mu}{u})d\xi^{(0)^2} - (1+\frac{\mu}{u})du^2 - u^2(d\theta^2 + sin^2\theta \, d\phi^2) + \frac{\beta}{u} sin^2\theta \, d\phi \, d\xi^{(0)} \quad (55)$$

$$d\sigma^{(-)^2} = (1+\frac{\mu}{u})d\xi^{(0)^2} - (1-\frac{\mu}{u})du^2 - u^2(d\theta^2 + sin^2\theta\; d\phi^2) - \frac{\beta}{u}sin^2\theta\; d\phi\; d\xi^{(0)} \tag{56}$$

Here again formula (26) is satisfied.

10 Conjugated, stationary, homogeneous, isotropic and linearized solutions

FLRW metrics in their dimensionless form are written as :

$$d\sigma^{(+)^2} = d\xi^{(0)^2} - \alpha^{(+)^2}[\frac{du^2}{1-k^{(+)}u^2} + u^2(d\theta^2 + sin^2\theta\; d\phi^2)] \tag{57}$$

$$d\sigma^{(-)^2} = d\xi^{(0)^2} - \alpha^{(-)^2}[\frac{du^2}{1-k^{(-)}u^2} + u^2(d\theta^2 + sin^2\theta\; d\phi^2)] \tag{58}$$

The introduction of these metrics in the system of field equations, modulo the hypothesis of negligible pressure (matter dominated era) leads to the system of differential equations:

$$\frac{\alpha^{(+)'2}}{\alpha^{(+)^2}} + \frac{k^{(+)}}{\alpha^{(+)^2}} = -\frac{1}{3}[\omega^{(+)} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\omega^{(-)}] \tag{59}$$

$$2\frac{\alpha^{(+)''}}{\alpha^{(+)}} + \frac{\alpha^{(+)'2}}{\alpha^{(+)^2}} + \frac{k^{(+)}}{\alpha^{(+)^2}} = 0 \tag{60}$$

$$\frac{\alpha^{(-)'2}}{\alpha^{(-)^2}} + \frac{k^{(-)}}{\alpha^{(-)^2}} = -\frac{1}{3}[\omega^{(-)} + (\frac{\alpha^{(+)}}{\alpha^{(-)}})^3\omega^{(+)}] \tag{61}$$

$$2\frac{\alpha^{(-)''}}{\alpha^{(-)}} + \frac{\alpha^{(-)'2}}{\alpha^{(-)^2}} + \frac{k^{(-)}}{\alpha^{(-)^2}} = 0 \tag{62}$$

which are analogous to the Einstein classic equations of general relativity. The condition of compatibility between these equations leads to the conservation of the mass :

$$\omega^{(+)}\alpha^{(+)^3} + \omega^{(-)}\alpha^{(-)^3} = 3\mu = cst \tag{63}$$

Such constant can be positive, negative or zero. We also get $k^{(+)} = k^{(-)} = -1$. The resulting differential equations are :

$$2\frac{\alpha^{(+)''}}{\alpha^{(+)}} = -[\omega^{(+)} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\omega^{(-)}] \tag{64}$$

$$2\frac{\alpha^{(-)''}}{\alpha^{(-)}} = [\omega^{(-)} + (\frac{\alpha^{(+)}}{\alpha^{(-)}})^3\omega^{(+)}] \tag{65}$$

Then we have the two equations

$$\alpha^{(+)''} = -\frac{\mu}{\alpha^{(+)^2}} \; ; \; \alpha^{(-)''} = \frac{\mu}{\alpha^{(-)^2}} \tag{66}$$

The observational evidence of the acceleration of cosmic expansion ([3], [4], [5]) indicates that the dynamics of the set is dominated by the negative mass content, i.e. $-\mu < 0$. We then have $\alpha^{(+)''} > 0, \alpha^{(-)''} < 0$.
These second derivatives tend towards zero when the dimensionless time marker $\xi^{(0)}$ tends towards infinity. The equation of the asymptote corresponds to the line:

$$\alpha = \xi^{(0)} \tag{67}$$

The goal of the Janus model is to take account of available observational data. As regards cosmic evolution (700 type Ia supernovae), these data relate to relatively low redshift values, and thus to a relatively recent past. This then corresponds to the linearization of the solutions, in the form of perturbation terms with respect to the asymptotic form

$$\alpha^{(+)} = \xi^{(0)} + \epsilon\zeta^{(+)} \; , \; \alpha^{(-)} = \xi^{(0)} + \epsilon\zeta^{(-)} \tag{68}$$

with schematical evolution curves like this (see Fig 1)
We get

$$\alpha^{(+)''} = \epsilon\zeta^{(+)''} \simeq \frac{|\mu|}{2\xi^{(0)^2}} \; ; \; \alpha^{(-)''} = \epsilon\zeta^{(-)''} \simeq -\frac{|\mu|}{2\xi^{(0)^2}} \tag{69}$$

$$\alpha^{(+)'} = \epsilon\zeta^{(+)'} \simeq 1 - \frac{|\mu|}{2\xi^{(0)}} \; ; \; \alpha^{(-)'} = \epsilon\zeta^{(-)'} \simeq 1 + \frac{|\mu|}{2\xi^{(0)}} \tag{70}$$

The comparison of the model with observational data has already been done and published [21] (see Fig. 2)
The purpose of this work is not to revisit this point but to justify the relationship (18).
The metrics in question are part of the sets of metrics describing an unsteady situation associated with isotropic and homogeneous media, in their linearized form (confronted with observations).
These metrics depend on only one parameter μ , the variation $\delta\mu$ of which determines the variations of $\delta\bar{\omega}^{(+)}_{\mu\nu}$ and $\delta\bar{\omega}^{(-)}_{\mu\nu}$. We have

$$\delta\bar{\omega}^{(+)}_{\mu\nu} \propto 2\alpha^{(+)}\delta\alpha^{(+)} \propto 2\xi^{(0)}\delta\alpha^{(+)} \tag{71}$$

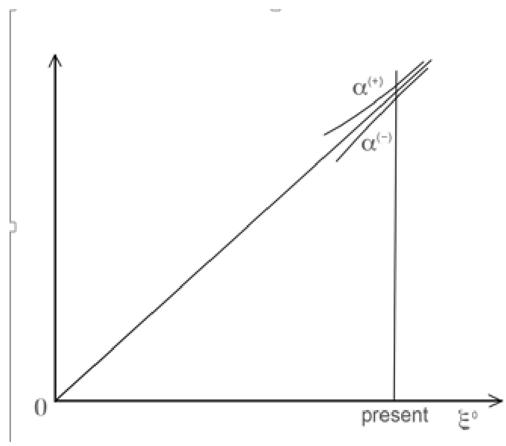

Figure 1: Schematical evolution of the spatial scale factor in the dimensionless linear representation

$$\delta\bar{\omega}^{(-)}_{\mu\nu} \propto 2\alpha^{(-)}\delta\alpha^{(-)} \propto 2\xi^{(0)}\delta\alpha^{(-)} \tag{72}$$

to the second order.
The variations being opposite, the relationship (18) is well satisfied. This study completes the examination of the mathematical basis of the model. The system of coupled field equations results from an action and satisfies, asymptotically, the conditions of zero divergence. The ratio of the dimensionless scale factors remains close to unity. How is it then possible that they differ so much in their dimensional form :

$$\frac{a^{(+)}}{a^{(-)}} \simeq 100 \tag{73}$$

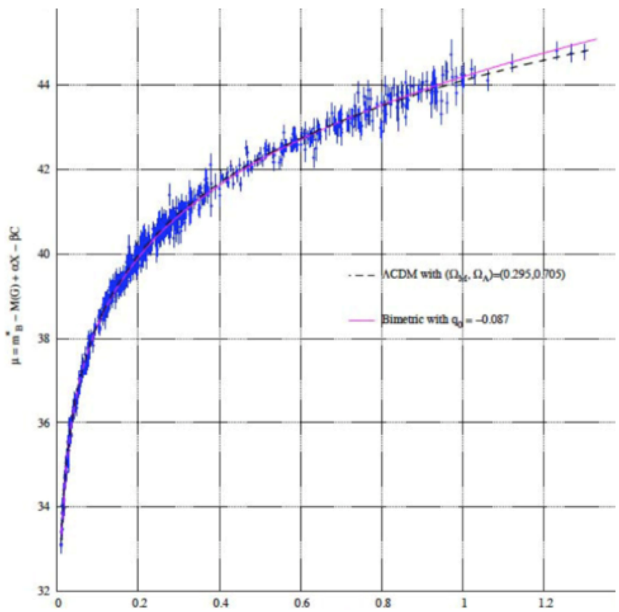

Figure 2: Dashed line: the model ΛCDM. Red line: the Janus model

11 Two different ways of reading physical phenomena

In his article [15] S.Hossenfelder insists on the fact that in his bimetric model there are two different observers, designated by the letters g and h .
It is the same in the Janus model. For us, who are made up of positive mass, it all comes down to making all quantities *dimensional* using the quantities

$$\{c^{(+)}, G^{(+)}, h^{(+)}, e^{(+)}, m^{(+)}, \mu_0^{(+)}, L^{(+)}, T^{(+)}\} \tag{74}$$

In this set we find

- The speed of light $c^{(+)}$
- The constant of gravity $G^{(+)}$
- The Planck's constant $h^{(+)}$
- The unit electric charge $e^{(+)}$

- The "elementary mass" $m^{(+)}$

- The magnetic permeability of vacuum $\mu_0^{(+)}$

- A space scale factor $L^{(+)}$

- A time scale factor $T^{(+)}$

These magnitudes are not totally independent since we assume that

$$T^{(+)} = \frac{L^{(+)}}{c^{(+)}} \tag{75}$$

This reduces the number of these quantities to seven. We will transform the metric into its dimensional form by multiplying left and right sides by $L^{(+)}$, which gives

$$L^{(+)^2} d\sigma^{(+)^2} = L^{(+)^2} d\xi^{(0)^2} - L^{(+)^2} \sum_{space} \bar{\omega}_{\mu\nu}^{(+)} d\xi^{\mu} d\xi^{\nu} \tag{76}$$

The proper time $t_{pr}^{(+)}$ and the time coordinate $t^{(+)}$ are displayed according to

$$t_{pr}^{(+)} = \frac{L^{(+)}\sigma^{(+)}}{c^{(+)}} \; ; \; t^{(+)} = \frac{L^{(+)}\xi^{(0)}}{c^{(+)}} \tag{77}$$

same for space variables

$$x^{(+)\mu} = L^{(+)}\xi^{\mu} \tag{78}$$

By the way can replace $\bar{\omega}_{\mu\nu}^{(+)}$ by $g_{\mu\nu}^{(+)}$, insofar as these coefficients of the metric are numbers. It comes, always by separating the temporal part and the spatial part

$$c^{(+)^2} dt_{pr}^{(+)^2} = c^{(+)^2} dt^{(+)^2} - \sum_{space} g_{\mu\nu}^{(+)} dx^{(+)\mu} dx^{(+)\nu} \tag{79}$$

In vacuum,

$$c^{(+)^2} dt_{pr}^{(+)^2} = c^{(+)^2} dt^{(+)^2} - \sum_{space} dx^{(+)\mu} dx^{(+)\nu} \tag{80}$$

If, in the manifold, where the points are marked by the coordinates $\{\xi^0, \xi^1, \xi^2, \xi^3\}$, we consider two distinct points A and B , with coordinates

$$\{\xi^0, \xi_A^1, 0, 0\} \; , \; \{\xi^0, \xi_B^1, 0, 0\}$$

they will be separated by different distances

$$D_{AB}^{(+)} = L^{(+)}(\xi_B^1 - \xi_A^1)\ ,\ D_{AB}^{(-)} = L^{(-)}(\xi_B^1 - \xi_A^1) \tag{81}$$

depending on whether this distance will be taken into account by an observer (+) or an observer (-).
The *histories* $\alpha^{(+)}(\xi^0)$ and $\alpha^{(-)}(\xi^0)$ will be *read* in a different way, both spatially and temporally according to the scales used in (abscissa/ordinate).

12 Poisson's equation and the action-reaction principle

This is known to derive from the linearization of the field equation. We will have two readings of the gravitational potential $\psi^{(+)}$ and $\psi^{(-)}$ with two different gravitational fields $-\frac{\partial\psi^{(+)}}{\partial x^{(+)}}$ and $-\frac{\partial\psi^{(-)}}{\partial x^{(-)}}$.But we can consider a gravitational potential without dimension noted and a gravitational field also without dimension

$$-\frac{\partial\Phi}{\partial\xi}$$

Let's come to the establishment of the Poisson equation, as a linearization of a field equation. Let us take again the system of coupled field equations (33), (34) written in covariant form where we operate a double development into a series in the vicinity of a Lorentzian metric

$$\bar{\omega}_{\mu\nu}^{(+)} = \eta_{\mu\nu} + \epsilon\ \gamma_{\mu\nu}^{(+)}\ ;\ \bar{\omega}_{\mu\nu}^{(-)} = \eta_{\mu\nu} + \epsilon\ \gamma_{\mu\nu}^{(-)} \tag{82}$$

The calculation analogy [22] leads us to

$$\epsilon\ \sum_{i=1}^{3}\ \gamma_{00|ii}^{(+)} = -[\omega^{(+)} + (\frac{\alpha^{(-)}}{\alpha^{(+)}})^3\omega^{(-)}] \tag{83}$$

$$\epsilon\ \sum_{i=1}^{3}\ \gamma_{00|ii}^{(-)} = [\omega^{(-)} + (\frac{\alpha^{(+)}}{\alpha^{(-)}})^3\omega^{(+)}] \tag{84}$$

Here we exploit the fact that

$$\frac{\alpha^{(-)}}{\alpha^{(+)}} \simeq 1 \tag{85}$$

$$\epsilon \sum_{i=1}^{3} \gamma_{00|ii}^{(+)} = -[\omega^{(+)} + \omega^{(-)}] \tag{86}$$

$$\epsilon \sum_{i=1}^{3} \gamma_{00|ii}^{(-)} = [\omega^{(-)} + \omega^{(+)}] \tag{87}$$

Knowing that

$$\sum_{i=1}^{3} \gamma_{00|ii}^{(+)} = \frac{\partial^2 \gamma_{00}^{(+)}}{\partial {\xi^1}^2} + \frac{\partial^2 \gamma_{00}^{(+)}}{\partial {\xi^2}^2} + \frac{\partial^2 \gamma_{00}^{(+)}}{\partial {\xi^3}^2} \tag{88}$$

we introduce gravitational potentials according to

$$-\epsilon\gamma_{00}^{(+)} = \frac{\Phi^{(+)}}{4\pi} \ , \ -\epsilon\gamma_{00}^{(-)} = \frac{\Phi^{(-)}}{4\pi} \tag{89}$$

which leads us to Poisson's first equation

$$\frac{\partial^2 \Phi^{(+)}}{\partial {\xi^1}^2} + \frac{\partial^2 \Phi^{(+)}}{\partial {\xi^2}^2} + \frac{\partial^2 \Phi^{(+)}}{\partial {\xi^3}^2} = 4\pi[\omega^{(+)} + \omega^{(-)}] \tag{90}$$

By posing

$$\Delta = \frac{\partial^2}{\partial {\xi^1}^2} + \frac{\partial^2}{\partial {\xi^2}^2} + \frac{\partial^2}{\partial {\xi^3}^2} \tag{91}$$

we have two Poisson equations, written in dimensionless form

$$\Delta\Phi^{(+)} = 4\pi[\omega^{(+)} + \omega^{(-)}] \ ; \ \Delta\Phi^{(-)} = -4\pi[\omega^{(+)} + \omega^{(-)}] \tag{92}$$

In other words,

$$\Phi^{(+)} = -\Phi^{(-)} \tag{93}$$

Hence the gravity fields and the accelerations

$$\frac{\partial \Phi^{(+)}}{\partial \xi^i} = -\frac{\partial \Phi^{(-)}}{\partial \xi^i} \tag{94}$$

Placed in a given field of gravity two test particles of equal and opposite mass have equal and opposite reactions. This reflects the action-reaction principle. The latter will *project* itself into the two coordinate systems, and this principle of action-reaction will also be observed by *plus* or *minus* observers. We will write the projection of this Poisson equation in the positive mass system. To do this we multiply the two sides by $G^{(+)}M^{(+)}L^{(+)^{-3}}$. The first member will become

$$G^{(+)}M^{(+)}L^{(+)^{-3}}(\frac{\partial^2\Phi^{(+)}}{\partial\xi^{1^2}}+\frac{\partial^2\Phi^{(+)}}{\partial\xi^{2^2}}+\frac{\partial^2\Phi^{(+)}}{\partial\xi^{3^2}})$$

$$=\frac{G^{(+)}M^{(+)}}{L^{(+)}}(\frac{\partial^2\Phi^{(+)}}{\partial x^{1^2}}+\frac{\partial^2\Phi^{(+)}}{\partial x^{2^2}}+\frac{\partial^2\Phi^{(+)}}{\partial x^{3^2}})=\frac{G^{(+)}M^{(+)}}{L^{(+)}}\Delta\ \Phi^{(+)} \tag{95}$$

With

$$\frac{1}{L^{(+)^2}}\Delta=\Delta^{(+)}\ ,\ \psi^{(+)}=\frac{G^{(+)}M^{(+)}}{L^{(+)}}\Phi^{(+)} \tag{96}$$

and

$$\rho^{(+)}=\frac{M^{(+)}}{L^{(+)^3}}\omega^{(+)} \tag{97}$$

we get

$$\Delta^{(+)}\psi^{(+)}=4\pi G^{(+)}[\rho^{(+)}+\rho^{(-,+)}] \tag{98}$$

The density $\rho^{(-,+)}$ represents the *apparent negative mass* as it contributes to the apparent gravitational potential $\psi^{(+)}$. For an observer with negative mass this equation would become

$$\Delta^{(-)}\psi^{(-)}=-4\pi G^{(-)}[\rho^{(-)}+\rho^{(+,-)}] \tag{99}$$

The construction of the mechanism of joint gravitational instabilities; described for example by a positive mass observer will involve the Poisson equation *projected into this particular frame of reference*. In the study of the dynamics of cosmic expansion we have assumed that $|\omega^{(-)}| >> \omega^{(+)}$. This leads to

$$|\rho^{(-)}| >> \rho^{(+)} \tag{100}$$

which justifies the drafts of the numerical simulations carried out since 1995 [23] as well as those leading to the spiral structure [7].

13 Conclusion

We have placed the Janus cosmological model on the required mathematical basis, by locating in which domain this model is currently valid, i.e. for all relativistic observations where the comparison is made with linearized metrics, which excludes situations with high gravitational field. The Bianchi equations are then satisfied and the fundamental relationship $\delta g_{\mu\nu}^{(-)} = -\delta g_{\mu\nu}^{(+)}$ between the metrics, which is involved in the Lagrangian derivation of the system, is justified. For the moment we are not in a position to go further into the past, i.e. to describe the radiative era, where pressures can no longer be neglected. Beyond that it will remain to justify the hypothesis of deep asymmetry $|\omega^{(-)}| >> \omega^{(+)}$ from which arise the phenomenon of acceleration of expansion, the phenomenon of confinement of galaxies, the flatness of their rotation curves, the confinement of clusters, the intensity of observed gravitational (negative) lens phenomenon, the spiral structure, the large-scale structure, the Great Repeller phenomenon, the low magnitude of galaxies with high redshift, as presented in previously published papers . In a future paper we will extend to the two cosmic entities the hypothesis of a double evolution with systems of *variable constants* evoked in [23]. In this scheme we start from a totally symmetrical situation, which turns out to be unstable and leads to an exponential break in symmetry.

References

[1] A. Benoit-Lévy and G.Chardin : Introducing the Dirac-Milne universe. Astronomy and Astrophysics. Vol. 537 (january 2012) A 78

[2] Milne E.A. : 1933 Zeitschrift fur Astrophysik , 6 , 1

[3] Perlmutter, S., et al. 1999, ApJ, 517, 565

[4] Riess A. G. 2000, PASP, 112, 1284

[5] Schmidt, B. P., et al., 1998, Astrophys. J. 507, 46.

[6] J.Farnes : A unifying theory of dark energy ans dark matter : Negative mass and matter creation within a modified ΛCDM framework. Astronomy ans Astrophysics 2018, 620, A 92

[7] J.P.Petit, P.Midy and F.Landsheat : Twin matter against dark matter. Intern. Meet. on Astrophys. and Cosm. *Where is the matter ?*, Marseille 2001 june 25-29

[8] S.Bondarenko : Negative mass and Schwarzschild Spacetime in general relativity Modern Physics Letters A 2019 july Vol. 34, 11

[9] H.Socas-Navarro : Can a negative mass explain dark matter and dark energy ? Astronomy and Astrophysics 625 A5 2019

[10] A.Stepanian : The invalidity of negative mass description of the dark sector. Mod. Phys. Lett. A Vol.34, 35, juin 2019

[11]] H. Bondi: Negative mass in General Relativity : Negative mass in General Relativity. Rev. of Mod. Phys., Vol 29, 3, July 1957

[12] J.P.Petit : Nuovo Cimento B. The missing mass problem. 109, 697 (1994).

[13] Damour T. , Kogan I I. Effective Lagrangians and universality classes of nonlinear bigravity Phys. Rev. D 66 (2002) 104024. hep-th/0206042.

[14] Damour T. , Kogan I. I. , Papazoglou A. Non-linear bigravity and cosmic acceleration Phys. Rev. D 66 (2002) 104025. hep-th/0206044.

[15] S. Hossenfelder : A bimetric Theory with Exchange Symmetry. Phys. Rev. D78, 044015, 2008 and arXiv : 0807.2838v1 (gr-qc)17 july 2008

[16] S.Hossenfelder : Antigravitation. Phys Letters B Vol. 636, issue 2 , may 2006

[17] S.Hossenfelder : Static Field Solutions in Symmetric Gravity. arXiv : 1603.07075v2 (gr-qc) 30 sept 2016

[18] J.P.Petit, G.D'Agostini : Negative Mass hypothesis in cosmology and the nature of dark energy. Astrophysics And Space Science,. A 29, 145-182 (2014)

[19] J.P.Petit, G. D'Agostini, N.Debergh : Physical and mathematical consistency of the Janus Cosmological Model (JCM). Progress in Physics 2019 Vol.15 issue 1

[20] Y.Hoffman , D.Pomarède, R.B.Tully and H.Courtois : The Dipole Repeller, Anture Astronomy 1 , 0036 (2017) DOI 10.1038/s4 1550-016-0036

[21] G. DAgostini and J.P.Petit : Constraints on Janus Cosmological model from recent observations of supernovae type Ia, Astrophysics and Space Science, (2018), 363:139.https://doi.org/10.1007/s10509-018-3365-3

[22] R.Adler, M.Bazin and M.Schiffer : Introduction to General Relativity Mc Graw Hill Book 1965-1975, pages 346-347.

[23] J.P.Petit, Twin Universe Cosmology, Astrophys. and Sp. Science, 226, 273-307, 1995

Appendix

In the case of spherical symmetry the two metrics are written

$$ds^{(+)^2} = e^{\nu^{(+)}} dx^{0^2} - e^{\lambda^{(+)}} dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) \tag{A1}$$

$$ds^{(-)^2} = e^{\nu^{(-)}} dx^{0^2} - e^{\lambda^{(-)}} dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) \tag{A2}$$

In the following, in order to lighten the writing, we will pose

$$g^{(+)}_{\mu\nu} \equiv g_{\mu\nu} \; , \; g^{(-)}_{\mu\nu} \equiv \bar{g}_{\mu\nu}$$

$$R^{(+)}_{\mu\nu} \equiv R_{\mu\nu} \; , \; R^{(-)}_{\mu\nu} \equiv \bar{R}_{\mu\nu}$$

$$R^{(+)} \equiv R \; , \; R^{(-)} \equiv \bar{R}$$

$$E^{(+)}_{\mu\nu} \equiv E_{\mu\nu} \; , \; E^{(-)}_{\mu\nu} \equiv \bar{E}_{\mu\nu}$$

$$\rho^{(+)} \equiv \rho \; , \; \rho^{(-)} \equiv \bar{\rho}$$

$$\nu^{(+)} \equiv \nu \; , \; \nu^{(-)} \equiv \bar{\nu}$$

$$\lambda^{(+)} \equiv \lambda \; , \; \lambda^{(-)} \equiv \bar{\lambda}$$

We will perform the calculations starting from an expression of the field equations presented in mixed form

$$E^\nu_\mu = R^\nu_\mu - \frac{1}{2} R g^\nu_\mu = \chi[T^\nu_\mu + \sqrt{\frac{g^{(-)}}{g^{(+)}}} \hat{T}^{(-)\nu}_\mu] \tag{A3}$$

$$\bar{E}^\nu_\mu = \bar{R}^\nu_\mu - \frac{1}{2} \bar{R} \bar{g}^\nu_\mu = -\chi[T^{(-)\nu}_\mu + \sqrt{\frac{g^{(-)}}{g^{(+)}}} \hat{T}^\nu_\mu] \tag{A4}$$

Let us consider the calculation of the geometry inside an object consisting of a sphere filled with a positive mass. The equations become

$$E^\nu_\mu = R^\nu_\mu - \frac{1}{2} R g^\nu_\mu = \chi T^\nu_\mu \tag{A5}$$

$$\bar{E}^{\nu}_{\mu} = \bar{R}^{\nu}_{\mu} - \frac{1}{2}\bar{R}\bar{g}^{\nu}_{\mu} = -\chi\sqrt{\frac{\bar{g}}{g}}\hat{T}^{(+)\nu}_{\mu} \qquad (A6)$$

The first equation can then be identified with Einstein's equation without cosmological constants.
The second equation accônts for an *induced geometry effect* (on the geodesics of the negative mass species, due to the presence of positive mass inside a sphere of radius and density $\rho^{(+)} = \rho$.
We will place ourselves in weak field conditions.
With the metric in this form the non-zero components of the Ricci tensor are

$$R_{00} = e^{\nu-\lambda}(-\frac{\nu''}{2} + \frac{\nu'\lambda'}{4} - \frac{\nu'^2}{4} - \frac{\nu'}{r}) \; ; \; R^0_0 = -e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu'}{r})$$

$$R_{11} = \frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} - \frac{\lambda'}{r} \; ; \; R^1_1 = -e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} - \frac{\lambda'}{r})$$

$$R_{22} = e^{-\lambda}(1 + \frac{\nu' r}{2} - \frac{\lambda' r}{2}) - 1 \; ; \; R^2_2 = -e^{-\lambda}(\frac{1}{r^2} + \frac{\nu'}{2r} - \frac{\lambda'}{2r}) + \frac{1}{r^2}$$

$$R_{33} = R_{22} sin^2\theta \; ; \; R^3_3 = R^2_2 \qquad (A7)$$

and the Ricci scalar

$$R = R^{\mu}_{\mu} = e^{-\lambda}[2(-\frac{\nu''}{2} + \frac{\nu'\lambda'}{4} - \frac{\nu'^2}{4}) - \frac{\nu'}{r} + \frac{\lambda'}{r} - \frac{2}{r^2} - \frac{2\nu'}{2r} + \frac{2\lambda'}{2r}] + \frac{2}{r^2} \quad (A8)$$

which gives the Einstein tensor

$$E^0_0 = e^{-\lambda}(\frac{1}{r^2} - \frac{\lambda'}{r}) - \frac{1}{r^2} \qquad (A9)$$

$$E^1_1 = e^{-\lambda}(\frac{1}{r^2} + \frac{\nu'}{r}) - \frac{1}{r^2} \qquad (A10)$$

$$E^2_2 = e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}) \qquad (A11)$$

Consider the first of the two equations

$$E^{\nu}_{\mu} = \chi \, T^{\nu}_{\mu} \qquad (A12)$$

$$e^{-\lambda}(\frac{1}{r^2} - \frac{\lambda'}{r}) - \frac{1}{r^2} = \chi \, T^0_0 \qquad (A13)$$

$$e^{-\lambda}(\frac{1}{r^2} + \frac{\nu'}{r}) - \frac{1}{r^2} = \chi \, T^1_1 \qquad (A14)$$

$$e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}) = \chi\ T_2^2 \quad (A15)$$

and

$$\chi\ T_0^0 - \chi\ T_1^1 = -\frac{\nu' + \lambda'}{r} e^{-\lambda} \quad (A16)$$

We will now consider the outer metric, where the right sides of the equations are zero. The method is described in reference [2] , in chapter 14, and it corresponds to

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2m}{r}$$

$$ds^2 = (1 - \frac{2m}{r})dx^{0^2} - \frac{dr^2}{1 - \frac{2m}{r}} - r^2(d\theta^2 + sin^2\theta\ d\phi^2) \quad (A17)$$

with

$$m = \frac{GM}{c^2} \quad (A18)$$

M being the (positive) mass of the star.
Let's move on to the classical construction of the inner metric [2] . We have

$$T_\mu^\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -\frac{p}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p}{c^2} \end{pmatrix} \quad (A19)$$

The equations are written

$$e^{-\lambda}(\frac{1}{r^2} - \frac{\lambda'}{r}) - \frac{1}{r^2} = \chi\ \rho \quad (A20)$$

$$e^{-\lambda}(\frac{1}{r^2} + \frac{\nu'}{r}) - \frac{1}{r^2} = -\chi\ \frac{p}{c^2} \quad (A21)$$

$$e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}) = -\chi\ \frac{p}{c^2} \quad (A22)$$

$$-\frac{\nu' + \lambda'}{r}\ e^{-\lambda} = \chi\ (\rho + \frac{p}{c^2}) \quad (A23)$$

from which we get

$$e^{-\lambda}(\frac{1}{r^2} + \frac{\nu'}{r}) - \frac{1}{r^2} = e^{-\lambda}(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}) \quad (A24)$$

$$\frac{e^{\lambda}}{r^2} = \frac{1}{r^2} - \frac{\nu''}{2} + \frac{\nu'\lambda'}{4} - \frac{\nu'^2}{4} + \frac{\nu' + \lambda'}{2r} \tag{A25}$$

For the resolution, we ask

$$e^{-\lambda} \equiv 1 - \frac{2m(r)}{r} \ \rightarrow \ 2m(r) = r(1 - e^{-\lambda}) \tag{A26}$$

We derive this expression

$$2m' = (1 - e^{-\lambda}) + r\lambda' e^{-\lambda} \tag{A27}$$

$$m' = -\frac{r^2 \chi \rho}{2} = 4\pi r^2 \frac{G}{c^2}\rho \tag{A28}$$

We get

$$m(r) = \int_0^r m'(r)dr = \frac{4}{3}\pi r^3 \rho \frac{G}{c^2} \tag{A29}$$

$$\nu' = 2\frac{m + 4\pi G p \frac{r^3}{c^4}}{r(r - 2m)} \tag{A30}$$

Deriving Eq(21) and combining with Eq (25) we get

$$-\chi \frac{p'}{c^2} = -e^{-\lambda} \frac{\nu'}{2r}(\nu' + \lambda') \tag{A31}$$

Using (23) we get

$$\frac{p'}{c^2} = -\frac{\nu'}{2}(\rho + \frac{p}{c^2}) \tag{A32}$$

At the end we get the well-known *TOV equation* [1] (Tolmann-Oppenheimer-Volkoff)

$$\frac{p'}{c^2} = -\frac{m + 4\pi G p \frac{r^3}{c^4}}{r(r - 2m)}(\rho + \frac{p}{c^2}) \tag{A33}$$

When moving to the Newtonian approximation ($p << \rho c^2$, $2m << r$)and taking into account that

$$m = \frac{4}{3}\pi r^3 \rho \frac{G}{c^2}$$

we get

$$\boxed{p' = -\frac{\rho m c^2}{r^2} = -\frac{G M \rho}{r^2}} \tag{A34}$$

[1] which corresponds to equation (14.22) of reference [22]

In spherical symmetry, the gravitational field that prevails at a distance $r < r_s$ (inside the star of supposed constant density) is equal to the field that would be created by the mass $M(r)$ contained in a sphere of radius r_s , concentrated in the center.
The calculation is then identical to that of the reference [22] and leads to the expression of the inside metric [2]

$$\boxed{[\frac{3}{2}(1-\frac{r_s^2}{\hat{R}}^2)^{\frac{1}{2}} - \frac{1}{2}(1-\frac{r^2}{\hat{R}}^2)^{\frac{1}{2}}]^2 dx^{0^2} - \frac{dr^2}{1-\frac{r^2}{\hat{R}}^2} - r^2(d\theta^2 + sin^2\theta\ d\phi^2)} \quad (A35)$$

We are now going to deploy the same calculation scheme, but this time adapting it to the metric describing the negative mass species, which is then the solution of the equation

$$\bar{E}^\nu_\mu \equiv \bar{R}^\nu_\mu - \frac{1}{2}\bar{g}^\nu_\mu \bar{R} = -\chi\ \frac{\sqrt{-g}}{\sqrt{-\bar{g}}} T^\nu_\mu \equiv -\chi\ \frac{w}{\bar{w}} \hat{T}^\nu_\mu \quad (A36)$$

The determinants ratio can be written

$$\frac{\sqrt{-g}}{\sqrt{-\bar{g}}} = \frac{\sqrt{-det(g_{\mu\nu})}}{\sqrt{-det(\bar{g}_{\mu\nu})}} = \frac{\sqrt{e^\nu e^\lambda r^4 sin^2\theta}}{\sqrt{e^\nu e^\lambda r^4 sin^2\theta}} = e^{\frac{\nu}{2}} e^{\frac{\lambda}{2}} e^{-\frac{\nu}{2}} e^{-\frac{\lambda}{2}} \equiv k_D \quad (A37)$$

k_D will be taken little different from 1 because we will always be in the Newtonian approximation.
Now we calculate the impact of the presence of positive masses on the geometry $\bar{g}_{\mu\nu}$ of the negative sector. It should be remembered that we are perfectly free to choose this tensor $\hat{T}^\nu_\mu$, insofar as this choice can result from a Lagrangian derivation. And we opt for

$$\hat{T}^\nu_\mu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \frac{p}{c^2} & 0 & 0 \\ 0 & 0 & \frac{p}{c^2} & 0 \\ 0 & 0 & 0 & \frac{p}{c^2} \end{pmatrix} \quad (A38)$$

The construction of the left side of the field equation is again based on a metric which this time is

$$d\bar{s}^2 = e^\nu dx^{0^2} - e^{\bar{\lambda}} dr^2 - r^2(d\theta^2 + sin^2\theta\ d\phi^2) \quad (A39)$$

[2]Equation (14.47) from reference [22]

The Ricci tensor coefficients are recalculated. The first members of the equations are the same, simply replacing (ν,λ) by $(\nu,\bar{\lambda})$. We then get

$$e^{-\lambda}(\frac{1}{r^2}-\frac{\bar{\lambda}'}{r})-\frac{1}{r^2}=-\chi\;\rho \qquad (A40)$$

$$e^{-\lambda}(\frac{1}{r^2}+\frac{\bar{\nu}'}{r})-\frac{1}{r^2}=-\chi\;\frac{p}{c^2} \qquad (A41)$$

$$e^{-\bar{\lambda}}(\frac{\nu''}{2}-\frac{\nu'\bar{\lambda}'}{4}+\frac{\nu'^2}{4}+\frac{\nu'-\bar{\lambda}'}{2r})=-\chi\;\frac{p}{c^2} \qquad (A42)$$

$$-\frac{\bar{\nu}'+\bar{\lambda}'}{r}\;e^{-\bar{\lambda}}=-\chi\;(\rho-\frac{p}{c^2}) \qquad (A43)$$

$$\frac{e^{\bar{\lambda}}}{r^2}=\frac{1}{r^2}-\frac{\bar{\nu}''}{2}+\frac{\bar{\nu}'\bar{\lambda}'}{4}-\frac{\bar{\nu}'^2}{4}+\frac{\bar{\nu}'+\bar{\lambda}'}{2r} \qquad (A44)$$

For the resolution, we ask

$$e^{-\lambda}\equiv 1-\frac{2\bar{m}}{r}\;\rightarrow\;2\bar{m}=r(1-e^{-\lambda}) \qquad (A45)$$

Similarly we derive this expression

$$2m'=(1-e^{-\lambda})+r\bar{\lambda}'e^{-\lambda}\;\rightarrow\;-\frac{2\bar{m}'}{r^2}=-\frac{1}{r^2}+e^{-\bar{\lambda}}(\frac{1}{r^2}-\frac{\bar{\lambda}'}{r}) \qquad (A46)$$

Using (40),

$$m'=-4\pi r^2\frac{G}{c^2}\rho\;\rightarrow\;\bar{m}(r)=\int_0^r\;\bar{m}'(r)dr=-\frac{4}{3}\pi r^3\rho\frac{G}{c^2}=-m$$

In conclusion, at this point

$$\bar{m(r)}=-m(r) \qquad (A47)$$

We get

$$\bar{\nu'}=2\frac{-m+4\pi Gp\frac{r^3}{c^4}}{r(r+2m)} \qquad (A47)$$

To eliminate $\bar{\nu''}$, we derive (A41). Combining to (A44) we get

$$\boxed{\frac{p'}{c^2}=-\frac{m-4\pi Gp\frac{r^3}{c^4}}{r(r+2m)}(\rho-\frac{p}{c^2})} \qquad (A48)$$

to be compared to what emerged from the analysis for positive masses, i.e. equation (A34)

$$\boxed{\frac{p'}{c^2} = -\frac{m + 4\pi G p \frac{r^3}{c^4}}{r(r-2m)}(\rho + \frac{p}{c^2})}$$

If we introduce the Newtonian approximation these equations become identical. The pressure terms are negligible. The variable r is large in front of $2m$ (Schwarschild radius).
Finally we have to introduce on the value of m

$$m = \frac{4}{3}\pi r^3 \rho \frac{G}{c^2}$$

Both equations lead to the same result

$$p' = -\frac{m\rho c^2}{r^2} \qquad (A49)$$

We can, as before, finalize the calculation of the inner metric of the negative species.
Hence the final expression of the inner metric $\bar{g}_{\mu\nu}$

$$\boxed{[\frac{3}{2}(1 + \frac{{r_s^2}^2}{\hat{R}})^{\frac{1}{2}} - \frac{1}{2}(1 + \frac{{r^2}^2}{\hat{R}})^{\frac{1}{2}}]^2 dx^{0^2} - \frac{dr^2}{1 + \frac{{r^2}^2}{\hat{R}}} - r^2(d\theta^2 + sin^2\theta \, d\phi^2)} \qquad (A50)$$

which links to the outside metric

$$d\bar{s}^2 = (1 + \frac{2GM}{c^2 r})c^2 dt^2 - \frac{dr^2}{1 + \frac{2GM}{c^2 r}} - r^2(d\theta^2 + sin^2\theta \, d\phi^2) \qquad (A51)$$

Under linearized forms

$$d\bar{s}^2 = (1 + \frac{3r_s^2}{2\hat{R}^2})dx^{0^2} - (1 - \frac{r^2}{\hat{R}^2})dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) \qquad (A52)$$

$$d\bar{s}^2 = (1 + \frac{2GM}{c^2 r})c^2 dt^2 - (1 - \frac{2GM}{c^2 r})dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2) \qquad (A53)$$