Physical and mathematical inconsistancy of the black holes models.
The alternative of the plugstars model.

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Abstract :

We review the different points on which the black hole model is based, showing in particular that the metric extension of Kruskal relies on an irregular change of variable, associated to the hypothesis that zero raised to the zero power is equal to unity. The naive obsession with the contractibility of space leads the black hole designers to a real geometrical aberration consisting in considering that inside the object the variables of time and space see their roles exchanged, whereas a structure equipped with a throat sphere avoids this choice. We show that a re-reading of the inner Schwarzschild solution, combined with the choice of another time coordinate admitting the presence of a cross term in drdt, leads to the consideration of a one-way membrane, as a communication interface between two Lorentzian spacetimes through which matter transits in a finite time, which invalidates the treatment of the outer Schwarzschild solution as a freeze-frame of a very fast process. Taking into account the inner Schwarzschild solution, where a physical criticality occurs before the geometrical criticality shows that this results in a local inversion, at the center of the object, of the time variable and the mass of the particles. This phenomenon can then be interpreted through the cosmological Janus model, this excess mass being immediately ejected from the star. An alternative model, of plugstar, emerges, presenting a gravitational redshift of 3, which fits with the data collected for the hypermassive objects of M87 and the Milky Way. We predict that this same value will accompany similar images collected in the future.

Key words: black hole, Schwarzschild, Janus Cosmological Model, Kruskal, Plugstar, neutron star, negative mass, negative energy,

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1 – **Introduction.**

Today, the existence of stellar or giant black holes is considered as an established fact [1]. But what do we see on the two images, presented in false colors, of hypermassive objects located at the center of the galaxy M87 and the Milky Way, immediately qualified as giant black holes? Just a ring of greater brightness, surrounding a dark spot. In the article, below the image on the left of figure 1, there is a bar to link the chromatic intensity to a brightness temperature, where black codes have a value close to zero, while a very bright yellow, tending to white, indicates a maximum brightness

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temperature reaching 6 billion degrees. Thanks to a set of radiotelescopes the images below have been reconstructed.

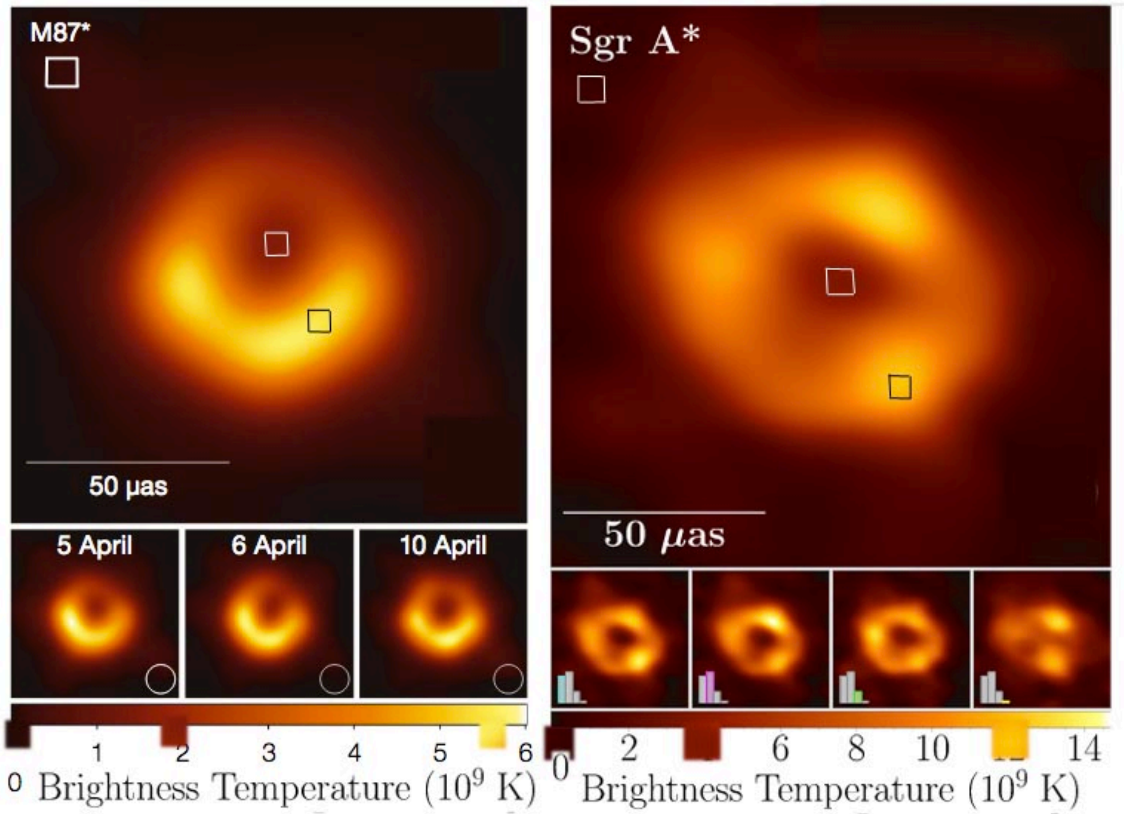

Figure 1: Images of the objects at the center of M87 and the Milky Way. Brightness temperature measurements. Ratio Tmax/Tmin: 3.

It turns out that the human eye perceives the chromatic differences with precision. Using it as a measuring instrument, if the brightness temperature of the environment is found to be in the range of zero, this is far from being the case for the central part of the object in the figure on the left, representing the object at the center of M87, which leads to a value of around 1.9 billion degrees, with the maximum temperature, in the ring, reaching 5.7 billion Kelvin. Although the authors of the article add "that the value of the brightness temperature measured should not be immediately identified with a temperature of the plasma surrounding the object", we are still entitled to think that there is a strong correlation between the two. Let's move on to the object in the figure on the right, representing the object Sgr A*, in the center of the Milky Way. Minimum temperature 4 billion degrees. Maximum temperature 12 billion degrees.

Here again the ratio of the maximum to minimum brightness temperature is 3.

If we refer to the synthetic images of black holes, based on the numerous simulations carried out, a much higher brightness temperature contrast should be observed, whatever the angle from which it is observed.

The black hole model was built from an exact solution [2], published in 1916 by the German mathematician Karl Schwarzschild of the particular form, without second member, referring to an empty portion of the universe, of the equation found by

A.Einstein a few months earlier [3]. It is of major importance to keep in mind the conditions under which Scharzcshild makes this calculation. All quantities are real: the coordinates and the value of the length s. Schwarzschild starts with a Cartesian coordinate system, then moves to a representation introducing the real and positive scalar r :

(1) $$\{ x , y , z \} \in \mathbb{R}^3 \qquad r = \sqrt{x^2 + y^2 + x^2} \in \mathbb{R}^+$$

All this being perfectly specified in the article:

Wenn man zu Polarkoordinaten gemäß $x = r \sin \vartheta \cos \phi$, $y = r \sin \vartheta \sin \phi$, $z = r \cos \vartheta$ übergeht, lautet dasselbe Linienelement:

$$\begin{aligned} ds^2 &= F dt^2 - G(dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\phi^2) - H r^2 dr^2 \\ &= F dt^2 - (G + H r^2) dr^2 - G r^2 (d\vartheta^2 + \sin^2 \vartheta d\phi^2) \end{aligned} \qquad (6)$$

Fig.1 : Schwarzschild introduces polar coordinates [1]

Schwarzschild chooses to express his metric solution, not in the coordinate system $\{ t , r , \theta , \varphi \}$ but by involving a quantity R, which he calls an "intermediate quantity" (Hilfstroβe), i.e. in the system $\{ t , R , \theta , \varphi \}$ in order to obtain an immediate comparison with the approximate, linear solution published by Einstein [4] . Hereafter is the corresponding passage from his article [2].

wobei die Hilfsgröße

$$R = (3 x_1 + \rho)^{1/3} = (r^3 + \alpha^3)^{1/3}$$

eingeführt ist.

Setzt man diese Werte der Funktionen f im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des Einsteinschen Problems bildet:

$$ds^2 = (1 - \alpha/R) dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2 (d\vartheta^2 + \sin^2 \vartheta d\phi^2), \quad R = (r^3 + \alpha^3)^{1/3}. \qquad (14)$$

Dasselbe enthält die eine Konstante α, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

Fig.1 : Extract from his article [2] *where he presents the solution*

It is therefore clear that this letter R does not designate a radial coordinate r:

(2) $$R = (r^3 + \alpha^3)^{1/3} \quad \rightarrow \quad R \geq \alpha$$

In these conditions it is clear that the portion of the space-time $\left\{ t , r , \theta , \varphi \right\}$ such that $R < \alpha$ goes out of the world of the real.

What we propose to show in this article is how the fundamental error, consisting in treating this quantity R as a radial variable, has produced a chimera, which has become the black hole model. Today, specialists are either not aware of this error, or they refuse to consider it.

It is impossible to understand how such a gross and damaging mistake could have been made after the Second World War, more than half a century ago, if one does not accurately retrace the history of what is considered today as cosmological science.

2 – **Historical review**

In 1916 the mathematician Karl Schwarzschild started from the field equation that Albert Einstein had just constructed [3] in 1915 and thanks to which Einstein was able to construct a stationary, linearized solution [4] of his equation, when the second member is zero, i.e. referring to a portion of empty space-time. From this solution, expressed as a metric, Einstein was able to produce geodesics by proposing that the planets follow these curves.

From the linearized solution that he built and published Einstein found the explanation of the phenomenon of the advance of the perihelion of the planet Mercury. He then sends the whole of these results to Karl Schwarzschild who is then in cantonment on the Eastern front, the Russian front. As a mathematician, well versed in physics and astronomy, he mastered the new tools of differential geometry. He immediately understood the significance of Einstein's work.

He then immediately considered completing them by aiming at a complete modeling of the problem in terms of physics. He decides to represent the source of the field, the stars, the Sun, as spheres filled with an incompressible fluid of constant density ρ_o. He then constructs, still under the assumption of stationarity, the two geometrical solutions, in a non-linear form [5] , describing :

- The geometry outside the star, in the form of a metric solution that we will designate by $g_{\mu\nu}^{(ext)}$
- The geometry inside the star, in the form of a metric solution that we will designate by $g_{\mu\nu}^{(int)}$

He undertakes to construct his solutions in a system of coordinates $\left\{ t , x , y , z \right\}$. At no time does he refer to the world of complexes. These coordinates are real, as is the length element s, the key to this geometric description. He quickly switches to polar coordinates $\left\{ t , r , \theta , \varphi \right\}$. See figure 1. The quantity r is therefore automatically real and positive.

He ensures the compatibility of these two solutions by giving the conditions of connection of the two geometrical systems, on the sphere.

His first goal is to match the result of Albert Einstein [4] , for whom he has a deep admiration. His goal is not to explore the ins and outs of the solutions he has constructed but to link his results, in their linearized form, with Einstein's.

The non-linear version of these solutions does not seem to him to be of interest in physics, insofar as the characteristic radius, which he calls α , to which we will give his name, the Schwarzschild radius Rs, turns out to be very small compared to the radius of the star (which he evaluates at 3 km in the case of the Sun, as does Einstein).

He therefore does not explicitly present his metric solution. He just put it in a suitable form in order to fit Einstein's approximate linear result, in a letter to Einstein, through a particular formulation, in spherical coordinates, expressed using an intermediate variable R, corresponding to $R = (r^3 + \alpha^3)^{1/3}$. When r is large in front of α this quantity R differs from r by a negligible amount. The expression of the metric using this coordinate system $\left\{ t , R , \theta , \varphi \right\}$ implying $R > \alpha$ thus allows, in the simplest way, an identification with Einstein's linearized solution.

And it is in this form that Schwarzschild presents his result in his article of January 1916 [3]. See figure 1. Below is an excerpt from his paper where he presents his solution :

wobei die Hilfsgröße

$$R = (3x_1 + \rho)^{1/3} = (r^3 + \alpha^3)^{1/3}$$

eingeführt ist.

Setzt man diese Werte der Funktionen f im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des Einsteinschen Problems bildet:

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Dasselbe enthält die eine Konstante α, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

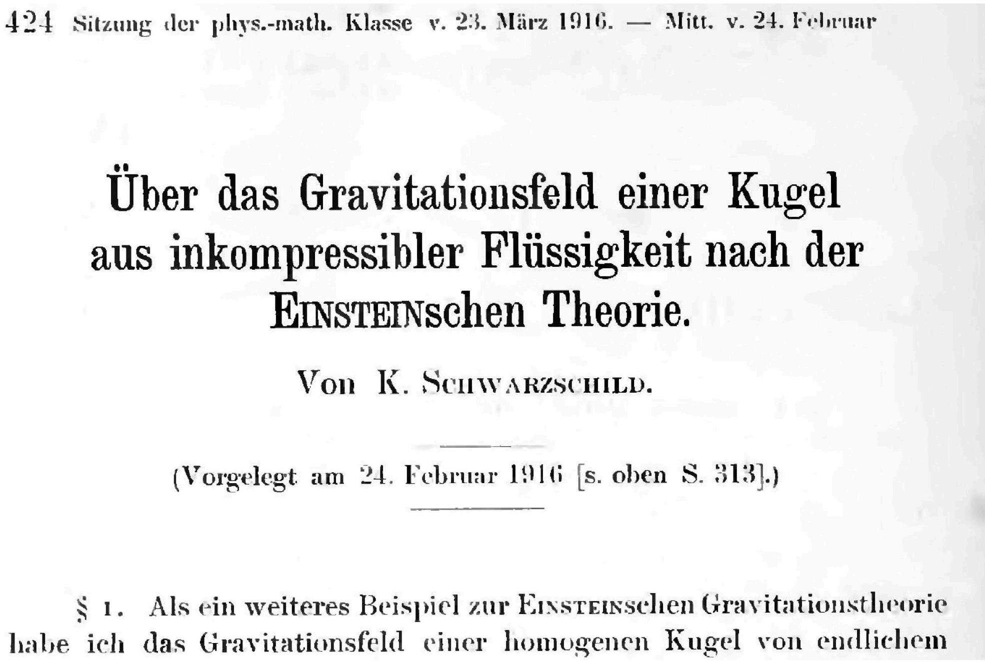
Fig.2 : Extract from his article where he presents the solution

Note the German word Hilfsgröβe, which translates as: "intermediate quantity". The succession of signs of the terms (+ - - -) constitutes what will later be called the signature of the metric. In this formulation, it is clear that if the coordinates have real values, so does the quantity if $R > \alpha$.

In fact, this is what Schwarzschild should have written, if he had wanted to remain consistent with the coordinate system he had chosen from the start.

$$(3) \qquad ds^2 = \left(1 - \frac{\alpha}{(r^3+\alpha^3)^{1/3}}\right)dt^2 - \frac{r^4(r^3+\alpha^3)^{-4/3}}{\left(1 - \frac{\alpha}{(r^3+\alpha^3)^{1/3}}\right)}dr^2 - (r^3+\alpha^3)^{2/3}(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Schwarzschild published a month later, still in the same journal, a second article [5] describing the geometry inside the mass of the star, still according to a metric solution with spherical symmetry and in stationary. Here is the title of this article.



424 Sitzung der phys.-math. Klasse v. 23. März 1916. — Mitt. v. 24. Februar

Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 24. Februar 1916 [s. oben S. 313].)

§ 1. Als ein weiteres Beispiel zur EINSTEINschen Gravitationstheorie habe ich das Gravitationsfeld einer homogenen Kugel von endlichem

Fig.3 "Gravitational field inside a sphere filled with an incompressible fluid, according to Einstein's theory".

In this article we find the expression of the second metric, always with this "intermediate " coordinate R, which is definitely not a "radial coordinate".

Das Linienelement im Innern der Kugel nimmt, wenn man statt x_1, x_2, x_3 (ix) die Variabeln χ, ϑ, ϕ benutzt, die einfache Gestalt an:

$$ds^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 dt^2 - \frac{3}{\varkappa\rho_0}\left[d\chi^2 + \sin^2\chi\, d\vartheta^2 + \sin^2\chi \sin^2\vartheta\, d\phi^2\right]. \quad (35)$$

Außerhalb der Kugel bleibt die Form des Linienelements dieselbe, wie beim Massenpunkt:

$$\left.\begin{aligned} ds^2 &= \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\vartheta^2 + \sin^2\vartheta\, d\phi^2\right) \\ \text{wobei:}\quad R^3 &= r^3 + \rho \end{aligned}\right\} \quad (36)$$

ist. Nur wird ρ nach (33) bestimmt, während für den Massenpunkt $\rho = \alpha^3$ war.

Fig.4 The inner and outer Schwarzschild metrics [5] .

The interior metric (Schwarzschild again takes c = 1) is expressed with the coordinates $\{ t, \chi, \theta, \varphi \}$. In this one intervenes a factor, which has the dimension of the square of a length.

(4) $$\frac{3}{\kappa\rho_o} = \frac{3c^2}{\kappa\rho_o} = \frac{3c^2}{8\pi G\rho_o} = \hat{R}^2$$

where G is the constant of gravitation and ρ_o the density, considered as constant in the star. This quantity $\hat{R}$ depends only on the density ρ_o of the star. The R coordinate is then replaced by an angle χ such that $R = \hat{R}\sin\chi$. In the current paper Schwarzschild evaluates the local energy density, always with c = 1 :

$$\rho_o + p = \rho_o \frac{2\cos\chi_a}{3\cos\chi_a - \cos\chi} \quad (30)$$

Fig.5 : Expression of the total energy, inside the sphere [5] .

The value $\chi = 0$ corresponds to the center of the star while its surface corresponds to the value χ_a. The energy density at the center is therefore :

(5) $$(\rho_o + p)_{\chi=0} = \frac{2\cos\chi_a}{3\cos\chi_a - 1}$$

Schwarzschild immediately notes that for a value $\cos\chi_a = 1/3$ this magnitude becomes infinite. For this pressure not to rise to infinity at the center of the star, the radius of the star, for a given density, must satisfy :

(6) $$\cos\chi_a = \sqrt{1 - \frac{R_a^2}{\hat{R}^2}} > \frac{1}{3} \qquad \rightarrow \qquad R_a < \sqrt{\frac{8}{9}}\,\hat{R} = \sqrt{\frac{8}{9}}\sqrt{\frac{3c^2}{8\pi G \rho_o}}$$

In fact of rise of the pressure in the center towards, it is rather a rise of the volumic density of energy (which creates the curvature). But a pressure is also a volume density of energy. In a fluid the pressure is given by $p = (\rho < v^2 >)/3$. At constant density, if the pressure in a fluid increases, it means that the average thermal agitation speed tends to c and $p = \rho c^2/3$, is then identified with the radiation pressure. At constant density and constant speed of light, this pressure is therefore limited by a maximum value.

As Schwarzschild remains in this hypothesis of a constant density ρ_o he deduces, as mentioned in his article that when the radius of the star tends to $\hat{R}$, in its center the speed of light becomes infinite

(7) $$R_a \rightarrow \sqrt{\frac{8}{9}}\,\hat{R} = \sqrt{\frac{8}{9}}\sqrt{\frac{3c^2}{8\pi G \rho}} \qquad c \rightarrow \infty$$

Historically, this is the first mention of a situation where the speed of light can cease to be an absolute quantity. The quantity α or Schwarzschild radius Rs was, by an identification based on the Newtonian approximation :

(8) $$\alpha = R_s = \frac{2GM}{c^2} = \frac{2GM}{c^2}\frac{4\pi G R_a^3 \rho}{3} = \frac{8\pi G R_a^3 \rho}{3c^2} = \frac{R_a^3}{\hat{R}^2}$$

In the outer metric, the presence of the term $-dR^2/(1-\alpha/R)$ suggests that a *geometric criticality* may occur when R tends to the Schwarzschild radius. This one is proportional to the mass M, thus to the cube of the radius of the star, at constant density.

(9) $$\alpha = R_s = \frac{2GM}{c^2} = \frac{2G}{c^2}\frac{4\pi R_a^3 \rho}{3} = \frac{8\pi G R_a^3 \rho}{3c^2}$$

When the radius R_a of the star becomes equal to the Schwarzschild radius R_s, it reaches the value :

(10) $$R_a = \sqrt{\frac{3c^2}{8\pi G \rho}}$$

which happens to be the value of the radius $\hat{R}$, which depends only on the density. It is by focusing on this idea of geometric criticality that astrophysicists will evaluate the maximum mass of a neutron star, assimilated to a star of constant density ρ_o, whereas it is quite clear from Karl Schwarzschild's second paper that a *physical criticality* would manifest itself before this value is reached. But before considering these considerations, let us return to the historical aspects.

Karl Schwarzschild died of an infection a few months after publishing his two papers.

Being fluent in German, the young Robert Oppenheimer, who came from a wealthy family, completed his studies at the University of Göttingen, the Mecca of theoretical science in the years 1900-1920. There he defended in 1927 a doctoral thesis under the direction of Max Born, focused on quantum mechanics, which had nothing to do with general relativity

At the beginning of 1933 Einstein stayed outside Germany. He learned that his house had been looted by the Nazis and decided not to return to Germany. At the invitation of Abraham Flexner, founder of the Institute for Advanced Study in Princeton, he emigrated to the United States and accepted a position at this institute, which he held until his death in 1955.

There he had exchanges with the American mathematician and theoretical physicist Richard Tolman, who was also familiar with the German language, which is far from being the case for all American scientists. Tolman, who was 52 years old at that time, had already a large part of his scientific career behind him. He became familiar with German general relativity at the same time as Robert Oppenheimer, who also met Einstein.

At that time there was no English translation of Schwarzschild's articles. The first one will be available in this language only in 1975 and the second in 1999. The presentation made by Tolman in a monograph, in 1934, thus takes the nature of a first "bible of general relativity". But in this work the confusion between the "Schwarzschild intermediate quantity R" and a radial coordinate r is realized, confusion which is also that of the mathematician David Hilbert. From then on, what is called the "Schwarzschild metric" is identified with the wellknown expression. In the literature this characteristic length is indifferently designated by R_g , R_s , 2m.

Hilbert's writings of 1915-1916, entitled "Fundamentals of Physics" ([6], [7]), whose English translations, still copyrighted by the German publisher Springer of works on relativity and theoretical physics, remain hardly accessible, are mostly unknown to theorists. In detailing them, we find the origin in particular of the change of signature $(+---) \quad \rightarrow \quad (-+++)$ which makes that today we find more commonly the form :

(11) $$ds^2 = -\left(1 - \frac{R_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{R_s}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

Let us notice that by giving real values to t and r one falls indifferently on real or imaginary pure values of ds. The explanation emerges when we detail the writings of Hilbert. At the time when he produced these works, it was not known that the universe had known "an instant zero", and that it was evolving. The geometry of space, and beyond that of space-time, thus correspond to an appearance ex nihilo, inspired by the theme of a biblical creation (Hibert is a believer). For Hilbert, what pre-exists is space

[6], with its three coordinates and their signature (+ + +). Then appears a time variable which he designates by the letter l. The tangent metric of Hilbert is then :

(12) $$ds^2 = dx^2 + dy^2 + dz^2 + dl^2$$

To satisfy the requirements of special relativity, Hilbert produces his own interpretation. While Einstein's genius was to understand that time was only a fourth dimension of the hypersurface and was initially measured in meters, for David Hilbert this fourth dimension, temporal, is in fact of a different essence, pure imaginary:

(13) $$l = i\,c\,t$$

So the tangent metric becomes:

(14) $$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

This is the origin of the sudden change of this signature, in the years after the war, and which ended up imposing itself, without one being able to find an article where this change is argued and justified In fact, Hilbert does not give himself the metric in this form, but simply considers the bilinear form:

(15) $$g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2 - g_{tt}dt^2$$

which he designates by the letter G. He then defines two "lengths" [7] . When G is negative, he constructs a first length, positive and real thanks to the introduction of a minus sign, which is identified with the proper time τ of special relativity:

heiße die *Länge der Strecke*; ein Kurvenstück, für welches

$$G\left(\frac{dx_s}{dp}\right) < 0$$

ausfällt, heiße eine *Zeitlinie* und das längs dieses Kurvenstückes genommene Integral

$$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}\,dp$$

Fig. 6 : The construction of the proper time, according to D.Hilbert.

When G is zero, this refers to geodesics of zero length, borrowed by the light. When G is positive, it defines a real length λ, according to :

nicht sein Vorzeichen ändert: ein Kurvenstück, für welches

$$G\left(\frac{dx_s}{dp}\right) > 0$$

ausfällt, heiße eine *Strecke* und das längs dieses Kurvenstücks genommene Integral

$$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\, dp$$

Fig.7 : A second length of space-time [7], defined by D.Hilbert whose meaning remains unknown to this day.

Thereafter, neither he, nor any of his commentators or successors will mention this "second real length" λ of Hilbert. But we can suppose that he may have had the idea of envisaging a kind of metaphysics where this magnitude would have been the "corresponding time". In any case, we owe him this inversion of the signature and the fact that modern theorists have lost all "sense of reality" in differential geometry, the expression used in this case being perfectly adequate to the situation. In the immediate post-war period the signatures (+ - - -) and (- + + +) can be found in various writings. But in 1934, and before the Second World War, it is the signature (+ - - -) that is found in all the articles, in particular in Tolman's[8]. As it will be evoked later in the works of Flamm [9] , 1916 and Weyl 1917 [10] (which will be the subject of English translations only in 2021 and 2015), some try to see in these solutions a way to represent massive particles. With his student Rosen, Einstein published in 1935 a paper in this direction [11], based on a presentation of the Schwarzschild solution, describing the vacuum, with other coordinates. From this emerges the idea of a kind of "bridge" between two spaces. But the metric reformulated in this way has the disadvantage of no longer being identified with Lorentz's metric to infinity. This concept, initially present in Flamm and Weyl, was taken up by many authors, including C.W. Misner and J.A. Wheeler in 1957 [16].

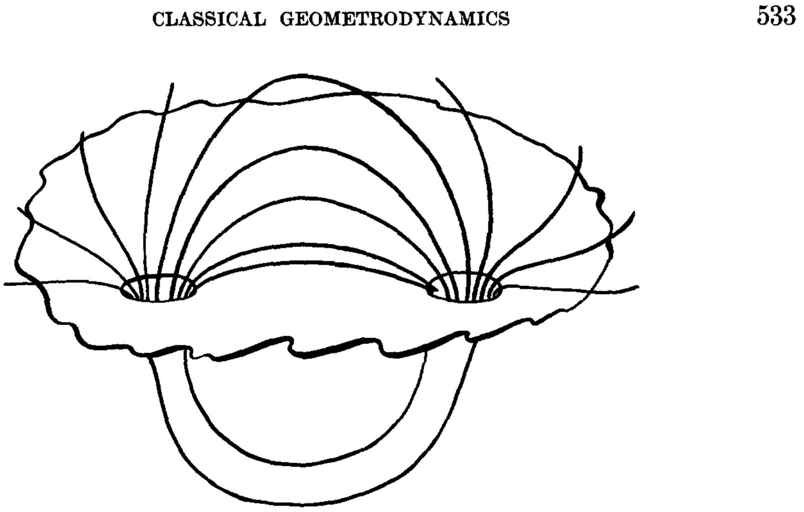


Fig. 8 : *The "Geon": the topological design of masses and charges of Misner and Wheeler. Figure taken from reference* [16].

From the beginning of the century, astronomy made giant leaps, linked to the development of spectroscopy and the progress of theoretical physics and quantum mechanics. The energy source of stars is identified. Spectroscopy provides precise observational data. Models of stars and their evolutionary scenarios are developed. In the 1930s, the American Fritz Zwicky sketches the brutal scenario that signals the end of life of massive stars and invents the supernova model.

The neutron having been discovered in 1932 by the American Chadwick, the structure of the remnant resulting from such brutal ends of massive stars is based on its properties. The concept of neutron star emerges. In 1939 Robert Oppenheimer publishes an article with Snyder [12] in which he suggests that when the counter-pressure exerted by the neutrons could not counterbalance the force of gravity, massive stars could implode without any other imitation than the fact of seeing all the matter converge towards a point, the geometric center of a spherically symmetric system.

He tries to construct the scenario of such an implosion, leading the matter to cross the Schwarzschild sphere, deduced from the simple mass of the star considered

This is the founding act of the black hole model.

He then published a second article [13], still in 1939, in connection with Tolman, where he undertook to construct the relativistic physics of hypermassive objects, such as neutron stars. In this article we find what will later be called the TOV equation, for Tolman, Oppenheimer and Volkoff, which allows us to construct the evolution of the pressure inside the star. It turns out that Richard Tolman was the author of a similar work. But neither Oppenheimer and Volkoff, nor Tolman mention the real author of this work: Karl Schwarzschild, in 1916.

Indeed it is nothing else than an identical reproduction of the solution published in 1916 by Karl Schwarzschild in his second paper [5]. It is easy to realize this by applying to his metric the inverse change of variable, going from the variable to the variable R (now r in the literature). But as nobody has read, or even knows about the existence of this second solution, the deception goes unnoticed.

On this point, the Second World War was an interlude. Already old, Tolman left the game completely and died in 1948 at the age of 67. Oppenheimer spends the war years working on interesting outdoor experiments. In the post-war period, preoccupied by the lawsuit brought by those who accused him of anti-American activity, he does not return to these questions of becoming stars.

Einstein remains skeptical about the scenario proposed by Oppenheimer. Trying in vain to extend his theory to the "unified fields", integrating in the same geometrical context gravitation and electromagnetism, he dies in 1958. It is then that a new generation of those who will be called "cosmologists" is born.

In 1963 a New Zealand mathematician, Roy Kerr, builds a solution of Einstein's equation without a second member, describing the vacuum, always stationary, which is no longer spherically symmetric, but axisymmetric [17]. It thus seems more suitable to describe astrophysical objects, observation having confirmed in 1967 the existence of neutron stars, while specifying that these objects are animated by very rapid rotations, up to a thousand revolutions per second.

(16)

$$ds^2 = \left(1 - \frac{2m\rho}{\rho^2 + a^2\cos^2\theta} \right) c^2 dt^2 - \frac{\rho^2 + a^2\cos^2\theta}{\rho^2 + a^2 - 2m\rho} d\rho^2 - \left(\rho^2 + a^2\cos^2\theta \right) d\theta^2$$
$$- \left[\left(\rho^2 + a^2 \right) \sin^2\theta + \frac{2m\rho a^2 \sin^4\theta}{\rho^2 + a^2\cos^2\theta} \right] d\varphi^2 - \frac{4m\rho a \sin^2\theta}{\rho^2 + a^2\cos^2\theta} c\, dt\, d\varphi$$

The Kerr metric has this time a cross term in $dt\, d\varphi$, whereas the classical expression of the Schwarzschild metric does not. This particularity makes the speed of light, for rays drawn tangentially to an orbit-circle, have two different values, depending on the direction chosen. This aspect is evoked by writing that, we quote :

- *Everything happens as if the source of the field, in rotation, were somehow competing with the distant Lorentzian portions of space-time, and thus dragging space-time along in its rotational movement.* ([19] ch. 7).

All this does not go beyond the stage of a simple remark, although it refers to the idea of the mathematician Ernst Mach, who denied the reality of an absolute space, thinking that space was fundamentally linked to matter. We will come back to this later.

In 1965, for example, Ronald Adler, Maurice Bazin and Menaheim Schiffer published a book entitled "Introduction to General Relativity", published by Mac Graw Hill, integrating the "state of the art" of the time [19]. In this book, we find the constructions of the exterior and interior metrics. At the end of chapter 14 where this construction is taken up again, leading to "the famous TOV equation", in the 1975 edition the authors quote Schwarzschild's second paper, from 1916, without explaining, in the text, that this work is in fact his own. It is not known whether this mention was already present in the 1965 edition, or even whether the content of this chapter was part of the book, since the work of one of the authors, Ronald Adler, only appeared in the specialized literature in 1971

In this pivotal period, between the sixties and seventies, the black hole model was built. The main architects are the physicist John Archibald Wheeler, a former member of the Manathan project, and his two students, Kip Thorne [19], future Nobel Prize winner in 2017, and Charles W. Misner [16]. Everything is set out in a lavishly illustrated book entitled "Gravitation", published in 1973. Enriched over the years by numerous additions, it is considered the reference work, reaching 1279 pages to date.

So what was going on at this pivotal time?

Wheeler and his collaborators took up Oppenheimer's seminal paper where most of the singular aspects of Schwarzschild's solution, as they thought it was expressed, were evoked. The coordinate r is considered as a radial coordinate. The object thus has, they thought, an exterior and an interior. An error that persists today.

The gravitational redshift effect tends to infinity, when a mass sees its radius merging with its Schwarzschild sphere. Beyond this point, it becomes impossible to construct a scenario describing this matter. Still, no light can emerge from such an "object". Hence the name given to it by Wheeler: Black hole.

When we consider the geodesic path followed by a witness mass, plunging towards this "object", it reaches, according to the proper time s, its geometric center in a finite time, which Oppenheimer had already located in days in 1939. On the scale of geological phenomena, this is an extremely short time.

On the other hand, the proponents of such a model base themselves on the time coordinate used by Schwarzschild, which, for them, is identified with the own time experienced by a distant observer. Such a paradox allows us to use this stationary solution to describe a phenomenon of extreme brevity. The distant observer is thus a spectator of a phenomenon which, for him, means in infinite time.

2 – **The question of the expression of the solution for** $r < R_s$

This question was solved relatively quickly in the sense that the Kretschman scalar was not zero at this point. A whole set of different choices of variables were successfully proposed, allowing to eliminate this coordinate singularity to the point that it is no longer a problem today. What remained was to succeed in exploring "the interior of this Schwarzschild sphere". In 1960 Martin Kruskal proposed a change of coordinates [18] which claimed to be able to make this jump. His project is, starting from coordinates $\left\{ t , r , \theta , \varphi \right\}$, to pass to a system $\left\{ v , u , \theta , \varphi \right\}$ where the metric takes the form :

$$ds^2 = f^2(u,v)(\, dv^2 - du^2\,) - r^2(\, d\theta^2 + \sin^2\theta d\varphi^2\,) \tag{17}$$

Under these conditions, if we consider the radial geodesics of zero length, followed by the light, we obtain a constant value of:

(18) $$\left(\frac{dv}{du^2}\right)^2 = 1$$

provided that the function f is nonzero. It introduces two changes of variable leading to an intermediate variable ξ

(19) $$r < R_s \qquad \xi = r + R_s\, L_n(1 - \frac{r}{R_s})$$

(20) $$r > R_s \qquad \xi = r + R_s\, L_n(\frac{r}{R_s} - 1)$$

We therefore have two formulas for changing the coordinate r, referring, one to the "inside", the other to the outside, which should in principle take over, which implies continuity. Now, when we draw the curve, we obtain this:

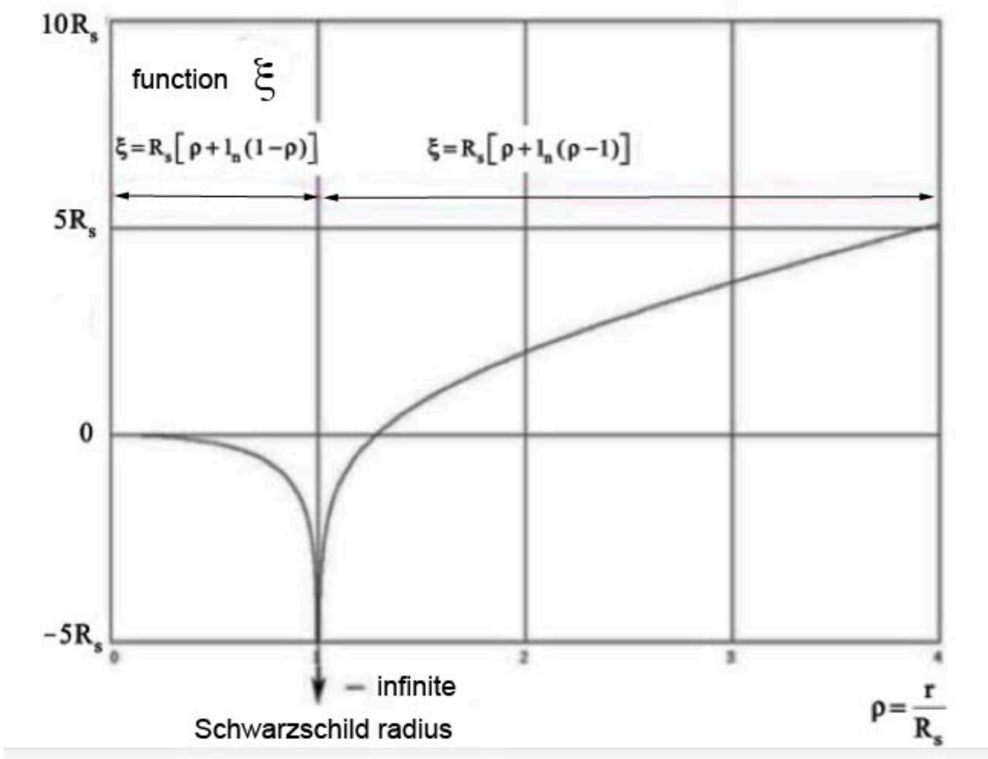


Fig. 9 : Lack of continuity in the change of variable of the article [18]

Kruskal develops his calculation by starting from the expression:

(21) $$ds^2 = \left(1 - \frac{2m}{r}\right)(dx^\circ)^2 - \frac{dr^2}{1 - \frac{2m}{r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

He then obtained:

(22)

$$r > 2m \qquad u = e^{\eta\xi}\, ch\, x^\circ \qquad v = e^{\eta\xi}\, sh\, x^\circ$$

$$e^{\eta\xi} = e^{\eta r}\left(\frac{r}{2m} - 1\right) \qquad f^2 = \frac{2m}{\eta^2 r}\left(\frac{r}{2m} - 1\right)^{1-4m\eta} e^{-2\eta r}$$

(23)

$$r < 2m \qquad u = e^{\eta\xi}\, sh\, x^\circ \qquad v = e^{\eta\xi}\, ch\, x^\circ$$

$$e^{\eta\xi} = e^{\eta r}\left(1 - \frac{r}{2m}\right) \qquad f^2 = \frac{2m}{\eta^2 r}\left(1 - \frac{r}{2m}\right)^{1-4m\eta} e^{-2\eta r}$$

where η is a constant of integration, to be defined. We note immediately that for r tending towards 2m, if the exponent of the quantity between brackets is different from zero, the function f cancels, which Kruskal wants to avoid.

He then resorts to an artifice that leaves both the physicist and the mathematician stunned:

For f to be non-zero for all the values of r considered it is simply necessary that

(24)
$$\eta = \frac{1}{4m}$$

Thus Kruskal bases his conclusions on a property of the numbers that we have framed, so aberrant is it:

(25)
$$\boxed{0^0 = 1}$$

Thanks to this artifice the continuity and the non nullity of the function is ensured, which becomes:

(26)
$$f = \frac{32\, m^2}{r}\, e^{-r/2m}$$

Kruskal describes this function as "transcendental" in his article [18]. Here it is:

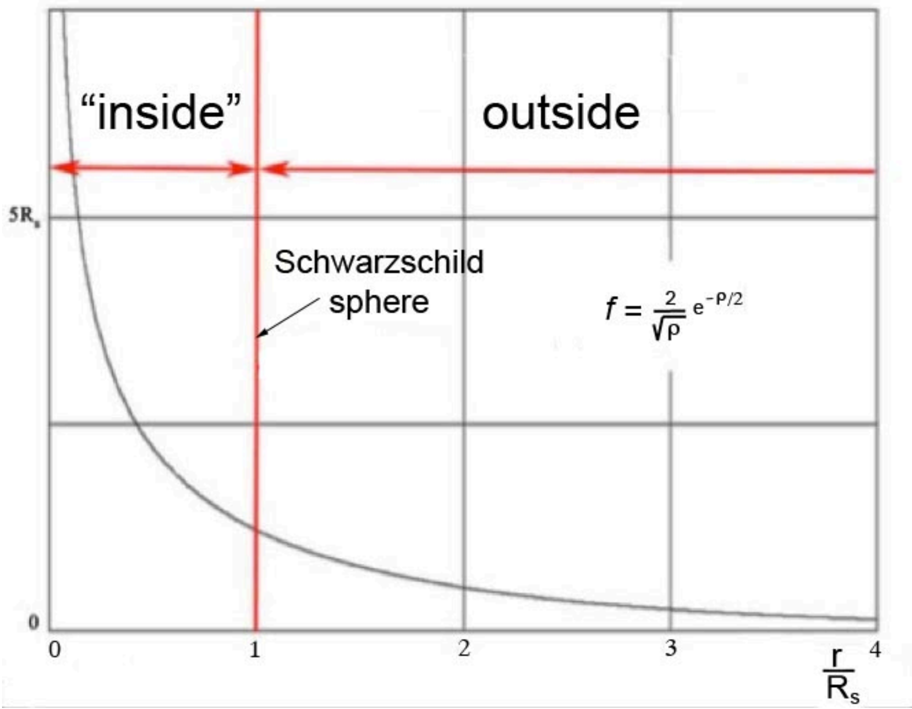


Fig.10 : The "transcendental Kruskal function"

The Kruskal representation then becomes:

(27) $\quad r > 2m \qquad u = \sqrt{\dfrac{r}{2m} - 1}\; e^{r/4m}\, ch\, x° \qquad \sqrt{\dfrac{r}{2m} - 1}\; e^{r/4m}\, sh\, x°$

(28) $\quad r < 2m \qquad u = \sqrt{1 - \dfrac{r}{2m}}\; e^{r/4m}\, sh\, x° \qquad \sqrt{1 - \dfrac{r}{2m}}\; e^{r/4m}\, ch\, x°$

The fact that, in these two expressions of coordinates, in a way, the new space and time coordinates exchange their respective roles gives support to what follows.

These Kruskal coordinates, (u,v) have then been used as a basis for a whole series of articles, based in particular on the "Kruskal diagram" and one has the right to wonder if there are any among the many authors of these articles who have taken the trouble to look at the basis of this 1960 construction. In this case on two points:

- The non-regularity of the mapping $r \to \xi$
- The fact that this work is based on the assumption that $0^0 = 1$

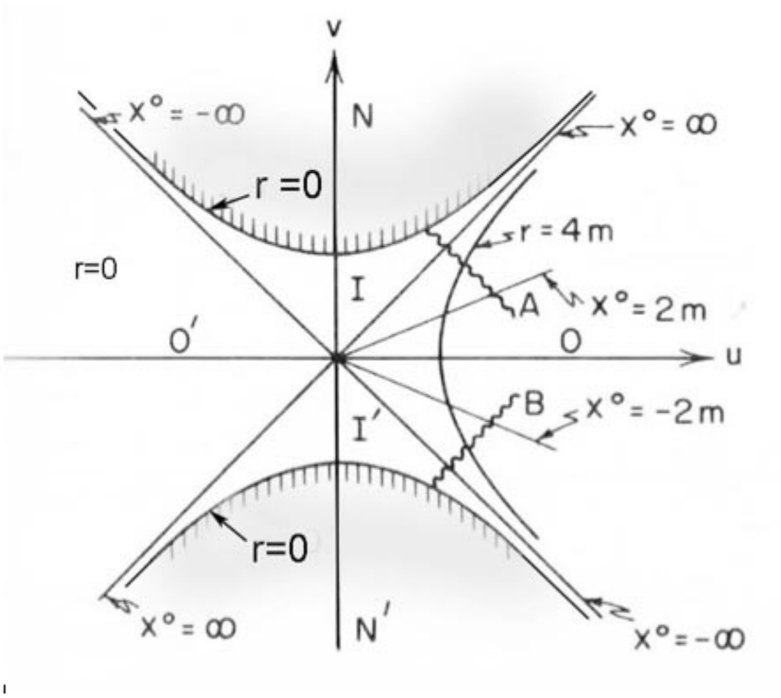


Fig.11 : The Kruskal diagram [19]

In science, the principle of Occam's razor applies, which encourages us to opt for the explanation that requires the least cumbersome and most coherent hypotheses. Kruskal's hypotheses do not go in this direction. We note in particular that the metric, expressed with the help of these coordinates, does not tend towards the Lorentz metric at infinity, far from it, since in this case the function f tends towards zero.

3 – **Geometries of real worlds and geometries of complex worlds.**

Cosmology is based on a modern geometry, which evokes the concept of Riemanian hypersurfaces, with 4 dimensions and a hyperbolic metric. From this metric it is then possible to construct a set of geodesic curves, the examination of which is supposed to allow to apprehend their geometrical structure. All the quantities that we manipulate are then supposed to belong to the world of the real, whether they are coordinates or the measurement of the element of length s.

What we lose sight of is the fact that we can consider curves, as sequences of points, which are not geodesics, but belong to the object. The metric tool, however, keeps its functionality. Let us give an example in two dimensions, through the metric, of signature (+ +) :

(29)
$$ds^2 = \frac{dr^2}{1-\frac{Rs}{r}} + r^2 d\varphi^2$$

Note that this represents a fragment of the Schwarzschild metric. It is associated with a surface of revolution, the "Flamm surface", this author, in 1916, having operated its plunge in and constructed the equation of its meridian [9].

(30)
$$r = R_s + \frac{z^2}{4R_s}$$

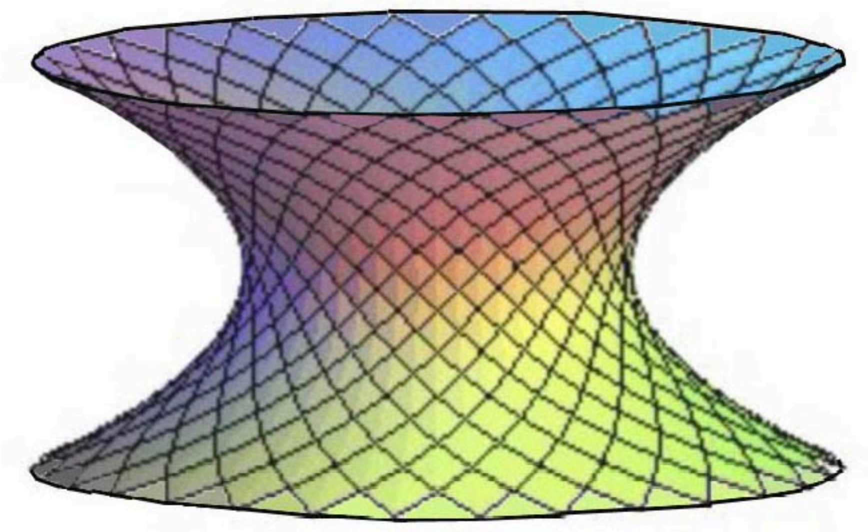


Fig.12 : The Flamm's surface.

This object is generated by the rotation of a parabola lying around an axis. Its parabolic meridians are also geodesics. Nevertheless one can analyze its topology by evaluating the perimeter of closed curves, at constant r, which are circles, but are not geodesics of the surface. This perimeter $2\pi r$, ceases to be real for r < Rs. We therefore consider that this one is minimal and that there are closed curves which, transformed by *regular homotopy*, have this property. The object is therefore not *contractible*. It is a geometrical structure which establishes a bridge between two 2D Euclidean spaces.

By passing in 3D we can consider the hypersurface defined by its metric

(31)
$$ds^2 = \frac{dr^2}{1-\frac{Rs}{r}} + r^2(\,d\theta^2 + \sin^2\theta d\varphi^2\,)$$

Note that this metric is nothig but the spatial part of the Schwarzschild metric. While the 3D object was invariant by the action of O(2), this one is invariant by the action of O(3). We can show again its non-contractibility by computing by integration the area of the object constituted by the subset of points corresponding to a constant value of the coordinate.

(32)
$$A(r) = \iint \sqrt{g_{\theta\theta}\, g_{\varphi\varphi}}\; d\theta d\varphi = r^2 \int_0^{\pi} \sin^2\theta d\theta \int_0^{2\pi} d\varphi = 4\pi r^2$$

Again, in the realm of the real we conclude that this area has a minimal value which is that of its *throat sphere*. The considered object can be interpreted as resulting from the regular deformation (homotopy) of a sphere S2 of area according to spheres of minimum area $4\pi R_s^2$ which then have their own network of geodesics ("great circles"). But these are not geodesics of the object corresponding to the metric (17). These two surfaces are also generated by the action of groups, O(2) in the case of the Flamm surface and O(3) in this 3D hypersurface, ensuring a bridge between two Euclidean 3D spaces, through the intermediary of a throat sphere of area $4\pi R_s^2$.

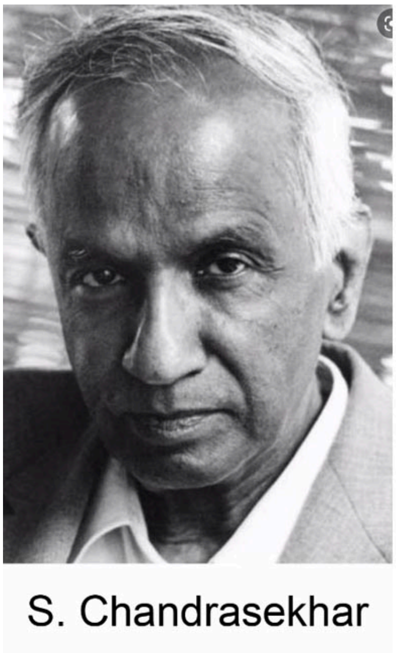
This 3D hypersurface thus presents two modes of generation. Either this one is done by a folding using spheres deduced from the throat sphere by homothety. The topology is then that of a bounded manifold and the geodesics of the spheres are not geodesics of the hypersurface. In a coordinate system the sections by "planes r = Cst are circles. Let this one be generated by the action of O(3) on a Flamm surface. The object then inherits the non-contractibility. To convince oneself of this, one must imagine that the Flamm surface, tilting according to the two angles θ and φ, sees all its points maintained at a distance from the geometric center greater than R_S. The infinite positions of this surface, according to the rotations, make that it always remains tangent to the throat circle, according to one of its great circles. The resulting 3D hypersurface thus corresponds to a bridge connecting two Euclidean 3D spaces, which is also the two-folds covering of a 3D bounded manifold, this one being bounded by the sphere of area $4\pi R_s^2$. This time curves resulting from the action of O(3) on the geodesics of the Flamm surface are also the geodesics of the 3D hypersurface

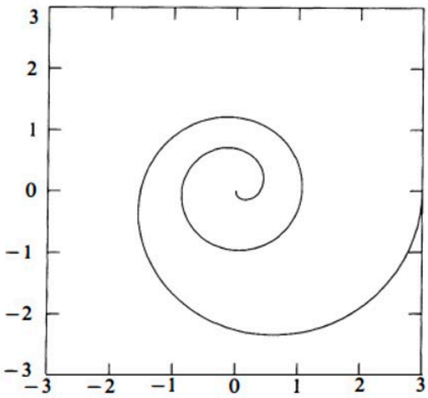
The Schwarzschild hypersurface is invariant by the action of two groups : O(3) and by R (translations), which conjugate through $O(3) \times R$. This hypersurface is deduced from the previous one by time translation. It inherits its property of non-contractibility. As suggested by Ludwig Flamm this 3D hypersurface can be considered as the intersection of hypersurface 4 by "planes t = Cst". But the geodesics of the 3D hypersurface are then not geodesics of the Schwarzschild 4D hypersurface. Moreover the hyperbolic character of the metric makes that the calculation of a perimeter corresponding to constant t , θ and r gives an imaginary value :

(33) $$p(r) = \int_0^{2\pi} \sqrt{g_{\varphi\varphi}}\, d\varphi = \iint \sqrt{-r^2}\ d\varphi$$

This leads to the eminently disconcerting character of the geometry of hypersurfaces defined by hyperbolic metrics. Everything depends then on the space in which we decide to represent the hypersurfaces, corresponding to what we consider as physical reality. In a space where everything is complex, the coordinates as well as the length element, everything can happen, everything is acceptable. A part of the hypersurface can correspond to real values of the coordinates (for $r > R_S$), leading to a real value of the length element. But, not less real values of these coordinates can lead to a pure imaginary value of this same length element : case of the Schwarzschild metric when $r < R_s$.

Instead of considering a topological interpretation, the post-war cosmologists, obsessed by the idea of a supposed contractibility of space, did not realize that they were operating an extension of the solution into the world of complexes. By the confusing game of algebraic calculation, the integration of the length according to a geodesic extended to r = 0 gives ... a real value! Here is a curve whose points are marked with real values of the coordinates, where the integrated length is real, which is located in a portion of space-time where the lengths are ... pure imaginary. One can find drawings of these so-called geodesics, spiraling to the geometric center, in Chandrasekhar's book entitled Mathematical Theory of Black Holes [21], published in 1983.





(h) $e = 0.1i,\ \ell = 1,\ M = \frac{3}{14}$

FIG. 7a. Various classes of time-like geodesics described by a test particle with $E^2 < 1$: (a), (b), (c): geodesics of the first and the second kind with eccentricity $e = 1/2$ and latera recta, $l = 11$, 7.5 and 3 respectively; (d): an example of a trajectory in which the orbits of the first and the second kind coalesce ($e = 1/2$, $l = 3/2$) for which $2\mu(3+e) = 1$); (e): an example of a circular orbit ($e = 0$, $l = 3/2$) and the associated orbit of the second kind; (f): the last unstable circular orbit when the orbit of the second kind spirals out of the orbit of the first kind ($e = 0$, $\mu = 1/6$); (g), (h): bound orbits with $l = 1$ and imaginary eccentricities $e = 0.01\,i$ and $0.1\,i$. (In all these figures $M = 3/14$ in the scale along the coordinate axes.)

Fig.13 : "geodesic » plunging towards the geometrical center of the Schwarzschild geometry of the Schwarzschild geometry [21].

It is interesting to read the commentary on his figure 7a, where he indicates that it is a "timelike" geodesic. In another part of the book we find what Chandrasekhar considers to be the Lagrangian associated to the Schwarzschild solution:

$$\mathscr{L} = \frac{1}{2}\left[\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - 2M/r} - r^2\dot{\theta}^2 - (r^2 \sin^2\theta)\dot{\varphi}^2\right]$$

Fig. 14 : What Chandrasekhar considers as the Lagrangian of the Schwarzschild solution.

Now, based on the current expression of the Schwarzschild metric ~~(Figure 4)~~ we should write, with a real and positive action:

(34) $$L = \sqrt{\left(1-\frac{2M}{r}\right)\dot{t}^2 - \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2 - r^2(\theta^2 + \sin^2\theta\varphi^2)}$$

New trap: The two expressions lead to the same system of Lagrange equations. The theorist, who forgets the condition r < Rs , thus switches to a complex geometry, without realizing it. This situation would have been avoided if the calculation of the geodesics had been based on the true formulation of the solution found by Schwarzschild, and not its transposition using the intermediate variable R. The metric is then written according to equation (3).

Hence the corresponding expression of the Lagrangian:

(35)

$$L = \frac{(r^3+R_s^{\ 3})^{1/3}-R_s}{(r^3+R_s^{\ 3})^{1/3}}\dot{dt}_s^{\ 2} - \frac{r^4}{(r^3+R_s^{\ 3})\left[(r^3+R_s^{\ 3})^{1/3}-R_s\right]}\dot{r}^2 - (r^3+R_s^{\ 3})^{2/3}(\dot{\theta}^2+\sin^2\theta\dot{\varphi}^2)$$

which then does not generate "chimerical geodesics".

While having neglected these precautions, cosmologists have opted for a representation, that is to say a choice of space and time coordinates, where the geodesics present a discontinuity at the crossing of the Scharzschild sphere at the same time as a 90° tilting of the cone of light, This has led Wheeler, Thorne and Misner [20] to the abracadabratic conclusion that "inside the Schwarzschild sphere r becomes a time coordinate and t a space coordinate". This is illustrated by the following figure

:

THE NULL GEODESICS 125

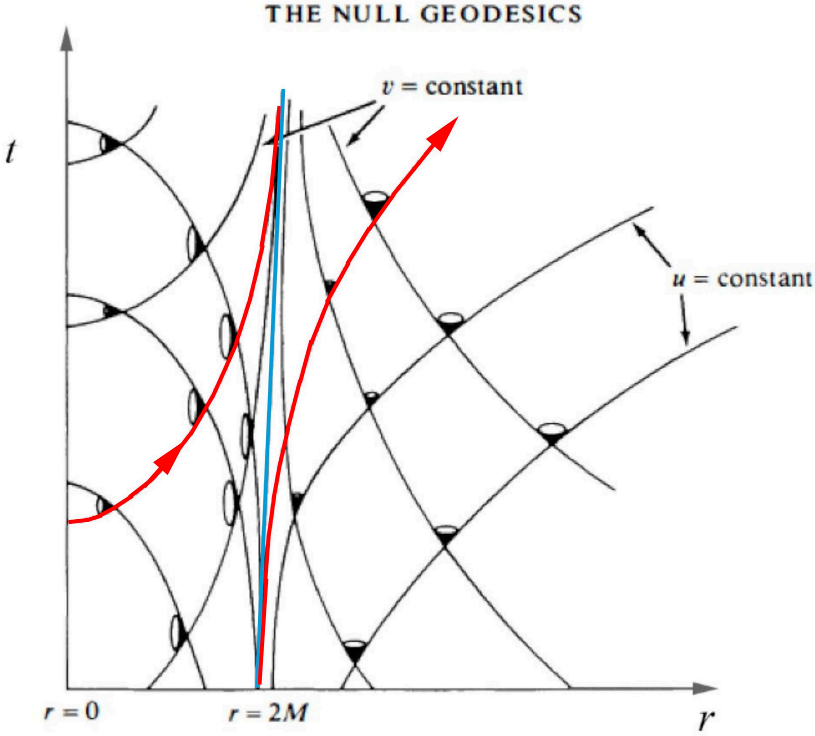


FIG. 8 Illustrating the ingoing and outgoing radial null-geodesics in the Schwarzschild coordinates.

Fig.15 : Extract from [20] , rotation of the light cone.

The plane projections of the geodesics correspond to the solution:

$$\varphi = \varphi_o + \int \frac{dr}{\sqrt{\frac{c^2\lambda^2 - 1}{h^2} r^4 + \frac{R_s r^3}{h^2} - r^2 + R_s r}} \tag{36}$$

which depends on two parameters, λ and h. The first one represents the ratio between the energy of the test particle, compared to its energy mc^2, the second one intervenes in the law of areas. The larger it is, the more the geodesic will deviate from a radial trajectory.

If we disregard this extension of the geodesics "inside the Schwarzschild sphere" there is, in a representation $\{ t , r , \theta , \varphi \}$, a very telling way to produce 3D images of the plane geodesics ($\theta = \pi / 2$) corresponding to this time-independent metric. The image of the Schwarzschild sphere is then that of a circle which develops, according to the time coordinate, according to a cylinder. Starting from a spatial projection of a geodesic, corresponding to given values of the parameters :

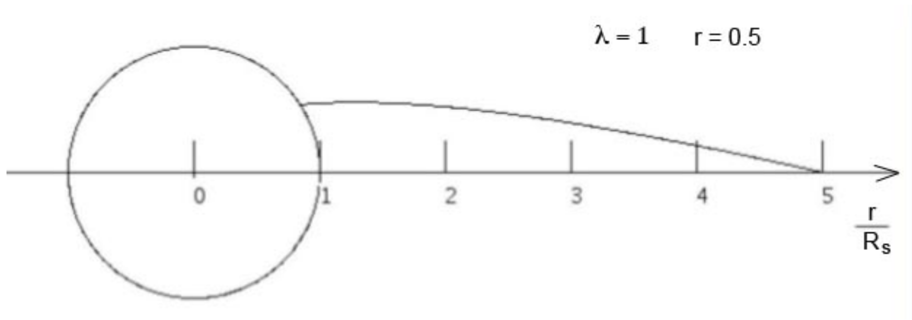


Fig.16 : Planar projection of the geodesic.

It is then easy to build a three-dimensional representation:

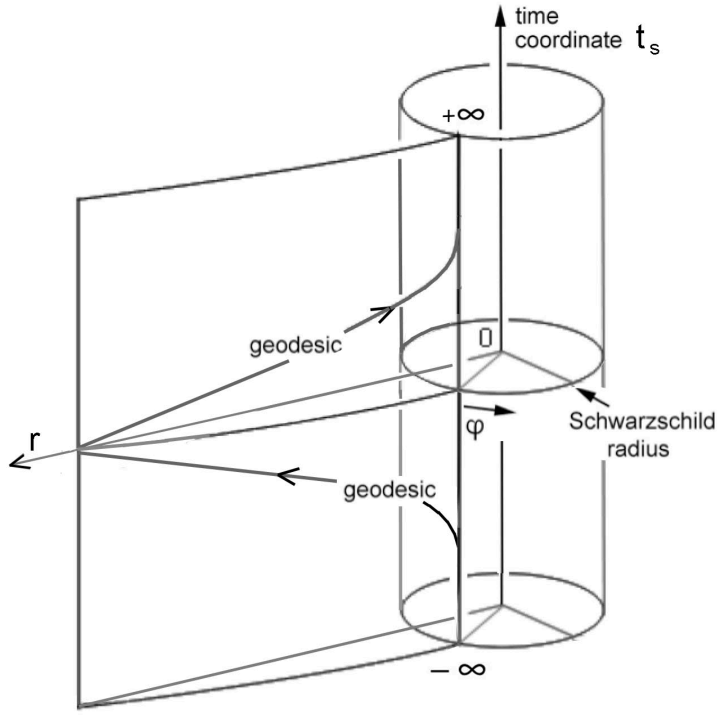


Fig.17 : Geodesics in the representation $\left(r , \varphi , t_s \right)$

Whatever the direction of travel of the geodesics, centripetal or centrifugal, with this choice of the time coordinate, reaching or extracting from the Schwarzschild sphere requires an infinite time. It is this aspect that constitutes one of the pillars of this black hole theory. But is there no other alternative?

4 – **About the hypothesis of the absence of a cross term in the metric**.

This assumption of no cross term is present in Schwarzschild's paper [3] and will be systematically renewed in the following decades, see his equation (14) reproduced in figure (2). This assumption is essential in the black hole model, because it gives infinite free-fall and escape times of a witness-mass, which allowed its designers to present the Schwarzschild solution, a stationary solution of the Einstein equation, expressed in the particular set of coordinates $\{ t_s , r , \theta , \varphi \}$ as a "snapshot" of an unsteady process, thus allowing to assimilate the resulting physical model to a "freeze-frame" extracted from an extremely fast implosion process. We have assigned the time variable an index s because, strictly speaking, it is the "Schwarzschild time t_s ".

(37) $$ds^2 = \left(1 - \frac{R_s}{r}\right) c^2 dt_s^2 - \frac{dr^2}{1 - \frac{R_s}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Kerr's metric (14), hailed by Chandrasekhar in his book [20] as a true triumph of mathematical research oriented towards the beauty of the mathematical solution, has a cross term in dr dϕ. We have seen that this had received an attempt at interpretation according to which the coordinate system, i.e. "space", was somehow "dragged along" by matter, which gave back corpps to the idea of Ernst Mach. Why then not consider a "radial frame dragging", a radial dragging of the coordinate system.

It is therefore perfectly licit to explore the properties of a representation accepting the presence of a term in dr dt. There have been several attempts in this direction, including that of Painlevé [21] Gullstrand [22].

(38) $$ds^2 = \left(1 - \frac{R_s}{r}\right) c^2 dt_{PG}^2 - dr^2 - \frac{2}{c}\sqrt{\frac{R_s}{r}}\, dt_{PG} dr - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

based on the change of time coordinate :

(39) $$t_s = t_{PG} - \frac{2 R_s}{c} \mathrm{Ln}\left(\frac{\sqrt{R_s / r} - 1}{\sqrt{R_s / r} + 1} \right)$$

The concern of the authors was to make the singularity disappear at r = R_S. We note that this metric tends to infinity towards the Lorentz metric.

Another change of variable was proposed [23], [24] by the Englishman Arthur Eddington in 1921:

(40) $$t_s = t_E^{(-)} + \frac{R_s}{c} \mathrm{Ln}\left(\frac{r}{R_s} - 1 \right)$$

A change of coordinate that applies to time ts, the "Schwarzschild time", to lead to another variable, which we will call "Eddington time-t minus: $t_E^{(-)}$ ". Expressed with this other set of coordinates, the metric becomes :

(41) $$ds^2 = \left(1 - \frac{R_s}{r}\right) c^2 dt_E^{(-)\,2} - \left(1 + \frac{R_s}{r}\right) dr^2 + 2\,\frac{R_s}{c\,r}\,dr\,dt_E^{(-)} - r^2\,(\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

Eddington's concern, like that of Painlevé, Gullstrand and many others, was to ensure the disappearance of the singularity in $r = R_s$, linked to the term in dr^2. Note that this approach can be completed by:

(42) $$t_s = t_E^{(+)} - \frac{R_s}{c}\,\mathrm{Ln}\left(\frac{r}{R_s} - 1\right)$$

(43) $$ds^2 = \left(1 - \frac{R_s}{r}\right) c^2 dt_E^{(+)\,2} - \left(1 + \frac{R_s}{r}\right) dr^2 - 2\,\frac{R_s}{c\,r}\,dr\,dt_E^{(+)} - r^2\,(\,d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

The geodesics, always with the constraint $r < R_s$ are deduced from the Lagrangian:

(44) $$L = \left(1 - \frac{R_s}{r}\right) c^2 \dot{t}_E^{(+)\,2} - \left(1 + \frac{R_s}{r}\right) \dot{r}^2 + 2\delta\,\frac{R_s}{c\,r}\,\dot{r}\,\dot{t}_E^{(+)} - r^2\,(\,\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\,)$$

with $\delta = \pm 1$.

This study was made by P.Koiran [25] . As this change of variable only affects the time coordinate, the projection of the geodesics is always deduced from equation (31). We will take as an example the geodesic corresponding to figure (18). The 3D representations in $\left\{\, t_E^{(+)}, r, \theta, \varphi \,\right\}$ and $\left\{\, t_E^{(-)}, r, \theta, \varphi \,\right\}$ correspond to the following figures. In the first one, a comparison is made between the Eddington time $t_E^{(+)}$, corresponding to the expression of the metric according to equation (39) and the Schwarzschild time t_s , which becomes infinite when r tends to R_s.

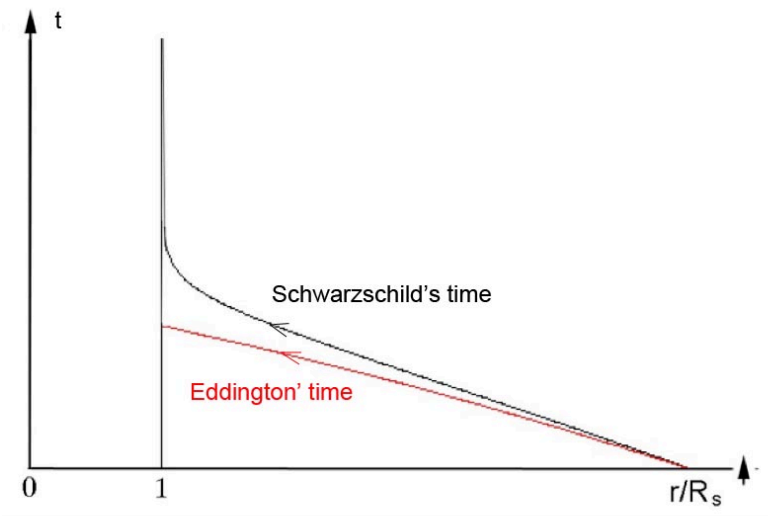


Fig. 18 : Evolution of the free fall time in representations $\left\{\, t_s, r, \theta, \varphi \,\right\}$ *and* $\left\{\, t_E^{(+)}, r, \theta, \varphi \,\right\}$

We see that the free fall time, expressed in the coordinate $t_E^{(+)}$, becomes finite. The curve is close to linearity which goes hand in hand with the entrainment of the coordinate system by imploding matter. We thus demonstrate that this hypothesis, which is the basis of the black hole theory, that the implosion of matter, for a distant observer living at the rate of the Schwarzschild time t_s, would last an infinite time $t_E^{(+)}$, disappears when we opt for another coordinate system where this free fall time, expressed with this new coordinate $t_E^{(+)}$ becomes finite. Indeed, when we make r tend to infinity in (36), it comes :

(45) $$ds = c\,d\tau = c\,dt_E^{(+)}$$

This time $t_E^{(+)}$ can thus perfectly be considered as time lived by a distant observer. With this mode of expression of the metric the escape time becomes infinite. The opposite conclusion with the metric (36). We can summarize these results with the help of three-dimensional representations:

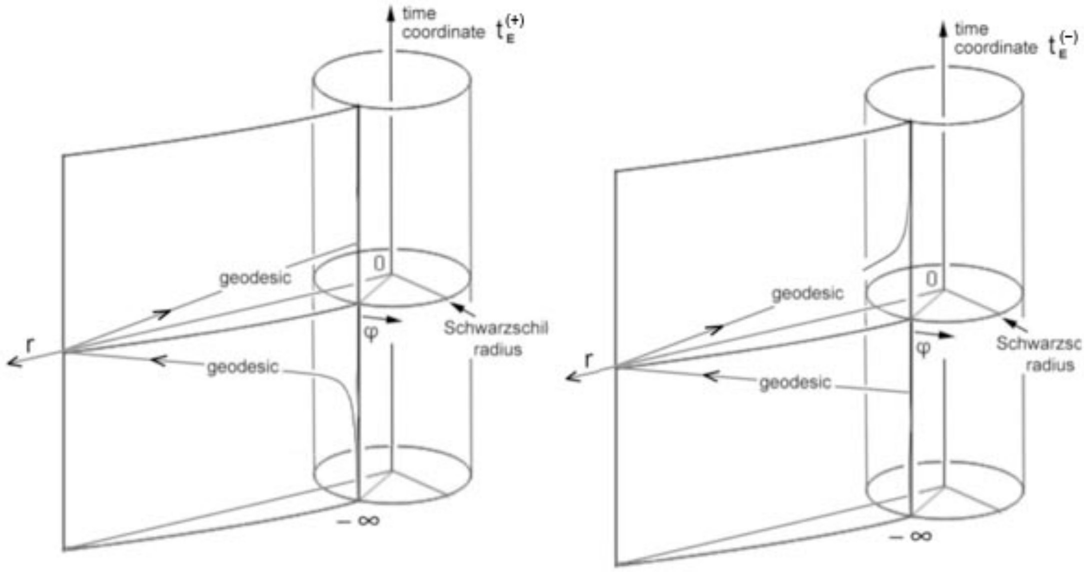


Fig. 19 : Time for free fall and escape
in the representations $\left\{ t_E^{(+)}, r, \theta, \varphi \right\}$ et $\left\{ t_E^{(-)}, r, \theta, \varphi \right\}$

5 – **Reinterpretation of the stop of geodesics on the Schwarzschid sphere by the topology**.

Immediately we imagine a structure with two folds $F^{(+)}$ and $F^{(-)}$, the fold $F^{(+)}$ being equipped with the metric (38) and the fold $F^{(-)}$ with the metric (36). This space-time geometry corresponds to a *one-way membrane*, which can only be crossed in one direction. The solution would then evoke a snapshot of a phenomenon translating the passage of matter from one sheet to the other, in a very short time. In spatial projection we would have, in (r , φ) :

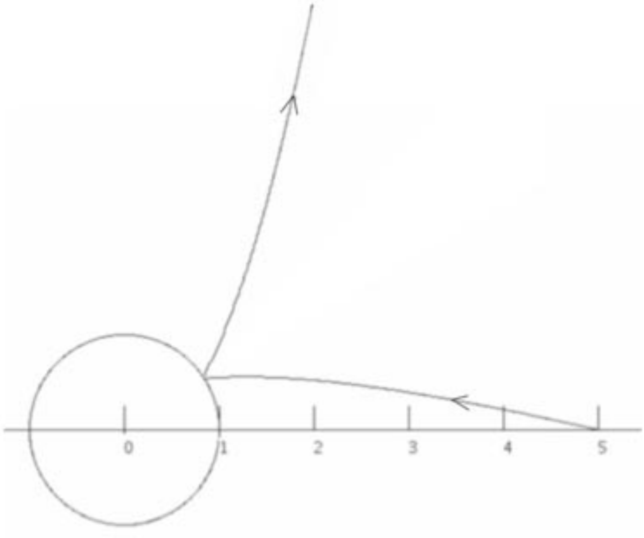


Fig.20 : Projection of the extended geodesics.

The geodesics continue, but undergo a break when crossing the Schwarzschild sphere, which then becomes a throat sphere.

Can we consider another coordinate system where the continuity of the geodesic can be established? One answer lies in the change of coordinate affecting the r-coordinate, introduced in [26].:

(46) $$r = R_s \left(1 + L_n \operatorname{ch}\rho \right)$$

With $d\Omega^2 = d\theta^2 + \sin^2 d\varphi^2$ the metrics become :

(47) $$ds^2 = \frac{L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} c^2 dt_E^{(+)2} - \frac{2 + L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} R_s^{\,2} \operatorname{th}^2 \rho \, d\rho^2 - \frac{2 R_s d\rho \, dt_E^{(+)}}{c(1 + L_n \operatorname{ch}\rho)} - R_s^{\,2}(1 + L_n \operatorname{ch}\rho)^2 d\Omega^2$$

(48) $$ds^2 = \frac{L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} c^2 dt_E^{(-)2} - \frac{2 + L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} R_s^{\,2} \operatorname{th}^2 \rho \, d\rho^2 + \frac{2 R_s d\rho \, dt_E^{(-)}}{c(1 + L_n \operatorname{ch}\rho)} - R_s^{\,2}(1 + L_n \operatorname{ch}\rho)^2 d\Omega^2$$

Is it possible to ensure the complete continuity of the geodesics translating the transit of the matter through the gorge sphere, corresponding to $\rho = 0$? Answer: it is enough that the time coordinate is reversed when crossing this gorge sphere

(49) $$t_E^{(-)} = - t_E^{(+)}$$

So we simply have the common metric, describing the two folds :

(50) $$ds^2 = \frac{L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} c^2 dt_E^2 - \frac{2 + L_n \operatorname{ch}\rho}{1 + L_n \operatorname{ch}\rho} R_s^{\,2} \operatorname{th}^2 \rho \, d\rho^2 - \frac{2 R_s d\rho \, dt_E}{c(1 + L_n \operatorname{ch}\rho)} - R_s^{\,2}(1 + L_n \operatorname{ch}\rho)^2 d\Omega^2$$

The projection $\{ \rho, \varphi \}$ of the geodesics in the representation $\{ t_E, \rho, \theta, \varphi \}$ becomes [26]:

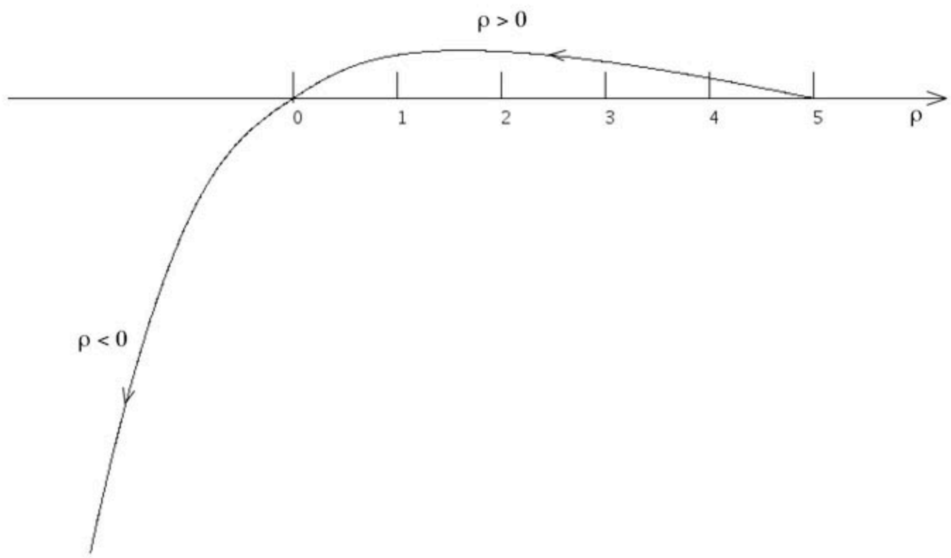


Fig.21 : Projection $\{ \rho, \varphi \}$ *from the representation* $\{ t_E, \rho, \theta, \varphi \}$

From (41) we get :

(51) $$dr = Rs\, th\rho\, d\rho$$

corresponding to the curve:

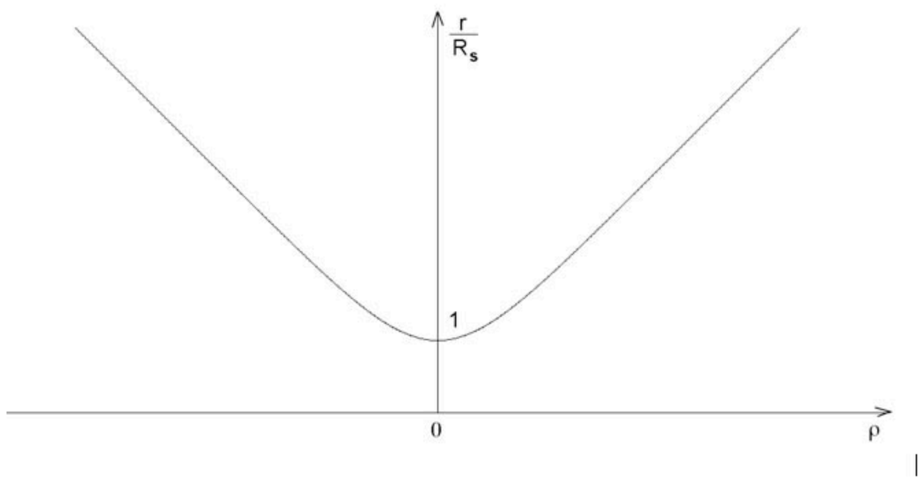


Fig.22 : Relationship between the two spatial coordinates r *and* ρ

The fact that thρ is null on the sphere of throat confers the continuity to the geodesics, in this system of reference, when we consider the movements of matter going from our sheet to the gemellar sheet. Hereafter the evolution of the transfer time, function of the coordinate ρ . Its continuity, as well as that of its spatial projection (figure 23), lead to the complete continuity of this plunging geodesic. Conversely, the emerging geodesic has two branches which tend towards infinity. We have a one-way membrane.

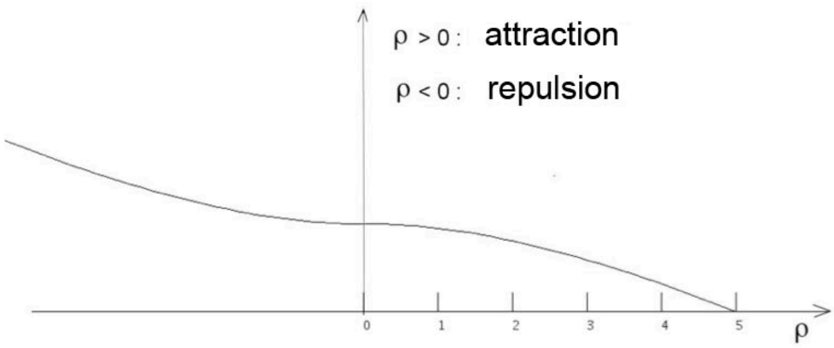


Fig.23 : Transit time in the coordinate system $\left\{ t_E , \rho , \theta , \varphi \right\}$

For a value of r we have two values of ρ corresponding to adjacent points. This corresponds to a topological covering structure with two folds F(+) and F(-). If we want to compare the projection of figure 22 with the diagram resulting from the passage to a coordinate ρ, we need to perform a P-symmetry in the graph of figure 24 (symmetry with respect to the origin) to understand how the portions of the curves are conjugated.

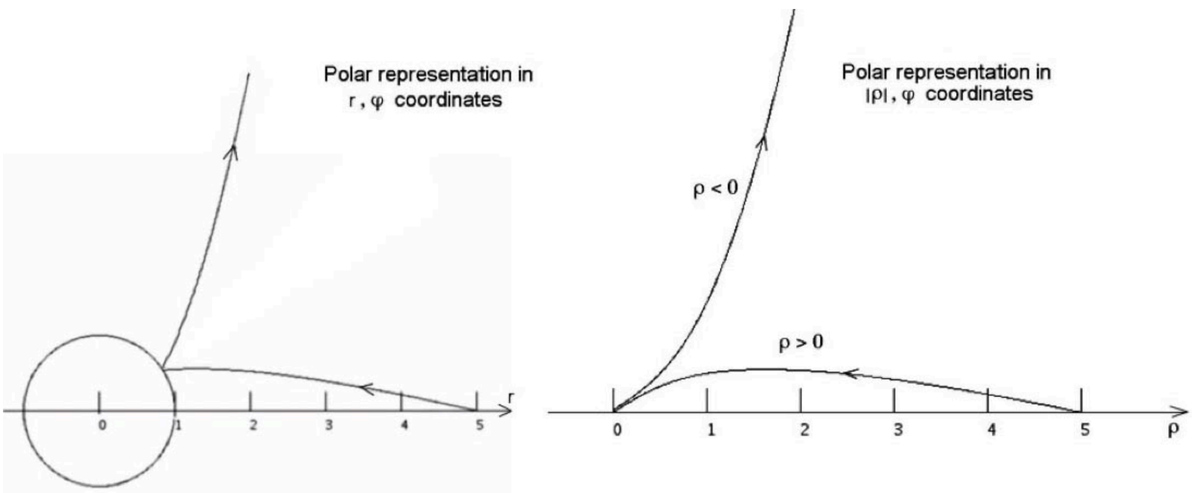


Fig. 24 : Projection of the geodesic in the two polar representations

To try to represent the phenomena, we have only images resulting from the choice of coordinates, each of these representations having their interest, with respect to the geometrical intuition, and their defects. If we opt for a representation (r , φ) according to the coordinates of plane geodesics, we have in mind the image of a sphere of throat of finite area but where the interior of this sphere does not belong to the hypersurface. When a mass plunges towards this gorge sphere, it seems to "bounce" on it. We have an image of the portion of trajectory corresponding to this "bounce" but the geodesics of this second sheet constitute, with respect to the starting sheet, a disjointed whole. As the perception that we can have only proceeds through the medium of electromagnetic waves, travelling on geodesics of zero length, we have to give up the idea of obtaining an evidence of these masses through observations, once they have passed on this kind of "reverse of the universe". In fact, the most telling 2D didactic image is that of a hole, with a circular edge, made in a sheet of paper. The universe, where we can walk, is the paper. In the hole, the paper is absent. A hole is the topological solution allowing to pass from the front to the back of a surface. This geometry, which we see being built, suggests that the notions of front and back apply to hypersurfaces with more than two dimensions. In 3D we can consider the non-contractible hypersurface, defined by the metric (27). The two 3D Euclidean sheets, thus put in communication, must be considered as the "front" and "back" of the 3D hypersurface constituted by the Euclidean space we are familiar with. Becoming aware of this must become as obvious as discovering that one can write on the front and back of a sheet of paper. Keeping in mind that "the inhabitants of the front side" cannot "see" the inhabitants of the back side, and vice versa.

If we opt for a representation $\left\{ t_E , \rho , \theta , \varphi \right\}$ the sphere of throat disappears, is reduced to a point. Continuity appears. But we must then consider that two points diametrically opposed with respect to the center of symmetry correspond in fact to the same region of space-time, to two points located "on either side" of the 4D hypersurface. Moreover, if we construct a sphere containing this point O and if we deform it by regular homotopy to bring it in contact with this point, its area tends to a finite limit!

6 – **Topology of space-time and associated dynamic group structure.**

This section builds on the material developed in reference [30]. The dynamical group associated to the Euclidean space, which is also its isometry group, is the Poincaré group:

(52) $$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}$$

where C is the space-time translation 4-vector:

(53) $$C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

and L is a matrix of format (4,4) representing the Lorentz group. Let us take as a metric matrix the Gramm matrix:

(54)
$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The axiomatic definition of the Lorentz group is:

(55)
$$^{t}L\,G\,L = G$$

A first action of this group, of dimension 10, is its action on the space-time. By positing:

(56)
$$X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

This action is written:

(57)
$$\begin{pmatrix} X' \\ 1 \end{pmatrix} = \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} X \\ 1 \end{pmatrix} = \begin{pmatrix} L\,X + C \\ 1 \end{pmatrix}$$

To this action is added the action on the dual of the group on its Lie algebra, on its space of moments, whose dimension is that of the group: 10. The moment of the Poincaré group is composed of:

- A scalar : energy E
- A 3-vector : impulsion $\begin{pmatrix} p_x & p_y & p_z \end{pmatrix}$
- Another 3-vector : spin $\begin{pmatrix} s_x & s_y & s_z \end{pmatrix}$
- An additional 3-vector, so called « passage » $\begin{pmatrix} f_x & f_y & f_z \end{pmatrix}$

which does not characterize the motion in the sense that it is cancelled out if we consider a system of axes linked to the particle. We arrange these components to form.

- The 4-vector energy-impulsion :

(58)

$$P = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

- An antisymmetrical matrix :

(59)
$$M = \begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix}$$

The action of the group on its momentum space is written [30]:

(60)
$$\begin{aligned} M' &= L M\, {}^tL + C\, {}^tP\, {}^tL - L P\, {}^tC \\ P' &= L P \end{aligned}$$

The Lorentz group has four related components, two orthochron (not reversing the time coordinate) and two "antichron" (reversing the time coordinate).

In physics, conceived through the prism of relativity, particles follow geodesics of space-time. On short distances and short intervals of time these geodesics are close to geodesics of tangent space, the Minkowski space. The elements of the group allow to decline the fans of the motions of the different particles. According to geodesics of zero length: these are photons. According to geodesics of non-zero length it is particles with a non-zero mass.

The antichronic components of the Lorentz group, and therefore of the Poincaré group, which inherits this property, reverse time and, correlatively, energy E and mass m. Mathematical formalisms sometimes suggest the existence of exotic objects. Sometimes this does not work. But sometimes it does work. The typical example is Dirac's prediction of the existence of antimatter, which aroused great skepticism, until the analysis of the motion of electrons from cosmic rays confirmed the Englishman's prediction.

The complete Poincaré group (whose subgroup, the restricted Poincaré group, is limited to its orthochron components) predicts the existence of particles of negative energy and mass. Does this correspond to a physical reality?

The answer lies in the evidence of the acceleration of the cosmic expansion, under the effect of a negative pressure. Now a pressure, if we usually perceive it as a force per unit area, is also a volumetric density of energy. Hence the birth of the concept of dark energy.

To a gas made of particles of negative mass is automatically associated the negative pressure :

(61) $$p^{(-)} = \frac{\rho^{(-)} < v^{(-)2} >}{3} < 0$$

whera $< v^{(-)2} >$ represents the root mean square speed of thermal agitation in the medium.

7- **Back to the inner Schwarzchild metric** [5]

We have already mentioned Schwarschid's result in Figure 4, where his interior metric is expressed using coordinates $\{ t_s , \chi , \theta , \varphi \}$. By simply performing the inverse variable change :

(62) $$\chi = \arcsin\left(\frac{r}{\hat{R}} \right)$$

We easily fall back on the classical expression:

(63) $$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{R_n^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{\hat{R}^2}} \right]^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

with:

(64) $$\hat{R} = \sqrt{\frac{3c^2}{8\pi G \rho}}$$

We are then in a proven stationary situation. There is therefore no need to use time coordinates with crossed terms. with (59) the exterior metric has the form (32). The first term of the interior metric is identified with the square of the time factor:

(65) $$ds^2 = f^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

with :

(66) $$f(r) = \frac{3}{2}\sqrt{1 - \frac{R_n^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{\hat{R}^2}}$$

The proper time flows, at the center of the star, according to:

(67) $$d\tau = f \, dt$$

Now, as Karl Schwarzchild noted in 1916, when the radius of the star becomes equal to $\sqrt{\frac{8}{9}}\,\hat{R}$ the pressure at the center of the star, so does the local value of the speed of light become infinite, and f becomes zero. The cone of light opens up, until it becomes a plane, and ends up turning over like an umbrella in a gust of wind.

Hereafter the evolution of the time factor

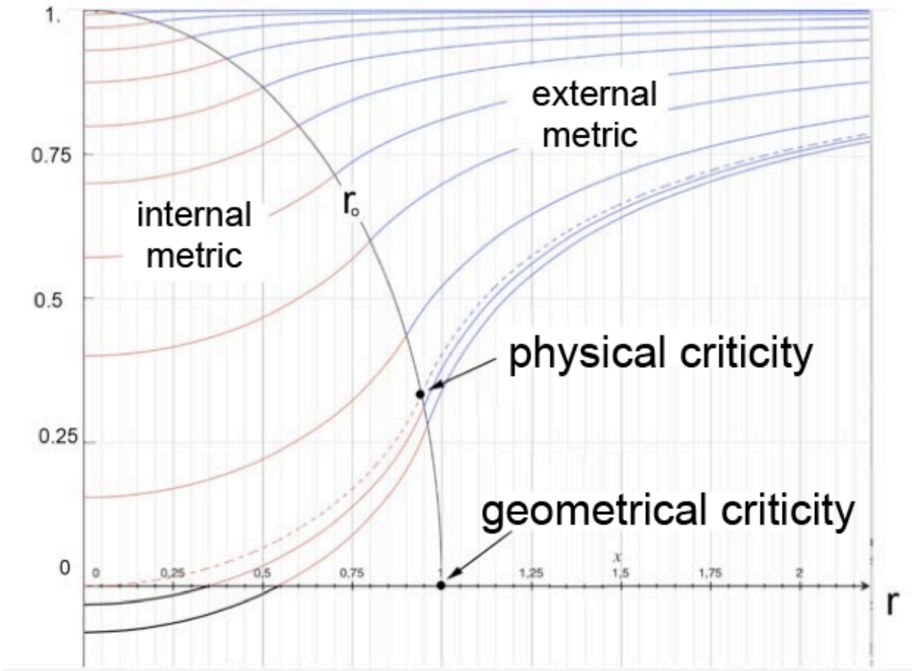


Fig.25 : Time factor versus radius for different mass values.

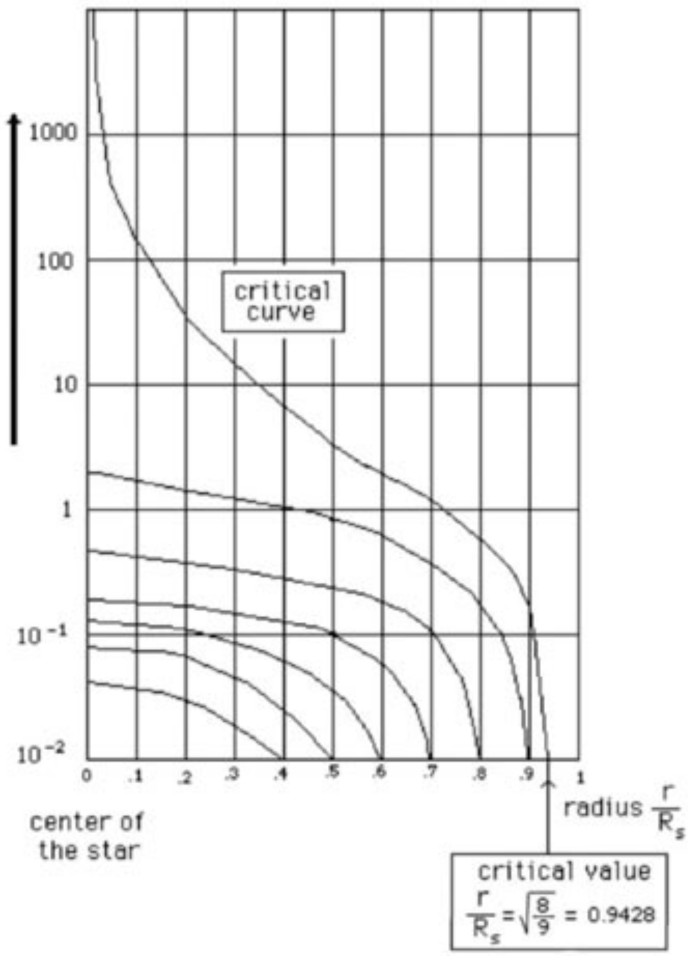


Fig.26 : Pressure evolution (TOV model)

If the mass is a little higher than this critical value, f changes sign. But the proper time τ cannot be reversed, so it is the time coordinate which is reversed, and beyond that the energy and the mass of the particles contained in this portion of space.

We can see that this theme of an inversion of the time, energy and mass coordinates manifests itself once again.

We note that the formalisms of quantum mechanics also generate states of negative energy [40], [41].

8- **A cosmology with inclusion of negative masses.**

The question then arises of how to introduce negative masses into the cosmological model. If we stick to the Einsteinian geometrical context, based on a single field equation, it is impossible. Indeed, locally, there is only one family of geodesics. Therefore, test particles, whether their mass is positive or negative, react in the same way when they are immersed in the gravitational field. In this context, if the appearance of a physical criticality would lead to a reversal of the mass of the particles located in the immediate vicinity of the geometric center of the star would remain confined within it. Moreover, as a general rule, the introduction of negative masses in the Einsteinian universe generates the unmanageable *runaway* phenomenon, which contradicts two principles:

- The action-reaction principle
- The principle of equivalence.

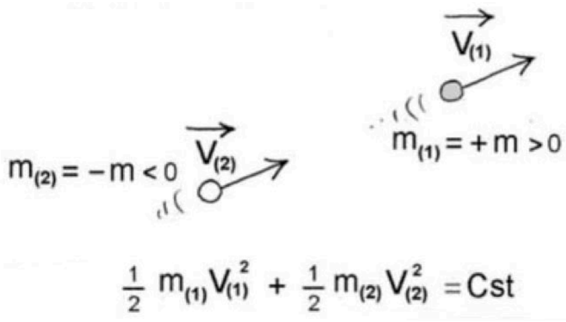


Fig. 27 : Runaway phenomenon.

A solution consists in restoring these two principles, which implies to consider that the negative masses react differently in the gravitational field, thus follow geodesics resulting from a second metric $g_{\mu\nu}^{(-)}$, while the particles of positive mass follow the geodesics resulting from the metric $g_{\mu\nu}^{(+)}$. From these metrics emerge two tensors $R_{\mu\nu}^{(+)}$ and $R_{\mu\nu}^{(-)}$ and two Ricci scalars $R^{(+)}$ and $R^{(-)}$. These elements then emerge from the system of coupled Janus field equations ([31] to [41]), here in their mixed form :

(68) $$R_{\nu}^{\mu(+)} - \frac{1}{2} R^{(+)} \delta_{\nu}^{\mu} = \chi \left[T_{\nu}^{\mu(+)} + \sqrt{\frac{g^{(-)}}{g^{(+)}}}\; \hat{T}_{\nu}^{\mu(-)} \right]$$

(69) $$R_{\nu}^{\mu(-)} - \frac{1}{2} R^{(-)} \delta_{\nu}^{\mu} = -\chi \left[\sqrt{\frac{g^{(+)}}{g^{(-)}}}\; \hat{T}_{\nu}^{\mu(+)} + T_{\nu}^{\mu(-)} \right]$$

$g^{(+)}$ and $g^{(-)}$ are the determinants of the two metrics. δ_{ν}^{μ} is the Kronecker symbol and χ the Einstein constant. The tensors of the second members are of the form

(70)
$$T^{\mu(+)}_{\nu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & -p^{(+)}/c^{(+)2} & 0 & 0 \\ 0 & 0 & -p^{(+)}/c^{(+)2} & 0 \\ 0 & 0 & 0 & -p^{(+)}/c^{(+)2} \end{pmatrix}$$

(71)
$$T^{\mu(-)}_{\nu} = \begin{pmatrix} \rho^{(-)} & 0 & 0 & 0 \\ 0 & -p^{(-)}/c^{(-)2} & 0 & 0 \\ 0 & 0 & -p^{(-)}/c^{(-)2} & 0 \\ 0 & 0 & 0 & -p^{(-)}/c^{(-)2} \end{pmatrix}$$

(72)
$$\widehat{T}^{\mu(+)}_{\nu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & p^{(+)}/c^{(+)2} & 0 & 0 \\ 0 & 0 & p^{(+)}/c^{(+)2} & 0 \\ 0 & 0 & 0 & p^{(+)}/c^{(+)2} \end{pmatrix}$$

(73)
$$\widehat{T}^{\mu(-)}_{\nu} = \begin{pmatrix} \rho^{(-)} & 0 & 0 & 0 \\ 0 & p^{(-)}/c^{(-)2} & 0 & 0 \\ 0 & 0 & p^{(-)}/c^{(-)2} & 0 \\ 0 & 0 & 0 & p^{(-)}/c^{(-)2} \end{pmatrix}$$

The shape of interaction tensors is determined by the satisfaction of the Bianchi conditions inside the masses. Outside, the shape of the first members is sufficient to satisfy this condition.

The Newtonian approximation leads to the interaction pattern:

- Masses of the same sign attract each other by Newton's law

- Masses of opposite signs repel each other by "anti-Newton".

9 – **Plugstars, an alternative model to the black hole model.**

Under these conditions, if a mass contribution brings the metric $g^{\mathrm{int}(+)\mu}_{\nu}$ into a state of physical criticality, an equivalent quantity of matter, in the center, sees its mass immediately reversed. The interaction between two particles of matter is essentially by exchange of photons (electromagnetic field). The photons $\varphi^{(+)}$ and $\varphi^{(-)}$ of the two subsystems follow the geodesics of zero length of the two metrics $g^{\mathrm{ext}(+)\mu}_{\ \ \ \ \ \ \ \nu}$ and $g^{\mathrm{ext}(-)\mu}_{\ \ \ \ \ \ \ \nu}$, which form disjoint systems. Thus the negative masses, no longer interacting with the positive masses, leave the star freely.

This negative feedback system from the system (63), (64) maintains neutron stars in a subcritical state, limiting their mass to two solar masses, while in the black hole model this "geometric" critical mass is around three solar masses. In this scheme it is never reached. The higher evaluation of the masses of so-called "stellar black holes" would then be attributable to erroneous interpretations of observational data.

The so-called "giant black holes", at the center of galaxies, would also be subcritical objects. These considerable masses, present at the center of many galaxies, do not form by accretion. The mechanism presiding over the formation of these objects, as residues of the quasar phenomenon, will be the subject of another article.

It is easy to construct the "joint" metrics that the inverted masses then follow. If we write the metrics $g^{\mathrm{int}(+)\mu}_{\ \ \ \ \ \ \ \nu}$ and $g^{\mathrm{ext}(+)\mu}_{\ \ \ \ \ \ \ \nu}$ using the density ρ_o:

(74)

$$ds^2 = \left[\frac{3}{2}\sqrt{1-\frac{8\pi G\rho_o R_n^2}{3c^2}} - \frac{1}{2}\sqrt{1-\frac{8\pi G\rho_o r^2}{3c^2}}\right]^2 c^2dt^2 - \frac{dr^2}{1-\dfrac{8\pi G\rho_o r^2}{3c^2}} - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

(75)
$$ds^2 = \left(1-\frac{8\pi G\rho_o R_n^2}{3c^2 r}\right)c^2dt_s^2 - \frac{dr^2}{1-\dfrac{8\pi G\rho_o R_n^2}{3c^2 r}} - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

We deduce the metrics $g^{\mathrm{int}(+)\mu}_{\ \ \ \ \ \ \ \nu}$ and $g^{\mathrm{ext}(+)\mu}_{\ \ \ \ \ \ \ \nu}$ by reversing the sign in front of ρ_o:

(76)

$$ds^2 = \left[\frac{3}{2}\sqrt{1+\frac{8\pi G\rho_o R_n^2}{3c^2}} - \frac{1}{2}\sqrt{1+\frac{8\pi G\rho_o r^2}{3c^2}}\right]^2 c^2dt^2 - \frac{dr^2}{1+\dfrac{8\pi G\rho_o r^2}{3c^2}} - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

(77)
$$ds^2 = \left(1+\frac{8\pi G\rho_o R_n^2}{3c^2 r}\right)c^2dt_s^2 - \frac{dr^2}{1+\dfrac{8\pi G\rho_o R_n^2}{3c^2 r}} - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

The non-zero negative mass geodesics can be deduced from the relation (31) where the sign of the field creating mass M has been inverted, which is equivalent to assigning Rs of a minus sign :

(78) $$\varphi = \varphi_o + \int \frac{dr}{\sqrt{\frac{c^2\lambda^2 - 1}{h^2} r^4 - \frac{R_s r^3}{h^2} - r^2 - R_s r}}$$

Hereafter is the characteristic shape of the geodesics:

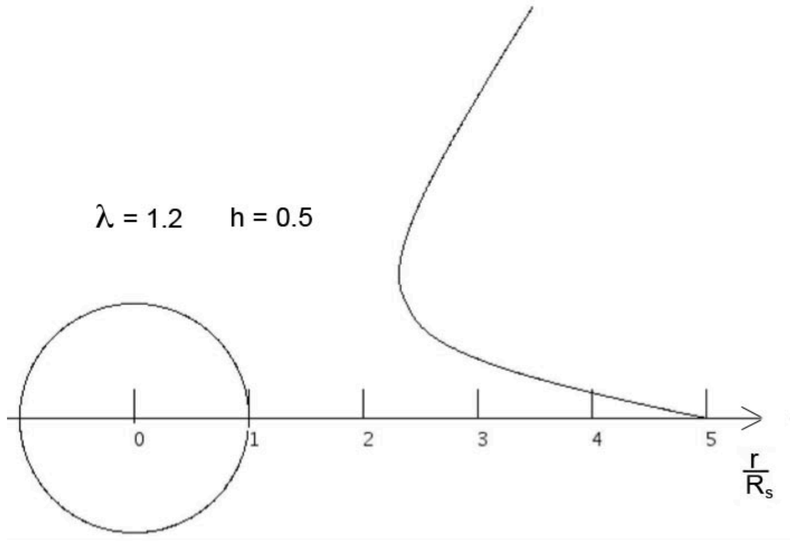


Fig.28 : Geodesic followed by a negative mass,
in the field created by the positive mass M

We note the necessary high value to give to the parameter λ. The explanation is simple. By "pulling" a negative mass towards the object, so as to allow it to approach it at a distance of only a few Schwarzschild radii, i.e. in regions where a gravitational field, in this case repulsive, is very intense, it is necessary to communicate to it, at a very great distance, a velocity such that the kinetic energy which accompanies it, negative, is no longer negligible in front of its also negative energy mc^2.

To give an idea of this acceleration and of the way in which the projection of the geodesic is modified, we indicate hereafter what emerges from a combination of the two geodesic arcs deriving from the external Schwarzschid metric, attractive, then repulsive in the second sheet in their polar representation (ρ, φ), a curve to be compared to that of figure 21

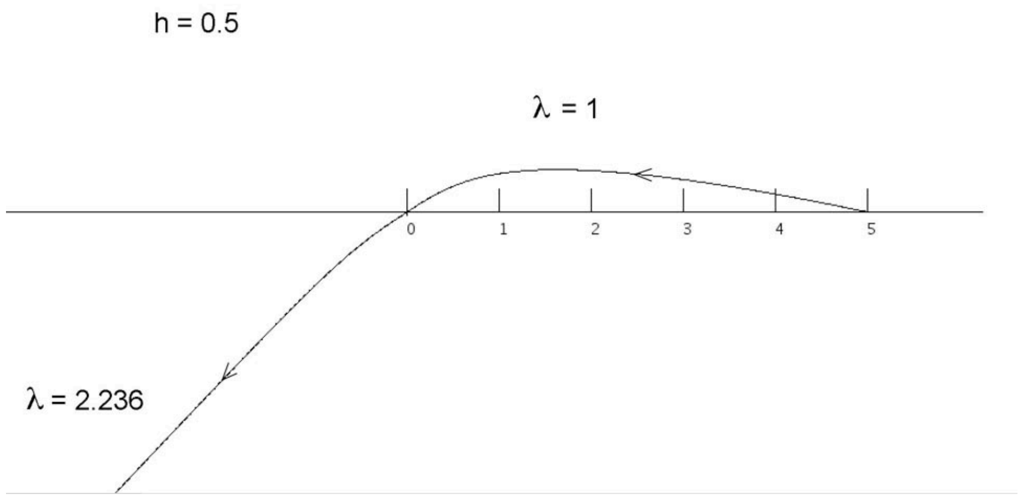


Fig 29 : Projection of the geodesic with succession of attraction and repulsion.

On the following figure, the evolution of the transit time, finite:

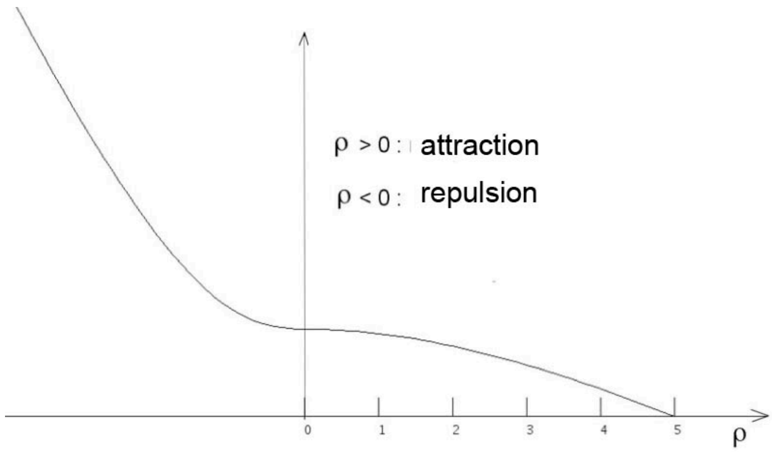


Fig.30 : Time evolution during mass transfer with inversion.

Curve to be compared with that of figure 23. This double continuity leads to the global continuity of the geodesic after crossing a sphere of throat.

As a conclusion the negative masses ejected will be given, under the effect of the repulsion, a non-negligible speed in front of c. The idea of a non conservation of the kinetic energy (in absolute value) induces a distorted vision of the phenomena. It is necessary to take into account the gravitational energy in the balance, implied by the dispersion of negative masses.

10 – **Falsifiability of the Plugstar model.**

No theoretical model can be considered if it does not fit with observational data. The Janus model, among the main effects explained:

- Accounts for the confinement of galaxies, undergoing the back pressure of the surrounding negative mass, which explains in passing the flatness of their rotation curves, as well as the strong gravitational lensing effects produced. A similar observation is made for galaxy clusters, explaining the high velocities of their components. Thus the Janus model ([31] to [41]), advantageously replaces the use of hypothetical positive mass dark matter by a negative mass content.

- Accounts for the acceleration of the cosmic expansion ([31], [33]), the dark energy being only the energy of negative mass particles.

- Accounts for the lacunar structure of negative mass, organized around invisible conglomerates of negative mass, the dipole repeller object being one of them [37].

- Provides a model of a perennial galactic spiral structure [31], a phenomenon reflecting a transfer of momentum (dynamical friction) by density waves towards the negative mass environment [32].

- Gives to the invisible components of the universe a precise identity: antiprotons, antineutrons, antielectrons of negative mass, this set constituting the primordial antimatter, escaping observation because emitting photons of negative energy [31].

- Makes interstellar travel non-impossible, as distances become a hundred times shorter in the second fold of the universe and the speed of light, of negative energy photons, ten times higher [38].

Although it will be a long time before we have sufficiently precise data on the X-ray sources, companions of stars, we have such data for two hypermassive objects located, one at the center of the galaxy M87, the other at the center of the Milky Way. The measurements of the ratios of the brightness temperatures, maximum and minimum, for these two objects, is found to be very close to 3.

Let us calculate the value of this gravitational redshift z which is then common to all plugstars, whatever their mass M.

Let R_a be the radius of the star, which is then $\sqrt{\frac{8}{9}}\,\hat{R} = \sqrt{\frac{8}{9}}\,\frac{3c^2}{8\pi G \rho_o}$ while the Schwarzschid radius is $R_s = \frac{2GM}{c^2} = \frac{8\pi G \rho_o R_a^3}{3c^2}$.

The gravitational redshift, affecting the light emitted at the surface of the star in a radial direction pointing to the observer, is then:

$$z = \frac{1}{\sqrt{1 - \frac{R_s}{R_a}}} = \frac{1}{\sqrt{1 - \frac{8}{9}}} = 3$$

This is exactly the value that can be deduced from the first two images of supermassive objects located at the center of the M87 galaxies and the Milky Way.

Other images of supermassive objects will appear. We conjecture that these brightness temperature ratios will be systematically equal or lower than 3. An ancillary consequence is the questioning of the evaluation of the masses of merging objects. The scenario of these mergers, as well as that of the creation of neutron stars in stars with the highest masses (up to 200 solar masses) remain to be written. In the supernova phenomenon, when the mass of the massive star collapses on its iron core. If the mass involved exceeds 2 solar masses the physical criticality is immediate and the excess mass is reversed. This inversion results in the emission of a gravitational wave of phenomenal intensity. The same scenario applies to the fusion of two subcritical neutron stars, if the sum of their masses exceeds 2 solar masses.

The non-inclusion of this alternative scenario casts doubt on the evaluation of the masses of "stellar black holes", resulting from the calculations of Kip Thorne

11 – **Conclusion**

We show that the construction of the black hole model was based on a failure to take into account the second paper of Karl Schwarzschild, describing the geometry inside the masses, which already at that time mentioned a physical criticality occurring before the mass corresponding to the classical geometric criticality appeared. The explanation lies in the fact that the first English translation of this second article appeared only in 1999. Many specialists are not even aware of its existence. The model, exclusively based on the "external" solution of Schwarzschild (and later of Kerr) has many weaknesses and incongruities on the mathematical and geometrical level, which are highlighted. With such errors at the beginning one cannot build a scenario leading to this vision of a geometry organized around a hypothetical "central singularity". Indeed, when one starts from a sound geometry, based on the two metrics constructed by Schwarzschild in 1916, describing the interior and the interior of a star, a physical criticality appears even before this geometrical criticality appears. It is shown that this results in the appearance of a region, in the vicinity of the center, where the inversion of the time coordinate leads to that of the energy and mass of the particles involved. This observation calls for the introduction of negative masses in the cosmological model. This, because of the appearance of the runaway phenomenon and the violation of the action-reaction and equivalence principles, is impossible in the framework of general relativity, based on the unique Einstein field equation. By opting for the Janus model, based on numerous observational confirmations, the laws of interaction, derived from Newtonian approximations, change. Masses of the same sign attract each other while masses of opposite signs repel each other. Sensitive to the gravitational field created by the negative mass, the negative masses are powerfully ejected by freely crossing the star. This leads to a feedback phenomenon which limits the masses to their subcritical value. For neutron stars: around 2 solar masses. The existence of stellar black holes is therefore questioned. Supermassive objects, whose formation will be explained in a future article, are also subcritical objects, Plugstars. The theory gives all plugstars a gravitational redshift of 3. This is exactly what is shown by the measurements of the images of two hypermassive objects located at the center of the galaxies M87 and the Milky Way.

We predict that this redshift of 3 will accompany future images of hypermassive objects that will appear in the future.

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