Janus, the only cosmological model that explains the acceleration of cosmic expansion

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We show that it is possible to integrate negative masses into the Janus cosmological model by describing the universe as a manifold equipped with two metrics, one referring to positive masses and the other to negative masses, as solutions of a system of two coupled field equations. The acceleration of the cosmic expansion then results from the dominance of the negative energy content, showing a good agreement with observational data.

I. INTRODUCTION

At the end of the 2000s, observational data began to accumulate, tending to show the movement of expansion, far from slowing down as previously expected, was on the contrary accelerating. The collected data were considered sufficiently reliable for this discovery to be rewarded with a Nobel Prize in 2011 [1–3]. However the cause of this acceleration remained unknown. The only approach proposed by specialists consisted in reintroducing the cosmological constant in the equation serving as the basis for general relativity, the famous Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \chi\, T_{\mu\nu} \tag{1}$$

In the past, before the Italian Torricelli discovered atmospheric pressure, it was considered that the cause of mercury rising in the barometers was "the horror of the vacuum", which we applied to quantify. But it did not constitute a real explanation, relying on clear physical foundations. Later on, it was evidently established that what caused the column of mercury to rise in the tube was nothing but than the impact of the very numerous collisions of air molecules on the free surface of the liquid metal, resulting which was called pressure. In the same way, that "the introduction of this cosmological constant in the equation, translates a repulsive power of the vacuum" cannot be considered as a satisfactory explanation.

The scientific community then gave it the qualifier "dark energy", which then has a negative value. On the dynamic level, this new unknown component is of considerable, majority importance. The specialists are since looking for a component of the cosmic soup which makes it possible to clarify this problem.

We present an explanation which introduces negative masses in the cosmological model. This has been attempted for a long time by the cosmologist Hermann Bondi [4]. This one concludes that the laws governing the interactions between positive and negative masses, within the framework of general relativity, do not lead to a true model, valid on the physical plan. They especially violate the fundamental principles of action-reaction and equivalence. Let's come back to this point.

General relativity is based on:

- The universe, the space-time, is a M_4 manifold, a four-dimensional hypersurface, defined at any point and at any time by its metric field $g_{\mu\nu}$.
- From this metric we can calculate two sets of geodesics. Those of non-zero length are followed by particles of matter, those of zero length by photons.
- If we know the source of the field, so the values at any point of the components of the tensor of the second member $T_{\mu\nu}$, we can calculate the metric solution $g_{\mu\nu}$.
- If we concentrate on those of non-zero length geodesics, the ones referring to the masses, for a given field $T_{\mu\nu}$, there is only one family of geodesics, along which the control particles will circulate, whether their masses are positive or negative.

To tackle this problem with precision, we resort to the two metric solutions constructed by Schwarzschild in 1916 [5, 6]. There is first the interior metric solution describing a mass M, likened to a sphere filled with an incompressible fluid of constant density ρ_0:

$$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{8\pi G\rho_0\, {r_0}^2}{3\, c^2}} - \frac{1}{2}\sqrt{1 - \frac{8\pi G\rho_0\, r^2}{3\, c^2}}\right]^2 c^2 dt^2 - \frac{dr^2}{1 - \dfrac{8\pi G\rho_0\, r^2}{3\, c^2}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 \tag{2}$$

which links to the solution referring to the vacuum surrounding this mass:

$$ds^2 = \left(1 - \frac{2\, G\, M}{c^2\, r}\right) c^2\, dt^2 - \frac{dr^2}{1 - \dfrac{2\, G\, M}{c^2\, r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \tag{3}$$

The non-zero length geodesics are given by:

$$\varphi = \varphi_0 + \int_{r_0}^{r} \frac{dr}{\sqrt{\dfrac{c^2\lambda^2 - 1}{h^2} r^4 + \dfrac{2\, G\, M\, r^3}{c^2} - r^2 + \dfrac{2\, G\, M\, r}{c^2}}} \tag{4}$$

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When this mass M is positive, the geodesics are of the ellipse, parabola, hyperbola type, all reflecting an attraction. We deduce, according to the model of general relativity that:

- The positive masses attract their fellow beings as well as the negative masses.

If this field is created by a negative mass, then to obtain the geodesics it suffices to change the sign in this Schwarzschild metric solution, the quantities M and ρ_0 being only simple constants of integration:

$$ds^2 = \left[\frac{3}{2}\sqrt{1+\frac{8\pi G|\rho_0|{r_0}^2}{3\,c^2}} - \frac{1}{2}\sqrt{1+\frac{8\pi G|\rho_0|r^2}{3\,c^2}}\right]^2 c^2dt^2 - \frac{dr^2}{1+\dfrac{8\pi G|\rho_0|r^2}{3\,c^2}} - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 \quad (5)$$

$$ds^2 = \left(1+\frac{2\,G\,|M|}{c^2\,r}\right)c^2\,dt^2 - \frac{dr^2}{1+\dfrac{2\,G\,|M|}{c^2\,r}} - r^2(d\theta^2+\sin^2\theta\,d\varphi^2) \quad (6)$$

The geodesics are then deduced from the expression:

$$\varphi = \varphi_0 + \int_{r_0}^{r}\frac{dr}{\sqrt{\dfrac{c^2\lambda^2-1}{h^2}r^4 - \dfrac{2G|M|r^3}{c^2} - r^2 - \dfrac{2G|M|r}{c^2}}} \quad (7)$$

Curves of the parabola and ellipse type disappear. Only geodesics of the hyperbole type remain, evoking a repulsion (see Fig. 1).

As a conclusion:

- Negative masses repel their fellow beings as well as positive masses.

Thus, if two masses of opposite signs are brought together, the positive mass flees, pursued by the negative mass. Both undergo a uniform acceleration without any input of energy, since that of the negative mass is negative. We have given the name *runaway* to this phenomenon, which violates both the principle of action-reaction and the principle of equivalence (Fig. 2).

II. AN ATTEMPT TO INTRODUCE NEGATIVE MASSES SATISFYING TO THE PRINCIPLES OF ACTION-REACTION AND EQUIVALENCE

In a purely heuristic approach, we could consider interaction laws corresponding to:

- Masses with same sign mutually attract by Newton's law.

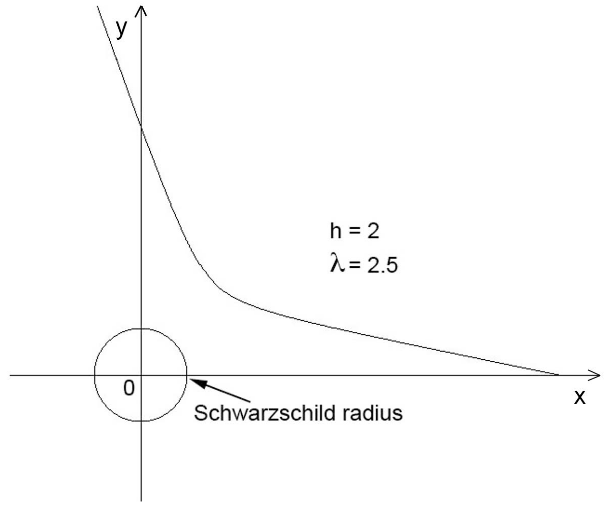

FIG. 1. Geodesics in a repelling field

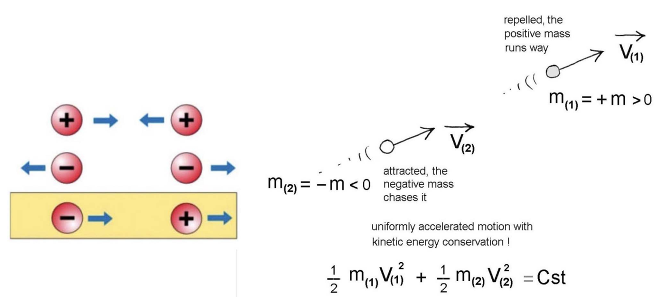

FIG. 2. The introduction of negative masses in general relativity results in the *runaway* phenomenon.

- Masses with opposite signs mutually repel by anti-Newton's law.

This was envisaged as early as 1995 [7, 8], with some profit, with respect to an attempt to explain the large-scale structure of the universe and the spiral structure of galaxies [8]. But we then ran into a serious pitfall: these laws do not match with general relativity. Indeed they implied that the witness masses, positive and negative, behave differently when they were immersed in the same gravitational field. To do this they had to follow different geodesic curves, from two different metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$

These cannot emerge from a single field equation. In the general relativity equation, Einstein's equation, the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R follow from the solution metric $g_{\mu\nu}$. However, two distinct metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$ generate different tensors $R_{\mu\nu}^{(+)}$ and $R_{\mu\nu}^{(-)}$ as well as scalars $R^{(+)}$ and $R^{(-)}$. Hence, we have a kind of bi-geometry, bimetric configuration.

III. FIRST BIMETRIC GEOMETRIES

In 2004 T. Damour and I. Kogan [9, 10] consider the interaction between two geometric entities, qualified as "right" and "left" "branes". A construction using a Lagrangian, inspired by the classical construction, leads them to propose the system of the following two coupled field equations:

$$2\,M_R^2\,(R_{\mu\nu}(g^R) - \frac{1}{2}g^R_{\mu\nu}\,R(g^R)) + \Lambda_R\,g^R_{\mu\nu} = T^R_{\mu\nu} + t^R_{\mu\nu}$$
$$2\,M_L^2\,(R_{\mu\nu}(g^L) - \frac{1}{2}g^L_{\mu\nu}\,R(g^L)) + \Lambda_L\,g^L_{\mu\nu} = T^L_{\mu\nu} + t^L_{\mu\nu} \tag{8}$$

In these equations the tensors $T^R_{\mu\nu}$ and $T^L_{\mu\nu}$ have the classical form, inspired by general relativity. The interaction relies entirely on the "interaction tensors" $t^R_{\mu\nu}$ and $t^L_{\mu\nu}$. Over the course of an article reaching forty pages, the two authors try different approaches, based on different models, without success. Moreover, the model is complicated by the assumption that the interactions involve gravitons with a mass spectrum.

The German Sabine Hossenfelder produces the second attempt in 2008 [11]:

$$^{(g)}R_{\kappa\nu} - \tfrac{1}{2}\,g_{\kappa\nu}\,^{(g)}R = T_{\kappa\nu} - \underline{V}\sqrt{\frac{h}{g}}\,a^{\underline{\nu}}_{\nu}\,a^{\kappa}_{\underline{\kappa}}\,\underline{T}_{\underline{\kappa}\,\underline{\nu}}$$
$$^{(h)}R_{\nu\kappa} - \tfrac{1}{2}\,g_{\nu\kappa}\,^{(h)}R = T_{\nu\kappa} - W\sqrt{\frac{g}{\underline{h}}}\,a^{\kappa}_{\underline{\kappa}}\,a^{\nu}_{\underline{\nu}}\,T_{\nu\,\kappa} \tag{9}$$

The two interacting geometric entities are then identified by the Greek letters κ and ν. The construction of the interaction terms is then based on the introduction of a functional, covariant relation:

$$\delta h_{\kappa\lambda} = -\left[a^{-1}\right]^{\mu}_{\kappa}\left[a^{-1}\right]^{\nu}_{\lambda}\delta g_{\mu\nu} \tag{10}$$

The author therefore limits herself to a systematic and binding hypothesis of symmetry between the two entities. This leads him to build an evolution scheme based on two FRLW metrics, simple copies of each other. The interaction laws are deduced from the two equations, through the Newtonian approximation. Under these conditions, the scheme once again leads to a violation of the principle of equivalence, and the author concludes that this is a property of all bimetric systems.

IV. ADAPTATION OF THE SYSTEM FOR A SATISFACTION OF THE PRINCIPLES ACTION-REACTION AND EQUIVALENCE

The Janus system of equations [12–19] can at this stage be considered as a variant of the model d'Hossenfelder, at the cost of a simple sign change (modulo the mode of writing the interaction tensors):

$$^{(g)}R_{\kappa\nu} - \frac{1}{2}\,g_{\kappa\nu}\,^{(g)}R \qquad = T_{\kappa\nu} - \underline{V}\sqrt{\frac{h}{g}}\,a^{\underline{\nu}}_{\nu}\,a^{\kappa}_{\underline{\kappa}}\,\underline{T}_{\underline{\kappa}\,\underline{\nu}}$$
$$^{(h)}R_{\nu\kappa} - \frac{1}{2}\,g_{\nu\kappa}\,^{(h)}R \quad = -\left[T_{\nu\kappa} - W\sqrt{\frac{g}{\underline{h}}}\,a^{\kappa}_{\underline{\kappa}}\,a^{\nu}_{\underline{\nu}}\,T_{\nu\,\kappa}\right] \tag{11}$$

Going back to the presentation in Eq. (11), with our sign conventions, this system is written:

$$R^{(+)\nu}_{\ \ \mu} - \frac{1}{2}R^{(+)}\delta^{\nu}_{\mu} \qquad = \chi\left[T^{(+)\nu}_{\ \ \mu} + \sqrt{\frac{g^{(-)}}{g^{(+)}}}\widehat{T}^{(-)\nu}_{\ \ \mu}\right]$$
$$R^{(-)\nu}_{\ \ \mu} - \frac{1}{2}R^{(-)}\delta^{\nu}_{\mu} \qquad = -\chi\left[\sqrt{\frac{g^{(+)}}{g^{(-)}}}\widehat{T}^{(+)\nu}_{\ \ \mu} + T^{(-)\nu}_{\ \ \mu}\right] \tag{12}$$

Such a system must satisfy the Bianchi conditions. These are satisfied identically for the first members. The tensors $T^{(+)\nu}_{\ \ \mu}$ and $T^{(-)\nu}_{\ \ \mu}$ are:

$$T^{(+)\nu}_{\ \ \mu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & -\frac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 & 0 \\ 0 & 0 & -\frac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 \\ 0 & 0 & 0 & -\frac{\mathtt{p}^{(+)}}{c^{(+)2}} \end{pmatrix}$$
$$T^{(-)\nu}_{\ \ \mu} = \begin{pmatrix} \rho^{(-)} & 0 & 0 & 0 \\ 0 & -\frac{\mathtt{p}^{(-)}}{c^{(-)2}} & 0 & 0 \\ 0 & 0 & -\frac{\mathtt{p}^{(-)}}{c^{(-)2}} & 0 \\ 0 & 0 & 0 & -\frac{\mathtt{p}^{(-)}}{c^{(-)2}} \end{pmatrix} \tag{13}$$

By applying the Bianchi condition to them, we obtain the relations reflecting, inside the masses, the balance between the force of pressure and the force of gravity. At this stage, the only constraint to which the interaction tensors $\widehat{T}^{(+)\nu}_{\ \ \mu}$ and $\widehat{T}^{(-)\nu}_{\ \ \mu}$ must obey is this Bianchi condition, which precisely determines their shape (see the detail of the calculations in the Appendix A).

$$\widehat{T}^{(+)\nu}_{\ \mu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & \dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 & 0 \\ 0 & 0 & \dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 \\ 0 & 0 & 0 & \dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} \end{pmatrix}$$
$$\widehat{T}^{(-)\nu}_{\ \mu} = \begin{pmatrix} \rho^{(-)} & 0 & 0 & 0 \\ 0 & \dfrac{\mathtt{p}^{(-)}}{c^{(-)2}} & 0 & 0 \\ 0 & 0 & \dfrac{\mathtt{p}^{(-)}}{c^{(-)2}} & 0 \\ 0 & 0 & 0 & \dfrac{\mathtt{p}^{(-)}}{c^{(-)2}} \end{pmatrix} \tag{14}$$

The system of coupled field equations [Eq. (12)] is valid only if it produces solutions consistent with observations. This had been outlined in [12]. While Hossenfelder has always placed herself in a perspective of symmetry between the two entities, the Janus model is based on the contrary on a fundamental asymmetry, the cosmic dynamics being under the control of the majority negative mass population. This asymmetry will be explained in a future article.

It had been exploited in [7] through numerical simulations. Under these conditions the negative masses, whose accretion time is then lower:

$$t_J^{(-)} = \frac{1}{\sqrt{4\pi G|\rho^{(-)}|}} \ll t_J^{(+)} = \frac{1}{\sqrt{4\pi G\rho^{(+)}}} \tag{15}$$

Thus, the negative masses form a regular system of spheroidal clusters, confining the positive mass in the remnant place, giving it a lacunar structure. This will be taken up and developed in a future article.

V. FIRST APPLICATION OF THE JANUS MODEL: EXPLANATION OF THE ACCELERATION OF COSMIC EXPANSION

Before bringing this asymmetry hypothesis into play, let's start by examining the compatibility of Eq. (12) when we assign them solutions, homogeneous and isotropic, of the FRLW type:

$$g^{(+)}_{\mu\nu} = c^{(+)2}dt^2 - \frac{a^{(+)2}}{1-k^{(+)}}\left[dr^2 + d\Omega^2\right]$$
$$g^{(-)}_{\mu\nu} = c^{(-)2}dt^2 - \frac{a^{(-)2}}{1-k^{(-)}}\left[dr^2 + d\Omega^2\right] \tag{16}$$

These equations become:

$$R^{(+)\nu}_{\ \mu} - \frac{1}{2}R^{(+)}\delta^\nu_\mu = \chi\left[T^{(+)\nu}_{\ \mu} + \frac{c^{(-)2}a^{(-)3}}{c^{(+)2}a^{(+)3}}\widehat{T}^{(-)\nu}_{\ \mu}\right]$$
$$R^{(-)\nu}_{\ \mu} - \frac{1}{2}R^{(-)}\delta^\nu_\mu = -\chi\left[\frac{c^{(+)2}a^{(+)3}}{c^{(-)2}a^{(-)3}}\widehat{T}^{(+)\nu}_{\ \mu} + T^{(-)\nu}_{\ \mu}\right] \tag{17}$$

In the classical treatment of the FRLW solution of Einstein's equation, we ended up with the conservation of energy:

$$E = \rho\, c^2\, a^3 = \mathtt{Cst} \tag{18}$$

A similar treatment (compatibility between the two equations) leads to the condition:

$$E = \rho^{(+)}\, c^{(+)2}a^{(+)3} + \rho^{(-)}c^{(-)2}a^{(-)3} = \mathtt{Cst} \tag{19}$$

which translates a conservation of global energy.

By focusing on the phase where these are universes of dust, where we can neglect the pressure, we can thus write the equations:

$$R^{(+)\nu}_{\ \mu} - \frac{1}{2}R^{(+)}\delta^\nu_\mu = \frac{\chi E}{c^{(+)2}a^{(+)3}} \tag{20a}$$

$$R^{(-)\nu}_{\ \mu} - \frac{1}{2}R^{(-)}\delta^\nu_\mu = -\frac{\chi E}{c^{(-)2}a^{(-)3}} \tag{20b}$$

In this system is the Einstein constant, which we take, in accordance with the form retained for the field tensors, equal to:

$$\chi = -\frac{8\pi G}{c^{(+)2}} \tag{21}$$

It is negative. The signs of the second members of the equations of the system shown in Eq. (20) will therefore be opposite. If the overall energy, dominated by the negative mass content, is negative, then Eq. (20b) will give an evolution of the Friedman model type, with an acceleration of this expansion. The solution of the Eq. (20a), then showing an acceleration of the expansion was studied by W.Bonnor [20]. This solution can be written in the following parametric form:

$$a^{(+)}(u) = \alpha^2 ch^2 u \tag{22}$$

$$t^{(+)}(u) = \frac{\alpha^2}{c^{(+)}}(1 + \frac{1}{2}sh(2u) + u) \tag{23}$$

To simplify the notations, we put: $a^{(+)} = a$, $c^{(+)} = c$. The "deceleration parameter" q becomes:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{2sh^2u} < 0 \tag{24}$$

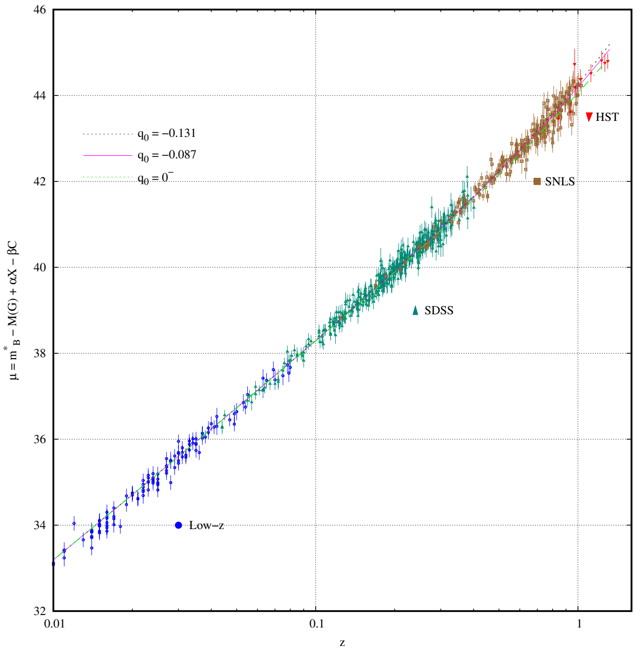

FIG. 3. Hubble diagram of the combined sample (log redshift scale)

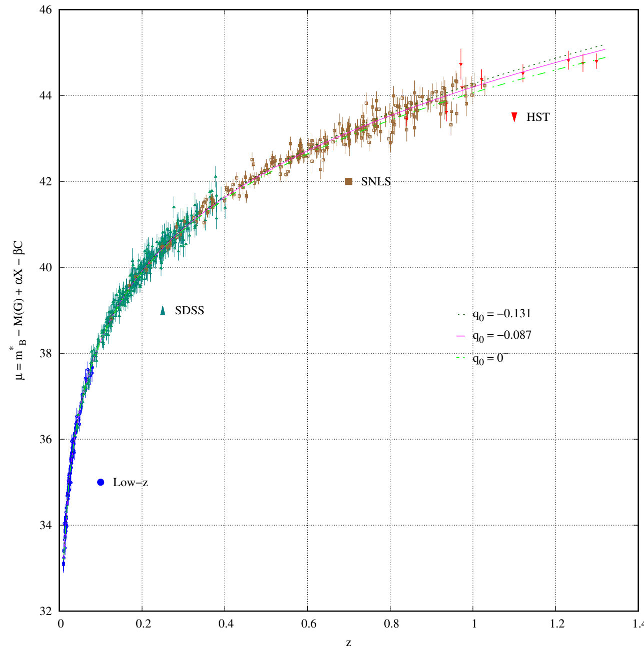

FIG. 4. Hubble diagram of the combined sample (linear redshift scale)

The construction of the relationship between the bolometric magnitude and the redshift is given in the appendix. It comes:

$$m_{bol} = 5\log_{10}\left[z + \frac{z^2(1-q_0)}{1+q_0z+\sqrt{1+2q_0z}}\right] + \texttt{cst} \quad (25)$$

where $q_0 < 0$ and $1 + 2q_0z > 0$. Sizing the values of q_0 and of the constant in order to fit available observational data [1], we get:

$$q_0 = -0.087 \pm 0.015 \quad (26)$$

Results presented below, show the standardized distance modulus $\mu = 5\log_{10}(d_L/10pc)$, linked to experimental parameters through the relation:

$$\mu = m_B^* - M_B + \alpha X_1 - \beta C \quad (27)$$

where m_B^* is the observed peak magnitude in rest frame B band, X_1 is the time stretching of the light curve and C the supernova color at maximum brightness.

Both M_B, α and β are nuisance parameters in the distance estimate.

We took the values given in [1] corresponding to the best fit of the whole set of combined data (740 supernovae) with ΛCDM model.

With the best fit we get χ^2/d.o.f. = 657/738 (740 points and 2 parameters). The corresponding curves are shown in Fig. 3, 4, 5, 6, in excellent agreement with the experimental data. The comparison with both model best fits are shown in Fig. 7.

We can derive the age of the universe (see Appendix B) with respect to q_0 and H_0 and some numerical values,

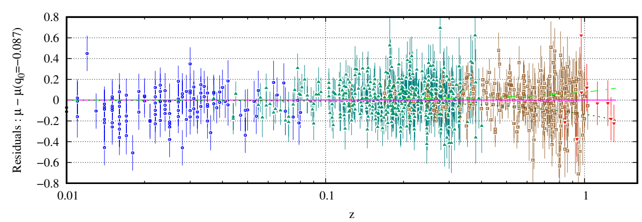

FIG. 5. Residuals of the combined sample (log redshift scale)

for different (q_0, H_0) values, are given in Table I. For our best fit, we get:

$$T_0 = \frac{1.07}{H_0} = 15.0\,Gyr \quad (28)$$

The content of the universe can then be revised. In this Cosmological Janus model, dark matter and dark energy are replaced by one component, a negative mass content, which will then be perfectly identified and which will be seen to account for all the phenomena for which these

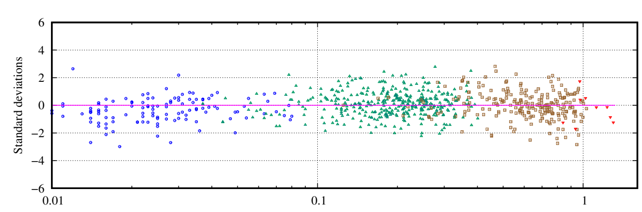

FIG. 6. Standard deviations of the combined sample (log redshift scale)

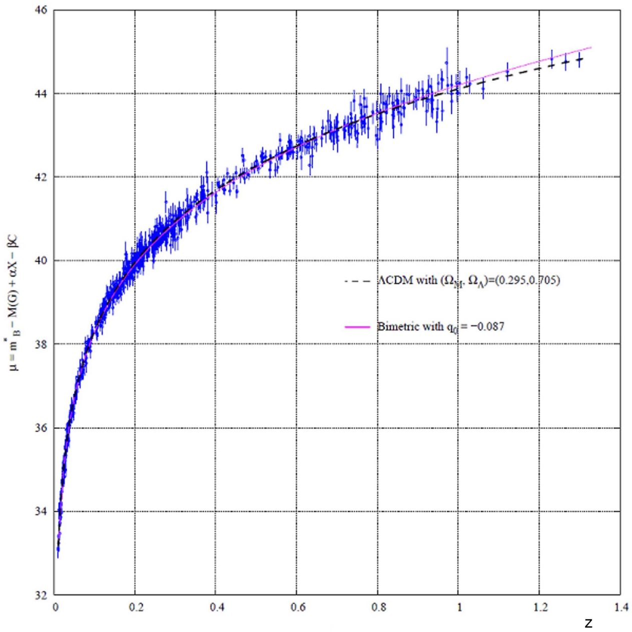

FIG. 7. Hubble diagram compared with the 2 models (linear redshift scale)

T_0 (Gyr)		q_0 0.000	-0.045	-0.087	-0.102	-0.117	-0.132
H_0	70	14.0	15.0	15.0	14.9	14.9	14.8
	73	13.4	14.4	14.4	14.3	14.3	14.2

TABLE I. T_0 values with respect to q_0 and H_0

two components, of an unspecified nature to date, were created. Hence, the composition of the universe is:

- 96 % (invisible) negative mass,
- 4 % positive mass.

When we try to interpret the phenomenon of acceleration of the expansion by invoking the cosmological constant Λ, this then evokes the presence of a negative energy independent of time, therefore independent of the expansion. Dynamically, it predicts an exponential expansion as a function of time. In the Janus model, the curvature indices k^+ and k^- are equal to -1. The geometries are therefore hyperbolic and the scale factors $a^{(+)}$ and $a^{(-)}$ tend asymptotically towards linear functions of the common chronological marker $x^o = c^{(+)}t^{(+)} = ct$.

VI. CONCLUSION

The model presented represents a profound change in the cosmological paradigm in the sense that we move from a description of the universe as a variety equipped with a single metric $g_{\mu\nu}$, solution of a single field equation, Einstein's equation, to a manifold equipped with two metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$, referring respectively to positive masses and negative masses, solutions of a system of two coupled field equations. The Newtonian approximation then provides interaction laws which, unlike what emerges from the attempt to introduce negative masses into the Einsteinian model (runaway phenomenon), satisfy the principles of action-reaction and equivalence. Masses of the same sign then attract each other according to Newton's law, while masses of opposite signs repel each other according to "anti-Newton". As will be developed in the following articles, gravitational instability tends to separate these two populations. Under these conditions, when one population dominates in one region of the universe, the other is mainly excluded.

Then the first equation of the proposed system is identified with Einstein's equation, which then appears as a limiting description, in the regions where the negative mass content can be neglected. The model thus agrees with the classic verifications of general relativity, of a local nature. As will be shown in subsequent articles, these negative mass particles are simply negative mass anti-hydrogen and anti-helium. We will show that the result of the interaction with the positive masses and the photons of positive energy accounts for all the phenomena that we attributed until now to the dark matter, which is then no longer necessary. The energy of this negative mass population being itself negative then identifies with what was called "dark energy". Thus, the two components of the model are thus replaced by a single component, ensuring these two functions, perfectly identified. As negative mass particles emit negative energy photons, they escape our observations in the field of optics. Incidentally, and this will also be the subject of another article, this negative mass antimatter is then identified with this great absentee: primordial antimatter, another problem for which the Janus cosmological model is the only one to provide a precise and consistent answer.

That said, we show that this new Janus cosmological model perfectly accounts for the acceleration of cosmic expansion, being the only one to provide a real explanation of this phenomenon.

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Appendix A:

The proposed Janus equations are:

$$G^{(+)}_{\mu\nu} = R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu} \qquad = \chi\left[T^{(+)}_{\mu\nu} + \sqrt{\frac{g^{(-)}}{g^{(+)}}}\widehat{T}^{(-)}_{\mu\nu}\right] \tag{A1}$$

$$G^{(-)}_{\mu\nu} = R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} \quad = -\chi\left[\sqrt{\frac{g^{(+)}}{g^{(-)}}}\widehat{T}^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}\right] \tag{A2}$$

By construction:

$$\nabla^{\nu}_{(+)}G^{(+)}_{\mu\nu} = \nabla^{\nu}_{(-)}G^{(-)}_{\mu\nu} = 0 \tag{A3}$$

Conservation laws give:

$$\nabla^{\nu}_{(+)}T^{(+)}_{\mu\nu} = \nabla^{\nu}_{(-)}T^{(-)}_{\mu\nu} = 0 \tag{A4}$$

It is therefore necessary, in order to ensure the mathematical and physical coherence of this system, that the interaction terms also satisfy these relations. Remember that Einstein's equation, the backbone of general relativity, provides only two types of solutions:

- Uniform and isotropic time dependent, solutions: Friedman models.
- Time independent solutions referring either to empty regions or to masses assimilated to volumes filled with an incompressible fluid of constant density.

The Bianchi conditions identify, in the first case, to the conservation of energy and mass. In the second case, we see emerging, within the masses, the relationship reflecting the balance between the force of gravity and the force of pressure. So this is analyzed by considering the inner Schwarzschild metric. The problem had thus been completely dealt with by Karl Schwarzschild in his second article of 1916 [6] where he even went so far as to explain the law r. This analysis was subsequently taken up by Tolman [21], Oppenheimer and Volkoff [22] and specialists then know this relationship in its differential form, as the so called "Tolman-Oppenheimer-Volkoff equation of state" or "TOV equation". For example, all the details of such a calculation can be found in [23], Chapter 14.

If we now shift to the system of the two coupled field equations, when we consider the unsteady "dust phases" where the pressures $\mathtt{p}^{(+)}$ and $\mathtt{p}^{(-)}$ can be neglected, a very simple case, in all respects analogous to what is done with a single species, in general relativity, leads to the following compatibility relation, reflecting a generalized conservation of energy:

$$\rho^{(+)}\, c^{(+)2}a^{(+)3} + \rho^{(-)}c^{(-)2}a^{(-)3} = \mathtt{Cst} \tag{A5}$$

In time independent conditions, the Bianchi condition is obviously satisfied in a trivial way in vacuum. The problem arises when we consider a region of space where there is matter. Let us consider for example a part of space corresponding to a sphere filled with a positive mass, assimilated to an incompressible fluid, of constant density. The system is written:

$$G^{(+)}_{\mu\nu} = R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu} = \chi\, T^{(+)}_{\mu\nu} \tag{A6}$$

$$G^{(-)}_{\mu\nu} = R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} = -\chi\sqrt{\frac{g^{(+)}}{g^{(-)}}}\widehat{T}^{(+)}_{\mu\nu} \tag{A7}$$

By taking the approach of Chapter 14 from [23], the metrics are given the forms:

$$ds^{(+)2} = e^{\nu}dx^{o\,2} - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta\varphi^2) \tag{A8}$$

$$ds^{(-)2} = e^{\underline{\nu}}dx^{o\,2} - e^{\underline{\lambda}}dr^2 - r^2(d\theta^2 + \sin^2\theta\varphi^2) \tag{A9}$$

From which we can calculate the components of the Ricci tensors $R^{(+)\nu}_{\ \ \mu}$ and $R^{(-)\nu}_{\ \ \mu}$ as in [23], Chapter 14. The positive mass tensor is:

$$T^{(+)\nu}_{\ \ \mu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & -\dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 & 0 \\ 0 & 0 & -\dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} & 0 \\ 0 & 0 & 0 & -\dfrac{\mathtt{p}^{(+)}}{c^{(+)2}} \end{pmatrix} \tag{A10}$$

To simplify the writing, we will put $c^{(+)} = c$; $\mathtt{p}^{(+)} = \mathtt{p}$; $\rho^{(+)} = \rho$. Equation (A6) leads to the system of equations:

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = \chi\,\rho \tag{A11}$$

$$e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)-\frac{1}{r^2}=-\chi\,\frac{\mathtt{p}}{c^2} \tag{A12}$$

$$e^{-\lambda}\left[\frac{\nu''}{2}-\frac{\nu'\lambda'}{4}+\frac{\nu'\,2}{4}+\frac{\nu'-\lambda'}{2r}\right]=-\chi\,\frac{\mathtt{p}}{c^2} \tag{A13}$$

We know that by setting:

$$e^{-\lambda}\equiv 1-\frac{2m_{(r)}}{r} \tag{A14}$$

We obtain the so-called Tolman-Oppenheimer-Volkoff equation:

$$\frac{\mathtt{p}'}{c^2}=-\frac{m+4\pi\,G\,\mathtt{p}\,r^3/c^4}{r(r-2m)}\left(\rho+\frac{\mathtt{p}}{c^2}\right) \tag{A15}$$

Consider the Newtonian approximation:

$$\mathtt{p}\ll\rho\,c^2\quad;\quad r\gg 2m \tag{A16}$$

This relation then identifies to Euler's:

$$\mathtt{p}'=-\frac{\rho\,m_{(r)}c^2}{r^2}=-\frac{G\,M_{(r)}\rho}{r^2} \tag{A17}$$

$M_{(r)}$ being the mass contained in a sphere of radius r. This relationship expresses the balance between the force of pressure and the force of gravity.

Passing to Eq. (A7) and placing ourselves again in the Newtonian approximation, for there to be compatibility between the two field equations we must find the same relationship. Let us give the interaction tensor $\widehat{T}^{(+)\nu}_{\ \ \mu}$ the form:

$$\widehat{T}^{(+)\nu}_{\ \ \mu}=\begin{pmatrix}\rho & 0 & 0 & 0\\ 0 & \frac{\mathtt{p}}{c^2} & 0 & 0\\ 0 & 0 & \frac{\mathtt{p}}{c^2} & 0\\ 0 & 0 & 0 & \frac{\mathtt{p}}{c^2}\end{pmatrix} \tag{A18}$$

It is the tensor $T^{(+)\nu}_{\ \ \mu}$, with inversion of the sign of the pressure terms. At this stage, let us specify that the choice of this tensor is completely free, its only virtue being to ensure that Equations A6 and A7 are compatible, under the conditions of the Newtonian approximation.

We get:

$$\frac{\sqrt{-g^{(+)}}}{\sqrt{-g^{(-)}}}=\frac{\sqrt{e^{\nu}e^{\lambda}r^4\sin^2\theta}}{\sqrt{e^{\bar\nu}e^{\bar\lambda}r^4\sin^2\theta}}=e^{\frac{\nu}{2}}\,e^{\frac{\lambda}{2}}\,e^{-\frac{\bar\nu}{2}}\,e^{-\frac{\bar\lambda}{2}}=K_{(r)} \tag{A19}$$

By calculating the components of the Ricci tensor $R^{(-)\nu}_{\ \ \mu}$, we then get the following system of equations.

$$e^{-\bar\lambda}\left(\frac{1}{r^2}+\frac{\bar\nu'}{r}\right)-\frac{1}{r^2}=-\chi\,K\,\frac{\mathtt{p}}{c^2} \tag{A20}$$

$$e^{-\bar\lambda}\left[\frac{\bar\nu''}{2}-\frac{\bar\nu'\bar\lambda'}{4}+\frac{\bar\nu'\,2}{4}+\frac{\bar\nu'-\bar\lambda'}{2r}\right]=-\chi\,K\,\frac{\mathtt{p}}{c^2} \tag{A21}$$

$$-\frac{\bar\nu'+\bar\lambda'}{r}e^{-\bar\lambda}=-\chi\,K\,\left(\rho+\frac{\mathtt{p}}{c^2}\right) \tag{A22}$$

Applying the Newtonian approximation upstream of this calculation, we can do $K\approx 1$. As in the above we will ask:

$$e^{-\lambda}\equiv 1-\frac{2\bar m}{r} \tag{A23}$$

We continue the calculation in a similar way to what was done above and we obtain the result:

$$\frac{\mathtt{p}'}{c^2}=-\frac{m-4\pi\,G\,\mathtt{p}\,r^3/c^4}{r(r+2m)}\left(\rho-\frac{\mathtt{p}}{c^2}\right) \tag{A24}$$

This differs from Eq. (A15). But these two equations come together when applying the Newtonian approximation. There is therefore compatibility, modulo this condition. We will conclude that the system of Equations A1 + A2 must be limited to the description of the universe under the conditions corresponding to the Newtonian approximation.

1. Remark

Building the interaction tensors $\widehat{T}^{(+)\nu}_{\ \ \mu}$ and $\widehat{T}^{(-)\nu}_{\ \ \mu}$ is not done in a day. The presence of the term $\frac{\sqrt{-g^{(+)}}}{\sqrt{-g^{(-)}}}$ comes from the Lagrangian construction ([11, 13]) of the system of coupled field equations, which involves hypervolumes:

$$\sqrt{-g^{(+)}}\,d^4x\quad\mathtt{et}\sqrt{-g^{(-)}}\,d^4x \tag{A25}$$

In a first, heuristic approach, we opted for the choice:

$$\widehat{T}^{(+)\nu}_{\ \ \mu}=T^{(+)\nu}_{\ \ \mu}\quad\mathtt{et}\quad\widehat{T}^{(-)\nu}_{\ \ \mu}=T^{(-)\nu}_{\ \ \mu} \tag{A26}$$

This did not pose a problem in the construction of the time-dependent solution, where the question of the balance between the forces of pressure and the force of gravity disappeared in a trivial way, because of the assumption of homogeneity and isotropic, which allowed a first construction of the exact unsteady solution (Eq. A26). In this case the system of field equations was reduced to:

$$R^{(+)}_{\mu\nu}-\frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu}=\quad\chi\left[T^{(+)}_{\mu\nu}+\frac{c^{(-)2}a^{(-)3}}{c^{(+)2}a^{(+)3}}T^{(-)}_{\mu\nu}\right] \tag{A27}$$

$$R^{(-)}_{\mu\nu}-\frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu}=\quad-\chi\left[\frac{c^{(+)2}a^{(+)3}}{c^{(-)2}a^{(-)3}}T^{(+)}_{\mu\nu}+T^{(-)}_{\mu\nu}\right] \tag{A28}$$

The application of the Newtonian approximation provided in passing the laws of interaction, initially introduced heuristically, in order to satisfy the principles of action-reaction and equivalence. It remained to modify the shape of the interaction tensors so that they satisfy, under non-homogeneous conditions, the Bianchi conditions, so that the bigeometry, inside the masses, does not lead to a contradiction. Indeed, in hypothesis given in Eq. (A26), the analysis presented above leads to:

- From the metric of the positive masses to the equation:

$$\frac{\mathtt{p}'}{c^2} = -\frac{m + 4\pi\, G\, \mathtt{p}\, r^3/c^4}{r(r-2m)} \left(\rho + \frac{\mathtt{p}}{c^2}\right) \tag{A29}$$

- From the metric of the negative masses to the equation:

$$\frac{\mathtt{p}'}{c^2} = +\frac{m - 4\pi\, G\, \mathtt{p}\, r^3/c^4}{r(r+2m)} \left(\rho - \frac{\mathtt{p}}{c^2}\right) \tag{A30}$$

In this case, the Newtonian approximation leads to the contradictory equations:

$$\mathtt{p}' = -\frac{G\, M_{(r)}\rho}{r^2} \qquad \mathtt{p}' = +\frac{G\, M_{(r)}\rho}{r^2} \tag{A31}$$

As shown in this article, this problem disappears when the interaction tensors are given adequate form like in Eq. (A18).

Appendix B: Bolometric magnitude derivation

Starting from the cosmological equations corresponding to positive species and neglectible pressure (dust universe):

$$a^{(+)2}\, \ddot{a}^{(+)} + \frac{8\pi\, G}{3} E = 0 \tag{B1}$$

with $E \equiv a^{(+)3}\rho^{(+)} + a^{(-)3}\rho^{(-)} = constant < 0$. For sake of simplicity we will write $a \equiv a^{(+)}$ in the following. A parametric solution of Eq. (B1) can be written as:

$$a(u) = \alpha^2 ch^2(u) \qquad t(u) = \frac{\alpha^2}{c}\left(1 + \frac{sh(2u)}{2} + u\right) \tag{B2}$$

with

$$\alpha^2 = -\frac{8\pi\, G}{3c^2}\, E \tag{B3}$$

This solution imposes $k = -1$. Writing the definitions $q \equiv -\dfrac{a\ddot{a}}{\dot{a}^2}$ and $H \equiv -\dfrac{\dot{a}}{a}$, we can write:

$$q = -\frac{1}{2sh^2(u)} = -\frac{4\pi\, G}{3}\frac{|E|}{a^3\, H^2} \tag{B4}$$

and also

$$(1-2q) = \frac{c^2}{a^2\, H^2} \tag{B5}$$

In terms of the time t used in the FRLW metric, the light emitted by G_e at time t_e is observed on G_0 at a time $t_0 (t_e > t_0)$ and the distance l travelled by photons ($ds^2 = 0$) is related to the time difference t and then to the u parameter through the relation:

$$l = \int_{t_e}^{t_0} \frac{c\, dt}{a(t)} = \int_{u_e}^{u_0} \frac{(1 + ch(2u))}{ch^2(u)}\, du = 2u_0 - 2u_e \tag{B6}$$

Using Friedman's metric with $k = -1$, we can also relate the distance l to the distance marker r by:

$$l = \int_{t_e}^{t_0} \frac{c\, dt}{a(t)} = \int_0^r \frac{dr'}{\sqrt{1 + r'^2}} = argsh(r) \tag{B7}$$

So we can write:

$$r = sh(2u_0 - 2u_e) = 2sh(u_0 - u_e)ch(u_0 - u_e) \tag{B8}$$

We need now to link u_e and u_0 to observable quantities q_0, H_0, z. From Eq. (B7), we get:

$$u = argch\left(\sqrt{\frac{a}{\alpha^2}}\right) \tag{B9}$$

We get the usual redshift expression:

$$a_e = \frac{a_0}{1+z} \tag{B10}$$

We get:

$$u_0 = argch\sqrt{\frac{2q_0 - 1}{2q_0}} = argsh\sqrt{-\frac{1}{2q_0}} \tag{B11}$$

and

$$u_e = argch\sqrt{\frac{2q_0 - 1}{2q_0(1+z)}} = argsh\sqrt{-\frac{1 + 2q_0 z}{2q_0(1+z)}} \tag{B12}$$

After a few technical manipulations, and considering the constraint that $1 + 2q_0 z > 0$, we get:

$$r = \frac{c}{a_0\, H_0} \frac{q_0 z + (1 - q_0)\left(1 - \sqrt{1 + 2q_0 z}\right)}{q_0^2(1+z)} \tag{B13}$$

Which is similar to Mattig's work with usual Friedman's solutions where $q_0 > 0$, here we have always $q_0 < 0$. The total energy received per unit area and unit time interval measured by bolometers is related to the luminosity:

$$E_{bol} = \frac{L}{4\pi a_0^2\, r^2 (1+z)^2} \tag{B14}$$

The bolometric magnitude can therefore be written as:

$$m_{bol} = 5\log_{10}\left[\frac{q_0 z + (1 - q_0)\left(1 - \sqrt{1 + 2q_0 z}\right)}{q_0^2}\right] + \mathtt{cst} \tag{B15}$$

This relation rewrites as:

$$m_{bol} = 5\log_{10}\left[z + \frac{z^2(1 - q_0)}{1 + q_0 z + \sqrt{1 + 2q_0 z}}\right] + \mathtt{cst} \tag{B16}$$

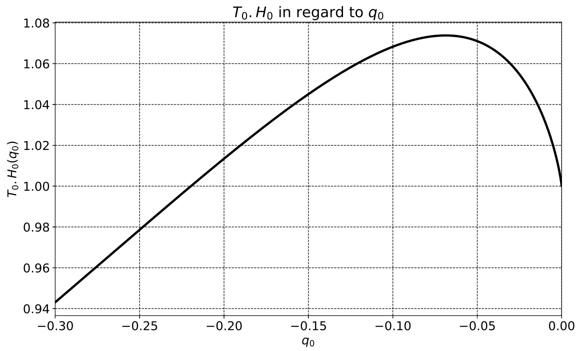

FIG. 8. Age of the Universe

1. Age of the universe

Below we will establish the relation between the age of the universe T_0 with q_0 and H_0 (see Fig. 8). This age is defined by:

$$T_0 = \frac{\alpha^2}{c}\left(\frac{sh(2u_0)}{2} + u_0\right) \tag{B17}$$

Then we get:

$$\frac{\alpha^2}{c} = -\frac{2q}{H}(1-2q)^{-\frac{3}{2}} = -\frac{2q_0}{H}(1-2q_0)^{-\frac{3}{2}} \tag{B18}$$

and so:

$$T_0 = -2q_0(1-2q_0)^{-\frac{3}{2}}\left(\frac{sh(2u_0)}{2} + u_0\right)\frac{1}{H_0} \tag{B19}$$

We finally get:

$$T_0.H_0 = -2q_0(1-2q_0)^{-\frac{3}{2}}\left(argsh\sqrt{\frac{-1}{2q_0}} - \frac{\sqrt{1-2q_0}}{2q_0}\right) \tag{B20}$$