# Geometric quantization in a fibered cosmology with CPT symmetry

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# 1. Introduction

Numerous approaches have attempted to explain the geometric origin of quantum structures, through symplectic geometry, principal fibers or non-trivial topological varieties. Particularly noteworthy is the work of Wu and Yang (1975), who propose a global formulation of gauge fields via non-integrable phases, or the developments of Cattaneo and Felder (2000) around deformation quantization. More recent approaches, such as those by Freed and Moore (2013), Hsieh and Ryu (2015), or Mazzoni (2018), show that discrete symmetries and twisted fibered structures can play a fundamental role in the emergence of quantum properties. In this context, our approach is based on a unifying geometric perspective, inspired by the Janus Model, in which the universe is postulated to possess a dual covering structure of an inorientable projective space. By extending this geometry with additional compact internal dimensions, we explore how CPT symmetry and the quantization of fundamental charges can emerge naturally.

# 2. Model structure

Consider a principal fibered of the form

(S⁶ × $S\_{B}^{1}$ × $S\_{L}^{1}$ × $Z$) → M¹² → S⁴/{±x}

* **Base**: the S⁴ sphere, modeling observable space-time. quotiented by antipodal identification to induce a global CPT symmetry.
* **Fiber**: composed of 6-spheres S6 encoding gauge charges (q,T3,Y,C1,C2,C3), two circles for the baryon and lepton numbers ($S\_{B}^{1}$,$S\_{L}^{1}$), and a discrete structure $Z^{3}$ representing the three particle generations.
* **Spin structure**: imposed over the entire fibered space to ensure the emergence of spin-½ representations.

Each $S^{6} $sphere in the fiber carries a symmetric structure enabling the identification of its isometries with internal gauge charges. The circles $S\_{B}^{1} $and $S\_{L}^{1} $each define a geometric phase whose winding number leads to the conservation of baryon and lepton number, respectively. The discrete factor $Z^{3}$​ encodes permutation symmetry among generations, suggesting the existence of an underlying flavor symmetry. Altogether, this defines a nontrivial fiber bundle whose connection may serve as the geometric origin of fundamental interactions.

**3. Geometric Quantization of Physical Quantities**

Each fundamental physical quantity is associated with a continuous symmetry and a corresponding compact dimension:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quantity | Dimension | Compactness | Symmetry | Quantization |
| spin | Angular Space | $$S^{2}$$ | SU(2) | Unitary representation |
| mass | Internal dimension | Fiber $$S^{6}$$ | Translations | Eigen modes ofthe Laplacian |
| Energy | Compact time | $$S\_{t}^{1}$$ | Temporal U(1) | Discrete frequency Spectrum |
| Gauges charges | $$S^{6}$$ | Compact | $$SU(2)×SU(3)×U(1)$$ | Noether invariants |
| Baryon,Lepton number | $S\_{B}^{1}$,$S\_{L}^{1}$ | Circle | U(1)B, U(1)L | Quantized holonomies |

Each compact dimension acts as a topological resonator. The spectrum associated with a physical quantity corresponds to the allowed modes on that dimension. Spin arises from the spherical harmonics on S2 ; mass corresponds to the eigenmodes of the Laplacian on S6; energy derives from harmonic modes on the temporal circle S1. The quantization of baryon and lepton numbers results from periodicity conditions imposed on the circles ​ $S\_{B}^{1} $and $S\_{L}^{1}$​.

Thus, the structure of the bundle not only encodes the charges but also their quantization — without relying on any external quantization postulate.

**4. CPT Symmetry as a Global Topological Property**

The antipodal identification on the base S4 and on the fibers S6 naturally induces:

* a **C-symmetry**: inversion in the internal dimensions (opposite charges),
* a **P-symmetry**: spatial inversion in S4,
* a **T-symmetry**: a locally orientable time loop,

and globally, a **non-orientability** that makes the space M12 both mathematically consistent and physically symmetric under CPT.

This symmetry stems from the global non-orientability of the fibered space, which prevents any absolute distinction between the two conjugate sheets: a global inversion corresponds to a CPT conjugation. As a result, physical laws remain invariant under the combined action of spatial reflection, time reversal, and internal charge conjugation — providing a geometric justification for CPT symmetry, beyond its role as a theorem in quantum field theory.

This framework proposes a natural unification of quantum physics and differential geometry. It offers a solid foundation for the quantization of fundamental constants, without resorting to arbitrary postulates. Every quantum quantity becomes a topological effect arising from a closed dimension endowed with symmetry.

**5. Differential Formalism**

To provide a rigorous foundation for this geometric construction, we apply the tools of differential geometry. The total space M12 is interpreted as a principal fiber bundle whose base is the quotient space S4 /{±x}, and whose fiber is a compact homogeneous space
F ≅ S6×$S\_{B}^{1} $×$S\_{L}^{1}$×$Z^{3}$ . A connection ω\omegaω is defined on this bundle as a Lie-algebra-valued 1-form. Its curvature Ω = dω+$\frac{1}{2}$[ω,ω] encodes the internal gauge fields and fundamental interactions.

The associated Chern classes can be interpreted as topological invariants corresponding to conserved quantum charges. Furthermore, Dirac operators defined on the spinor sections of associated vector bundles describe the quantum states of matter fields. This formulation connects internal symmetries directly to the topology and geometry of the bundle — in the spirit of geometric quantization as formulated by Jean-Marie Souriau.

**6. Effective Lagrangian**

The structure of the bundle M12 allows for a unified Lagrangian formulation in which physical fields arise from the components of a connection ω defined over the principal bundle. The effective Lagrangian takes the form of a sum of gauge-invariant densities that respect the structure and symmetries of the bundle:

$$L=L\_{grave}+L\_{int}+L\_{matter}$$

* $L\_{grave}$**​**: a density inspired by the generalized Einstein-Hilbert action, defined on the base S4 (or its covering), coupled to an effective metric induced by the fiber structure.
* $L\_{int}$**​**: a Yang-Mills-type term constructed from the curvature Ω of the connection. It encodes the internal interactions corresponding to the gauge group SU(3)×SU(2)×U(1), as well as the U(1)B​ and U(1)L ​symmetries:

$L\_{int}$**​** = - $\frac{1}{4}$ Tr $\left(Ω∧\*Ω\right)$

* $L\_{matter}$**​**: the minimal coupling of matter fields — modeled as spinors, i.e. sections of an associated vector bundle — with the connection components, via a generalized Dirac operator Dω​.

This Lagrangian framework allows the derivation of field equations via the usual variational principle and links the dynamics of physical fields to the topology of the fibered manifold. Quantization then emerges naturally from the compactness of internal dimensions, and coupling constants can be interpreted as geometric invariants (e.g. curvature radii or Chern classes).

**7. Cosmology and Observational Implications**

This article refines and deepens the proposals introduced in our December 2024 publication (Topological Extensions and Quantized Invariants in Fibered Cosmological Models, Reviews of Mathematical Physics), particularly regarding the geometric structure responsible for CPT symmetry and the quantization of fundamental constants.

The primary observational motivation lies in the complete absence of primordial antimatter in the observable universe — a mystery that has remained unresolved since the foundational work of Andrei Sakharov (1967). The model presented here offers a geometric alternative to this asymmetry: the global non-orientability of spacetime implies the existence of two conjugate sheets linked by CPT symmetry — one dominated by matter, the other by antimatter.

The compactness of spatial and temporal dimensions is not limited to the cosmological scale. It may also play a fundamental role in particle physics by naturally imposing quantization on energy and mass.

More precisely, a compact time — modeled as a circle S1 — implies dynamic periodicity, resulting in a discrete energy spectrum, much like the standing modes of a resonator. Similarly, the spatial or internal compactification of fiber dimensions — notably within S6 — imposes quantization on field solutions: elementary particles would then correspond to eigenmodes of the Laplacian on these compact spaces. This geometric interpretation accounts both for the discretization of physical quantities and the possible origin of fundamental constants.

The proposed geometric model, based on a base S4 with antipodal identification and compact internal fibers, leads naturally to a nonstandard cosmology. The fibered structure allows a geometric reinterpretation of key cosmological puzzles. In particular:

* The global non-orientability, tied to CPT symmetry, implies a double-universe configuration in which matter and antimatter coexist on conjugate sheets —
* offering a geometric resolution to the baryon asymmetry problem.

# References

- A.D.Sakharov , (1980). Cosmological Model of the Universe with a Time Vector Inversion. ZhETF (Tr. JETP 52, 349-351) (79): 689–693

- J.P.Petit, F.Margnat, H.Zejli : A bimetric cosmological model on Andreï’s twin universe approach. Th European Physical Journal. Vol. 84 :N°1126 (2024)

- **J.P.Petit, H.Zejli : Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, n001, 244004.**

- Wu, T. T., & Yang, C. N. (1975). \*Concept of Nonintegrable Phase Factors and Global Formulation of Gauge Fields\*. Physical Review D, 12(12), 3845–3857.

- Cattaneo, A. S., & Felder, G. (2000). \*A Path Integral Approach to the Kontsevich Quantization Formula\*. Communications in Mathematical Physics, 212(3), 591–611.

- Freed, D. S., & Moore, G. W. (2013). \*Twisted Equivariant Matter\*. Annales Henri Poincaré, 14(8), 1927–2023.

- Hsieh, C.-T., Cho, G. Y., & Ryu, S. (2015). \*Global Anomalies on the Surface of Fermionic Symmetry-Protected Topological Phases in (3+1) Dimensions\*. Physical Review B, 91(19), 195135.

- Mazzoni, L. (2018). \*A Fibre Bundle Approach to U(1) Symmetries in Physics\*. Università di Bologna.

- Swanson, N. (2019). \*Deciphering the Algebraic CPT Theorem\*. Studies in History and Philosophy of Modern Physics, 66, 1–13.