Geometric quantization in a fibered cosmology with CPT symmetry

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We propose a geometric framework in which the quantization of fundamental physical quantities — spin, charge, mass, and energy — emerges from the topology and compactness of a twelve-dimensional fibered manifold. The model is based on a non-orientable base space derived from the projective 4-sphere and extended by compact internal dimensions forming a structured fiber. In this construction, CPT symmetry arises naturally as a global topological feature due to the double-sheeted covering of the non-orientable base. Quantization results from the resonance conditions imposed by closed spatial and temporal dimensions, while gauge interactions emerge from connections on the fiber bundle. The model unifies internal quantum numbers and spacetime geometry without appealing to postulated quantization rules. This approach provides a coherent geometric origin for discrete charges and suggests a new path toward integrating quantum physics and cosmology..

Keywords: geometric quantization, CPT symmetry, principal fibers, non-orientable space, Janus model, compact dimensions, quantum charges, topological cosmology

I. INTRODUCTION

Numerous approaches have attempted to explain the geometric origin of quantum structures, through symplectic geometry, principal fibers or non-trivial topological varieties. Particularly noteworthy is the work of Wu and Yang ref.[1], who propose a global formulation of gauge fields via non-integrable phases, or the developments of Cattaneo and Felder ref.[2] around deformation quantization. More recent approaches, such as those by Freed and Moore ref.[3], Hsieh and Ryu ref.[4], or Mazzoni ref.[5], show that discrete symmetries and twisted fibered structures can play a fundamental role in the emergence of quantum properties. In this context, our approach is based on a unifying geometric perspective, inspired by the Janus Model ref.[7, 8], in which the universe is postulated to possess a dual covering structure of an inorientable projective space. By extending this geometry with additional compact internal dimensions, we explore how CPT symmetry and the quantization of fundamental charges can emerge naturally.

II. MODEL STRUCTURE

Consider a principal fibered of the form

$$(S^6 \times S^1_B \times S^1_L \times \mathbb{Z}) \to M^{12} \to S^4/\pm x$$

- **Base**: the S^4 sphere, modeling observable space-time. quotiented by antipodal identification to induce a global CPT symmetry.
- **Fiber**: composed of 6-spheres S^6 encoding gauge charges $(q, T3, Y, C1, C2, C3)$, two circles for the baryon and lepton numbers (S^1_B, S^1_L), and a discrete structure $\mathbb{Z}^3$ representing the three particle generations.
- **Spin structure**: imposed over the entire fibered space to ensure the emergence of spin-½ representations.

Each S^6 sphere in the fiber carries a symmetric structure enabling the identification of its isometries with internal gauge charges. The circles S^1_B and S^1_L each define a geometric phase whose winding number leads to the conservation of baryon and lepton number, respectively. The discrete factor $\mathbb{Z}^3$ encodes permutation symmetry among generations, suggesting the existence of an underlying flavor symmetry. Altogether, this defines a nontrivial fiber bundle whose connection may serve as the geometric origin of fundamental interactions.

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III. GEOMETRIC QUANTIZATION OF PHYSICAL QUANTITIES

Each fundamental physical quantity is associated with a continuous symmetry and a corresponding compact dimension:

Quantity	**Dimension**	**Compactness**	**Symmetry**	**Quantization**
spin	Angular Space	S^2	$SU(2)$	Unitary representation
mass	Internal dimension	Fiber S^6	Translations	Eigen modes of the Laplacian
Energy	Compact time	S^1_t	Temporal $U(1)$	Discrete frequency Spectrum
Gauges charges	S^6	Compact	$SU(2) \times SU(3) \times U(1)$	Noether invariants
Baryon,Lepton number	S^1_B, S^1_L	Circle	$U(1)_B, U(1)_L$	Quantized holonomies

Each compact dimension acts as a topological resonator. The spectrum associated with a physical quantity corresponds to the allowed modes on that dimension. Spin arises from the spherical harmonics on S^2 ; mass corresponds to the eigenmodes of the Laplacian on S^6; energy derives from harmonic modes on the temporal circle S^1. The quantization of baryon and lepton numbers results from periodicity conditions imposed on the circles S^1_B and S^1_L. Thus, the structure of the bundle not only encodes the charges but also their quantization — without relying on any external quantization postulate.

IV. CPT SYMMETRY AS A GLOBAL TOPOLOGICAL PROPERTY

The antipodal identification on the base S4 and on the fibers S^6 naturally induces:

- a **C-symmetry**: inversion in the internal dimensions (opposite charges),
- a **P-symmetry**: spatial inversion in S^4,
- a **T-symmetry**: a locally orientable time loop,

and globally, a **non-orientability** that makes the space M^{12} both mathematically consistent and physically symmetric under CPT.

This symmetry stems from the global non-orientability of the fibered space, which prevents any absolute distinction between the two conjugate sheets: a global inversion corresponds to a CPT conjugation. As a result, physical laws remain invariant under the combined action of spatial reflection, time reversal, and internal charge conjugation — providing a geometric justification for CPT symmetry, beyond its role as a theorem in quantum field theory.

This framework proposes a natural unification of quantum physics and differential geometry. It offers a solid foundation for the quantization of fundamental constants, without resorting to arbitrary postulates. Every quantum quantity becomes a topological effect arising from a closed dimension endowed with symmetry.

V. DIFFERENTIAL FORMALISM

To provide a rigorous foundation for this geometric construction, we apply the tools of differential geometry. The total space M^{12} is interpreted as a principal fiber bundle whose base is the quotient space $S^4/\{\pm x\}$, and whose fiber is a compact homogeneous space $F \cong S^6 \times S^1_B \times S^1_L \times \mathbb{Z}^3$. A connection ω is defined on this bundle as a Lie-algebra-valued 1-form. Its curvature $\Omega = d\omega + \frac{1}{2}[\omega, \omega]$ encodes the internal gauge fields and fundamental interactions.

The associated Chern classes can be interpreted as topological invariants corresponding to conserved quantum charges. Furthermore, Dirac operators defined on the spinor sections of associated vector bundles describe the quantum states of matter fields. This formulation connects internal symmetries directly to the topology and geometry of the bundle — in the spirit of geometric quantization as formulated by Jean-Marie Souriau.

VI. EFFECTIVE LAGRANGIAN

The structure of the bundle M^{12} allows for a unified Lagrangian formulation in which physical fields arise from the components of a connection ω defined over the principal bundle. The effective Lagrangian takes the form of a sum of gauge-invariant densities that respect the structure and symmetries of the bundle:

$$L = L_{grave} + L_{int} + L_{matter}$$

- L_{grave}: a density inspired by the generalized Einstein-Hilbert action, defined on the base S^4 (or its covering), coupled to an effective metric induced by the fiber structure.
- L_{int}: a Yang-Mills-type term constructed from the curvature Ω of the connection. It encodes the internal interactions corresponding to the gauge group $SU(3)\times SU(2)\times U(1)$, as well as the $U(1)_B$ and $U(1)_L$ symmetries:

$$L_{int} = -\tfrac{1}{4}Tr(\Omega \wedge *\Omega)$$

- L_{matter}: the minimal coupling of matter fields — modeled as spinors, i.e. sections of an associated vector bundle — with the connection components, via a generalized Dirac operator D_ω.

This Lagrangian framework allows the derivation of field equations via the usual variational principle and links the dynamics of physical fields to the topology of the fibered manifold. Quantization then emerges naturally from the compactness of internal dimensions, and coupling constants can be interpreted as geometric invariants (e.g. curvature radii or Chern classes).

VII. COSMOLOGY AND OBSERVATIONAL IMPLICATIONS

This article refines and deepens the proposals introduced in our December 2024 publication (Topological Extensions and Quantized Invariants in Fibered Cosmological Models, Reviews of Mathematical Physics), particularly regarding the geometric structure responsible for CPT symmetry and the quantization of fundamental constants.

The primary observational motivation lies in the complete absence of primordial antimatter in the observable universe — a mystery that has remained unresolved since the foundational work of Andrei Sakharov (1980) ref[6]. The model presented here offers a geometric alternative to this asymmetry: the global non-orientability of spacetime implies the existence of two conjugate sheets linked by CPT symmetry — one dominated by matter, the other by antimatter.

The compactness of spatial and temporal dimensions is not limited to the cosmological scale. It may also play a fundamental role in particle physics by naturally imposing quantization on energy and mass.

More precisely, a compact time — modeled as a circle S^1 — implies dynamic periodicity, resulting in a discrete energy spectrum, much like the standing modes of a resonator. Similarly, the spatial or internal compactification of fiber dimensions — notably within S^6 — imposes quantization on field solutions: elementary particles would then correspond to eigenmodes of the Laplacian on these compact spaces. This geometric interpretation accounts both for the discretization of physical quantities and the possible origin of fundamental constants.

The proposed geometric model, based on a base S^4 with antipodal identification and compact internal fibers, leads naturally to a nonstandard cosmology. The fibered structure allows a geometric reinterpretation of key cosmological puzzles. In particular:

- The global non-orientability, tied to CPT symmetry, implies a double-universe configuration in which matter and antimatter coexist on conjugate sheets
- offering a geometric resolution to the baryon asymmetry problem.

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