Geometrical Quantum Gravity (GQG). Quantization in Phase Space with Discrete Volume Elements

JP Levy[1]

Abstract

We propose a geometrical approach to quantum gravity based on a covariant formulation in phase space. In this framework, mass is no longer associated with point particles but with a probabilistic distribution function $f(x_\mu, u_\mu)$ over a six-dimensional phase space, where x^i are spatial coordinates and u^i are velocity components. The field equations are derived from a variational principle involving an action built from a quantized volume element. This yields a quantized description of the gravitational field, free from classical singularities and ultraviolet divergences. The vacuum is modeled as a structured medium filled with mass dipoles of opposite signs$(+m, -m)$, generating a gravitational polarization analogous to Debye screening in plasma physics. This leads to the emergence of a characteristic screening length, comparable to the Jeans length, regularizing the self-energy of the gravitational field. The uncertainty principle emerges naturally as a topological constraint on the phase-space volume element. This framework, which we refer to as Geometrical Quantum Gravity (GQG)**,** offers an alternative to quantization via field operators and provides a new foundation for a quantum theory of gravity grounded in statistical geometry.

Keywords: Geometrical Quantum Gravity (GQG), phase space, quantized volume element, gravitational screening, probabilistic matter, Jeans length, CPT symmetry, structured vacuum

1. Introduction

The reconciliation of general relativity and quantum mechanics remains one of the deepest challenges in theoretical physics. While general relativity is formulated on smooth manifolds governed by differential geometry, quantum mechanics introduces intrinsic discreteness and probabilistic behavior. A central difficulty lies in the fact that general relativity describes matter through continuous stress-energy tensors, whereas quantum theory assigns probabilistic, operator-based properties to discrete states.

This article proposes a geometric and statistical reformulation of gravitation based on an extended six-dimensional phase space, where positions and velocities (or momenta) are treated on equal footing. Matter is described not by point masses but by a distribution function $f(x^i, u^i)$, encoding the probability of presence of mass at each phase-space point. This reformulation avoids singularities by replacing delta-function sources with smeared, normalized densities.

Furthermore, we introduce a geometric quantization principle on the volume elements of phase space, leading to a reinterpretation of the uncertainty principle as a topological constraint. The curvature of spacetime becomes a statistical response to this distributed matter, resulting in modified field equations. In the weak-field limit, the model yields a Poisson equation where the

[1] Levy.gqg@gmail.com

density source is integrated over velocities, and exhibits gravitational screening similar to Debye shielding in plasma physics.

The framework naturally accommodates a structured vacuum composed of mass dipoles $(+m\,,-m)$, as proposed in Janus-type cosmologies. The action functional can be generalized to include a covariant formulation, which opens the path to a nonperturbative quantum theory of gravitation—here referred to as Quantum Gravito-Dynamics (QGD).

2. Distribution Function and Probabilistic Matter Description

We define a distribution function over the 6D phase space $f(x^i, u^i) \in [0,1]$, where x^i are spatial coordinates and u^i are velocity components. This function represents the probability density of the presence of matter (of mass m) at each phase-space point. The local mass density is obtained by integrating over velocities:

$$\rho(x^i) = \int m\, f(x^i, u^i)\, d^3u$$

In this framework, the gravitational field is no longer sourced by a singular distribution such as a delta function, but by a continuous density reflecting the spatial and kinematic dispersion of matter. This probabilistic approach provides a natural smoothing of sources, avoiding divergence in the field equations. The function $f(x^i, u^i)$ thus plays a dual role: it serves as both a statistical representation of matter and a dynamical quantity constrained by the geometry of phase space.

This probabilistic matter description aligns with the statistical interpretation of quantum mechanics and is particularly suited to cosmological models in which structure formation emerges from fluctuations in an initially homogeneous medium.

3. Modified Gravitational Field Equations

We now formulate the gravitational field equations based on the probabilistic distribution $f(x^i, u^i)$. Instead of sourcing curvature through a classical stress-energy tensor $T_{\mu\nu}(x)$, we replace this by a velocity-averaged energy-momentum density derived from the distribution function. In the covariant formulation, the Einstein field equations become:

$$G_{\mu\nu}(x) = 8\pi G \int f(x,u)\, m\, u_\mu , u_\nu\, d^4u$$

This generalizes the classical coupling between matter and curvature by integrating over the velocity degrees of freedom, treating matter as a statistical ensemble.

In the Newtonian limit, where the metric reduces to a scalar potential and velocities are non-relativistic, this reduces to a generalized Poisson equation:

$$\nabla^2\Phi(x) = 4\pi G \int m \ f(x,u)d^3u = \ 4\pi G \ \rho(x)$$

This equation governs the gravitational field created by a continuous distribution of mass in phase space, and avoids the singular behavior of classical point masses. The resulting potential can also exhibit screening effects due to the structure of the vacuum, especially if the background consists of a mixture of positive and negative mass elements.

This modification sets the stage for describing gravitational polarization phenomena analogous to those found in electromagnetism, and leads naturally to the concepts discussed in the next section.

4. Newtonian Limit and Screening

In the non-relativistic regime, where the spacetime metric is weakly perturbed and particle velocities are small compared to the speed of light, the covariant formulation reduces to a scalar theory governed by a modified Poisson equation. The gravitational potential $\Phi(x)$satisfies:

$$\nabla^2\Phi(x) = 4\pi G \int m \ f(x,u)d^3u = \ 4\pi G \ \rho(x)$$

This formulation ensures that the gravitational field reflects the phase-averaged mass distribution rather than idealized point sources. Furthermore, when the vacuum is considered as a medium filled with virtual mass dipoles (analogous to electric dipoles in a dielectric), this equation supports screened solutions.

For instance, in the presence of a structured vacuum with opposing mass signs, the effective gravitational potential may exhibit an oscillatory or exponentially damped form, such as:

$$\Phi(r) \ \propto \ \frac{1}{r} \ cos\left(\frac{r}{\lambda_J}\right)$$

where λ_J is the Jeans length associated with the background mass distribution. These expressions mirror known behaviors from plasma physics and condensed matter, where similar screening effects regularize interaction potentials. In cosmological contexts, this screening may help resolve issues related to singularities and long-range divergence of Newtonian gravity.

This completes the foundation for a deeper formulation of gravitational quantization and prepares the ground for geometric quantization principles in the following sections.

5. Quantization as a Topological Constraint

The formulation of gravity within phase space naturally leads to a reinterpretation of quantum principles as emerging from geometric constraints. In particular, we postulate that phase space is endowed with a minimal volume element, such that the product of position and velocity (or momentum) uncertainties is bounded below. This yields a generalized quantization condition:

$$\sqrt{|\det g|} \cdot \Delta x \cdot \Delta y \cdot \Delta z \cdot \Delta u \cdot \Delta v \cdot \Delta w \geq \hbar^3$$

This condition implies that the volume of a six-dimensional cell in phase space cannot fall below Planck's constant cubed, aligning with the uncertainty principle of quantum mechanics. In the one-dimensional case, this reduces to:

$$\Delta x \cdot \Delta u \geq \hbar$$

This phase space quantization is not imposed by operators or commutation relations, but rather emerges from the topology and metric structure of the fibered manifold. In this sense, quantum behavior is interpreted as a consequence of a granular geometry. The discreteness of observables such as energy or mass arises from the admissible phase space configurations respecting the minimal cell volume.

This interpretation offers a unification between quantum principles and classical geometry, suggesting that quantization is a global topological effect rather than a local operator algebra.

6. Gravitational Screening and Jeans Stability

To assess the stability of the gravitational potential under small perturbations in a medium described by the distribution function $f(x,u)$f, we linearize the generalized Poisson equation around a uniform background. In analogy with the Jeans instability analysis in classical astrophysics, we consider small perturbations δf and $\delta\Phi$ and derive a dispersion relation for the evolution of these fluctuations.

The resulting equation reveals the existence of a characteristic length scale λ_J , the Jeans length, beyond which gravitational collapse can occur. In the quantized phase-space framework, this scale is modified by the intrinsic granular structure and the polarization response of the vacuum. The effective potential due to perturbations exhibits a damped or oscillatory behavior:

$$\Phi(r) \propto cos\left(\frac{r}{\lambda_J}\right)$$

These solutions demonstrate the presence of gravitational screening, akin to Debye shielding in plasma physics. This mechanism prevents divergence of the potential at large distances and regulates the growth of perturbations, thereby introducing a natural cutoff to gravitational interaction scales.

This screening is not imposed phenomenologically but arises from the structure of the vacuum itself, shaped by the statistical distribution of virtual dipoles and the geometric quantization of phase space. It provides an elegant resolution to long-standing issues of divergence in Newtonian and relativistic gravity, while offering new insight into early-universe structure formation and the cosmic microwave background.

Appendix A. On the Physical Meaning of Renormalization

In standard quantum field theory, divergences often arise from treating particles as point-like objects interacting in a vacuum devoid of structure. Renormalization counteracts these divergences by redefining parameters such as mass and charge in terms of observed quantities. In this model, divergences are naturally avoided by replacing singular sources with smooth distributions in phase space, and by considering the vacuum as a medium composed of virtual dipolar structures. This perspective suggests that renormalization may have a physical rather than merely computational basis: it reflects the response of a structured vacuum to external sources, analogous to screening effects in electromagnetism.

Appendix B. Vacuum Structure in Janus Cosmology and Gravitational Quantization

In Janus cosmology, the universe is modeled as a two-sheeted structure with opposite time and mass orientation, connected via a compact projective manifold. Each "sheet" hosts matter of a given mass sign, and both evolve with independent but correlated metrics. The structured vacuum proposed in this paper is a natural extension of this idea: fluctuations in one sheet may polarize the vacuum, thereby influencing the other sheet. Gravitational quantization in this context emerges from geometric constraints on this dual structure, where mass dipoles and curvature distributions encode non-local interactions between the sheets.

Appendix C. Toward a Theory of Quantum Gravito-Dynamics (QGD)

The framework presented opens a path toward a new theory of gravity rooted in quantum principles without relying on the graviton. Quantum Gravito-Dynamics (QGD) seeks to describe gravity as an emergent property of a geometrically quantized, statistically structured phase space.

In this setting, the vacuum is conceived as a polarizable medium filled with virtual dipoles of mass $\pm m$. When a real mass is introduced, it induces a polarization analogous to that observed in dielectric media, leading to a modified potential and effective screening of the gravitational field. This approach regularizes the gravitational self-energy and aligns with expectations from quantum field theory where the vacuum responds dynamically to fields.

Moreover, the discrete nature of mass and other conserved quantities can be interpreted as arising from topological constraints on admissible configurations in the phase space. Quantization emerges not from operator algebra but from the underlying geometry and symmetry group of the manifold.

This suggests that the curvature of spacetime, rather than being a classical geometric object, is itself a statistical field, defined as the averaged response of a quantized structure. This statistical curvature couples to a probability distribution function that plays the role of the gravitational source, eliminating the need for point singularities.

QGD thus departs from traditional quantization schemes by building a theory directly on geometric and statistical principles. The combination of phase space quantization, vacuum polarization, and probabilistic sources offers a coherent, divergence-free alternative to the graviton-based approach.

Appendix D. Variational Formulation in Quantized Phase Space

To facilitate comparison with conventional field theory, we present a variational principle consistent with our statistical-geometric framework. Let $f\ (x,u)$ be a normalized distribution function in the six-dimensional phase space, and let Φ(x) be the gravitational potential. We define the action:

$$S[\,f,\Phi\,] = \int d^3x\, d^3u\, \left[f(x,u)\left(\frac{1}{2}mu^2 + m\Phi(\mathrm{x})\right) + 18\,\pi G \nabla^2\Phi \right]$$

The first term represents the kinetic and potential energy of the matter distribution, while the second is the field energy. The equations of motion are derived by extremizing S with respect to variations in Φ and *f*, under the constraint:

$$\int f(x,u) d^3u \;=\; \varrho(x) \qquad \int \rho(x) d^3\,x = M$$

Variation with respect to Φ yields the Poisson equation:

$$\nabla^2\Phi = 4\pi\mathrm{G} \int m\, f(x,u) d^3\,u = 4\pi G\; \rho(x)$$

This action principle provides a field-theoretic backbone to the statistical formulation. It allows for the application of standard techniques such as path integrals (over distribution functions), perturbation theory, and symmetry analysis, within the phase-space framework.

Appendix E. Toward a Quantum Evolution Equation in Phase Space

A natural question in the context of this statistical-geometric framework is whether a quantum dynamical equation—analogous to Schrödinger or Dirac—can be formulated within the extended phase space. We propose that the dynamics of a complex-valued wavefunction ψ, defined over the six-dimensional phase space, obeys a generalized Schrödinger-type equation:

$$i\,\hbar\partial\Psi/\partial\mathrm{t} = \left[-\frac{\hbar^2}{2m}\, \nabla_u^2 + m\;\Phi(x) \right] \Psi\;(\,x\,,u\,,t\,)$$

This equation describes the evolution of the probability amplitude for finding a mass at position with velocity u, under the influence of the gravitational potential $\Phi(x)$, sourced by the distribution function $f\ (x,u)$. Unlike canonical quantization, the operator ∇_u^2 acts on the fiber of velocities, reflecting the geometry of the tangent bundle.

This evolution equation admits stationary solutions of the form:

$$\Psi\,(\,x\,,u\,,t\,)=\;\psi\,(x,u)\,e^{-\frac{Et}{\hbar}}$$

suggesting the existence of bound states in a gravitationally structured vacuum. In the classical limit, the modulus squared of Ψ converges to the probability density $f\ (x, u)$.

This formulation provides a bridge between the classical distribution function and a quantized wave description, aligning the QGD framework with conventional quantum mechanics while retaining its geometric origin.

References

[1] B. S. DeWitt, *Quantum Theory of Gravity. I. The Canonical Theory*, Phys. Rev. 160, 1113 (1967).

[2] S. Weinberg, *The Quantum Theory of Fields*, Vols. I–III, Cambridge University Press (1995).

[3] C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, W. H. Freeman (1973).

[4] A. Ashtekar, *New Variables for Classical and Quantum Gravity*, Phys. Rev. Lett. 57, 2244 (1986).

[5] J.-M. Souriau, *Structure des systèmes dynamiques*, Dunod (1970); English translation: *Structure of Dynamical Systems*, Birkhäuser (1997).

[6] L. Landau and E. Lifshitz, *The Classical Theory of Fields*, 4th ed., Butterworth-Heinemann (1980).

[7] F. Winterberg, *Negative Mass and Gravitational Polarization of the Quantum Vacuum*, Z. Naturforsch. A 26, 1157 (1971).

[8] M. H. P. M. van Putten, *Gravitational Polarization of the Quantum Vacuum and the Cosmological Constant Problem*, Sci. Rep. 7, 12205 (2017).

[9] J.-P. Petit, F. Margnat, H. Zejli, *A Bimetric Cosmological Model on Andrei's Twin Universe Approach*, Eur. Phys. J. C 84, 1126 (2024).

[10] J.-P. Petit, G. D'Agostini, C. Gepetto, *Geometric Quantization in a Fibered Cosmology with CPT Symmetry*, submitted to Nature Communications (2025).