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J-P Petit, G d'Agostini

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Abstract : The current black hole model is based on a contractile interpretation of Schwarzschild's outer solution, influenced by the choice of Lagrangian introduced by Hilbert and the application of Birkhoff's theorem, which excludes the presence of cross terms. This work challenges this paradigm by returning to Schwarzschild's original solution, which implies a non-contractible topology, and reintroducing the inner solution, revealing a physical criticality. We show that inversion of the time factor beyond this criticality naturally leads to PT symmetry and mass inversion, paving the way for a two-sheet topology. Within this geometric framework, we introduce an alternative model of compact object - the plugstar - whose self-stability keeps it in a subcritical state. This stability leads to a mass limitation of 2.5 solar masses in the case of neutron stars, where density is constrained. The gravitational redshift associated with these objects is consistent with observations from the M87* and SgrA* sources.

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1 – Introduction: On what basis did the black hole model emerge?

a – The founding article is that of Oppenheimer and Snyder [1], in 1939. It is based on the fact that the free-fall time of a test mass toward the Schwarzschild sphere, when measured with the coordinate ttt, assumed to represent the proper time experienced by a distant observer, is infinite.

b – Moreover, the vanishing of the g_{tt} term on the Schwarzschild sphere implies that any light emitted from this surface undergoes an infinite gravitational redshift and thus behaves like a cosmological horizon.

c – The hypothesis is that the local topology is $\mathbb{R}_+ \times \mathbb{R}^3$. Thus, the universe is supposed to be locally contractile.

d – It is then assumed that in the bilinear form—solution of Einstein's field equation without source term, invariant under time translation, known as the "Schwarzschild solution":

(1) $$G_{ext}(t, r, \theta, \varphi, \dot{t}, \dot{r}, \dot{\theta}, \dot{\varphi}) = \left(1 - \frac{r}{\alpha}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{r}{\alpha}} - r^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2\right)$$

the letter r is supposed to represent a radial variable.

e – It is assumed that the solution curves derive from the Lagrangian:

(2) $$L = G_{ext}(t, r, \theta, \varphi, \dot{t}, \dot{r}, \dot{\theta}, \dot{\varphi})$$

involved in the construction of the action:

(3) $$J = \int L \, dp$$

It is assumed that these curves describe the geodesics of the entire contractile spacetime hypersurface, with topology $\mathbb{R}_+ \times \mathbb{R}^3$.

2 – The so-called "Schwarzschild solution" is not the original solution.

In his first article [2] from 1916, Schwarzschild clearly defines his initial coordinates $\{ t , x , y , z \}$ as well as a coordinate r defined by:

(3) $$r = \sqrt{x^2 + y^2 + z^2}$$

although he does not state it explicitly $\{ t , x , y , z \} \in \mathbb{R}_+ \times \mathbb{R}^3$. Thus:

(4) $$r \geq 0$$

By expressing that the solution is spherically symmetric, which introduces a constant of integration α (later called the "Schwarzschild radius"), he effectively chooses to express his solution not with the coordinates $\{ t , r , \theta , \varphi \}$ but with the coordinates $\{ t , R , \theta , \varphi \}$ where R is an intermediate variable (called "*Hilfsgröße*") defined by:

(5) $$R = (r^3 + \alpha^3)^{1/3} \geq \alpha$$

Which gives :

(6) $$ds^2 = \left(1 - \frac{R}{\alpha}\right) c^2 dt^2 - \frac{dR^2}{1 - \frac{R}{\alpha}} - R^2 (d\theta^2 + sin^2\theta \, d\varphi^2) \qquad R = (r^3 + \alpha^3)^{1/3}$$

The true Schwarzschild solution, from January 1916, expressed using his coordinates $\{ t , r , \theta , \varphi \}$, is therefore:

(7) $$ds^2 = \frac{(r^3+\alpha^3)^{1/3} - \alpha}{(r^3+\alpha^3)^{1/3}} c^2 dt^2 - \frac{r^4 \, dr^2}{(r^3+\alpha^3)\left[(r^3+\alpha^3)^{1/3} - \alpha\right]} - (r^3 + \alpha^3)^{2/3} (d\theta^2 + sin^2\theta d\varphi^2)$$

If we take dt = dr = 0 what remains is the spatial part of the metric:

(8) $$d\sigma^2 = (r^3 + \alpha^3)^{2/3} (d\theta^2 + sin^2\theta d\varphi^2)$$

This is the metric of a family of spheres with radius: $(r^3 + \alpha^3)^{1/3}$, with a minimal area value:

(9) $$A = 4 \pi (r^3 + \alpha^3)^{2/3}$$

The geometric object is therefore non-contractile. In terms of the spatial coordinates $\{ r , \theta , \varphi\}$ the topology of the geometric object is $\mathbb{R}_+ \times \mathbb{R}^3$ ut, base d on the system $\{ R , \theta , \varphi\}$ it a manifold with boundary, the boundary being the Schwarzschild sphere.

What happens to the metric coefficients as $r \longrightarrow 0$?

(10) $$g_{tt} = \frac{(r^3 + \alpha^3)^{1/3} - \alpha}{(r^3 + \alpha^3)^{1/3}} c^2 \longrightarrow +0$$

This vanishing implies that on the Schwarzschild sphere, one cannot define a volume form, and thus no orientation.

For the g_{rr} , we must perform a Taylor expansion:

(11) $$g_{rr} \approx - \frac{r^4}{\alpha^3 \left[\left(1 + \frac{r^3}{\alpha^3} \right)^{1/3} - 1 \right]} \approx - \frac{r^4}{\alpha^3} \longrightarrow -0$$

At r = 0 or $R = \alpha$, spacetime is locally non-orientable. The regions of space corresponding to values $R < \alpha$ lie outside the realm of real numbers. In this space, Flamm [3] constructs the meridional section in the form of a lying half-parabola, with equation:

(12) $$z = \pm 2\sqrt{\alpha(r - \alpha)}$$

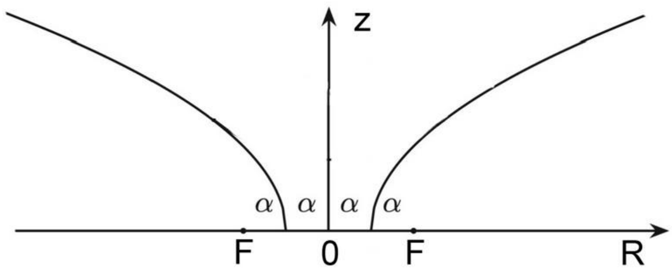

Fig.1 : Half-meridian of Flamm’s surface [3]

3 – **origin of the treatment of r as a radial variable**.

Who initiated this choice? It was David Hilbert [4]. He treats the solutions to the field equation—of which he had published his own version [5] a few days before Einstein [6]—not as metrics but as bilinear differential forms:

(13) $$G\left(x^i,\ \frac{dx^i}{dp} \right)$$

His field equation [5,6] is:

(14) $$K_{\mu\nu} - \frac{1}{2} K\, g_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}\, L}{\partial g^{\mu\nu}}$$

Where $K_{\mu\nu}$ is the Ricci tensor and K the corresponding Ricci scalar.

He places his solution in a space { x_1 , x_2 , x_3 , x_4 }, with the fourth coordinate referring to time. He considers the space to be quasi-Euclidean, writing (his equation (37)):

(14) $$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon\, h_{\mu\nu}$$

He then studies a stationary solution where the $g_{\mu\nu}$ are independent of x_4. He assumes the $g_{\mu\nu}$ are centrally symmetric with respect to the origin of the coordinates ("*zentrisch symmetrisch*"). On page 67, he chooses spherical coordinates:

(15) $$x_1 = r\, cos\theta$$
(16) $$x_2 = r\, sin\theta\, cos\varphi$$
(17) $$x_3 = r\, sin\theta\, sin\varphi$$
(18) $$x_4 = l$$

Taking symmetry into account, he writes the bilinear form (equation (42)):

(19) $$F(r)\, dr^2 + G(r)\,(d\theta^2 + sin^2\theta\, d\varphi^2) + H(r)\, dl^2$$

At this stage, he decides to set:

(20) $$r * = \sqrt{G(r)}$$

Then he writes:

> *Züren wir in (42) r anstatt r ein und lassen dann wieder das Zeichen * weg**
> (*Let us insert r∗ into (42) instead of t and then drop the asterisk again*)

In doing so, without realizing it, he expresses his solution not in terms of the coordinate

(21) $$r = \sqrt{{x_1}^2 + {x_2}^2 + {x_3}^2}$$

but according to Schwarzschild's intermediate quantity (« *Hilfsgrobe* »):

(22) $$R = (r^3 + \alpha^3)^{1/3} \geq \alpha$$

The first to point out this discrepancy was the Canadian mathematician Abrams [7]. Thus, Hilbert's bilinear form becomes, in his equation (43):

(23) $$M(r) + r^2 d\theta^2 + sin^2\theta\, d\varphi^2 + W(r)\, dl^2$$

4 – Hilbert's legacy: the survival of a purely imaginary time.

After determining the form of his functions M(r) and W(r), he writes (equation (45), p. 70)

(23) $$G(\,dr, d\theta, d\varphi, dl\ \ ) = \frac{r}{r-\alpha} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta\, d\varphi^2 + \frac{r-\alpha}{r}\, dl^2$$

Then, after writing:

"... so ergibt sich aus (43) *für l=it die gesuchte Maßbestimmung in der von Schwarzschild gefundenen Gestalt*"

He writes:

(24) $$G(\,dr, d\theta, d\varphi, dt) = \frac{r}{r-\alpha} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta\, d\varphi^2 - \frac{r-\alpha}{r}\, dt^2$$

One must recall that before Einstein presented his formulation of special relativity in 1905 [8], the majority of scientists adhered to the interpretation of the French mathematician Henri Poincaré, who in 1902 proposed the idea that the time variable was purely imaginary [9]. Even when Hermann Minkowski developed what would become spacetime in special relativity [10], he retained the idea of imaginary time—though he would soon abandon it [11], adopting Einstein's view that time is ultimately measured in meters.

In 1915, in his article on the advance of Mercury's perihelion [12], Einstein explicitly presents his Gram matrix, on page 832:

… der unspünglichen Relativitätstheorie entsprechende Schama gegeben:

... given the original theory of relativity corresponding to the shape:

(25) $$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5 – Everything depends on the meaning given to the word "geodesic"

One may say that at the time, before World War II, most authors (Einstein, Schwarzschild, Droste, Weyl, and others) explicitly opted for the signature (+ - - -), equipping the solution hypersurface with a metric where the length element s, identified with the proper time t through the relation s = c t, is real. One might consider that it was Hilbert's influence that suggested the shift (or return) to a signature inheriting the interpretation of time dt as a purely imaginary quantity.By contrast, with Schwarzschild everything is clear from the start, when he writes at the beginning of his article [2]:

(26) $$ds = \sqrt{\sum g_{\mu\nu}\, dx_\mu\, dx_\nu} \quad \mu = 1\,,2\,,3\,;\,4$$

The terms are set out very clearly. s is a length, essentially real. As for the constructed curves—the geodesics—they are the shortest paths, materialized by the variational equation:

(27) $$\delta \int ds = \delta \int \sqrt{\sum g_{\mu\nu}\, dx_\mu\, dx_\nu} = 0$$

Thus, Schwarzschild only considers curves that have a real length.But with Hilbert, things change completely when he writes [4], page 68:

> *Der erste Schritt hierzu ist die Aufstellung der Differentialgleichungen der geodätischen Linien durch Variation des Integrals*
>
> (*The first step is to formulate the differential equations of the geodesic lines by varying the integral*)

(28) $$\int M \left(\frac{dr}{dp}\right)^2 + r^2 \left(\frac{d\theta}{dp}\right)^2 + r^2 sin^2\theta \left(\frac{d\varphi}{dp}\right)^2 + H \left(\frac{dl}{dp}\right)^2$$

This integral is Hilbert's action J. The variational calculus is applied to it $\delta J = 0$, leading to the Lagrange equations. But then, what does one call a geodesic?

For Schwarzschild, Einstein, Droste, and Weyl, they are curves along which the real length s is minimized.

For Hilbert, these curves minimize the *square* of that length. The resulting Lagrange equations are the same, but in one case there is a constraint on the domain of validity of the solution, while in the other this constraint disappears.

Later in his article, the fact that his bilinear form $G\left(\frac{dx_s}{dp}\right)$ may be positive or negative does not bother Hilbert much, since he defines two *real* lengths:

– In the region where the bilinear form $G\left(\frac{dx_s}{dp}\right)$ is positive, he defines a real length referring to curve segments he calls *segments*.

(29) $$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\ \mathrm{dp}$$

– In the region where the bilinear form $G\left(\frac{dx_s}{dp}\right)$ is negative, he defines a second real length referring to curve portions he calls *time lines*.

(30) $$\tau = \int \sqrt{- G\left(\frac{dx_s}{dp}\right)}\ \mathrm{dp}$$

– Finally, when the bilinear form $G\left(\frac{dx_s}{dp}\right)$is null, it corresponds to “null lines” (*Nulllinien*),

6 – when the choice of lagrangian determines the topology of the object

This identification of the bilinear form with the Lagrangian appearing in the action will become generalized. One may cite, as an example, the mention in the relatively recent (1992) book by S. Chandrasekhar, *The Mathematical Theory of Black Holes* [13].

After presenting on page 92, in his equation (60), the Schwarzschild metric in the form:

(31) $$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1-\frac{2M}{r}} - r^2 d\theta^2 - r^2 sin^2\theta\ d\varphi^2$$

with signature (+ - - -), he defines on page 96, in equation (80), what he calls "the Lagrangian of the Schwarzschild spacetime":

(32) $$L = \frac{1}{2}\left[\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1-\frac{2M}{r}} - r^2\dot{\theta}^2 - r^2 sin^2\theta\ \dot{\varphi}^2\right]$$

This allows him to trace the geodesic curves located *“inside the Schwarzschild sphere”*, that is, with a purely imaginary ds:

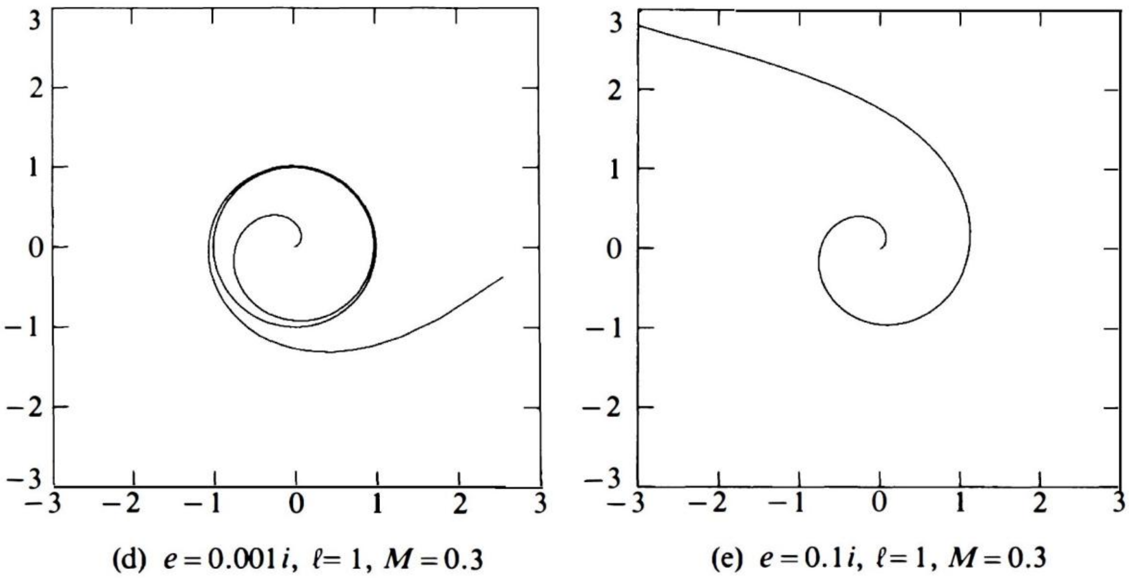

FIG. 7*b*. Various classes of time-like geodesics described by a test particle with $E^2 > 1$: (a), (b), (c): orbits of the first and the second kind with eccentricity $e = 3/2$ and latera recta, 4.5, 2.5, and 1.94 respectively ($M = 3/14$ in the scale along the coordinate axes); (d), (e): unbound orbits with $l = 1$ and with imaginary eccentricities $e = 0.001i$ and $0.1i$ ($M = 0.3$ in the scale along the coordinate axes).

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Figure 2: Geodesics spiraling *"inside the Schwarzschild sphere"* according to S. Chandrasekhar, from his book [13], p. 121.

It is clear that this choice of Lagrangian, differing from that of Schwarzschild, *"extends the expression of the solution to the entire spacetime"*, which goes hand in hand with the analytic continuation constructed by Kruskal [26]. It is no longer a manifold with boundary, but a space whose topology is $\mathbb{R}_+ \times \mathbb{R}^3$.

7 – everything depends on what one considers the physical world to be

This choice of Lagrangian—a "modern" choice—is at the root of the black hole model. Since it is accompanied by an alteration of the metric signature when crossing the Schwarzschild sphere, the interpretation accepted by all specialists is that in this region ttt becomes a spatial coordinate and rrr becomes a time coordinate.

As an indication only, let us now examine the various interpretations of the two-dimensional geometric object defined by the bilinear form:

(32)
$$G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2\dot{\varphi}^2$$

If, like Chandrasekhar, one chooses the Lagrangian:

(33)
$$L = G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2\dot{\varphi}^2$$

And the action :

(34) $$J = \int L(r, \dot{r}, \varphi, \dot{\varphi}) ds$$

then the Lagrange equations yield the differential equation:

(35) $$\frac{d\varphi}{dr} = \pm \frac{h}{r^2} \frac{1}{\sqrt{\left(1 - \frac{\propto}{r}\right)\left(1 - \frac{h^2}{r^2}\right)}}$$

Let us assign the constant α the value 1. The following figure shows different solution curves:

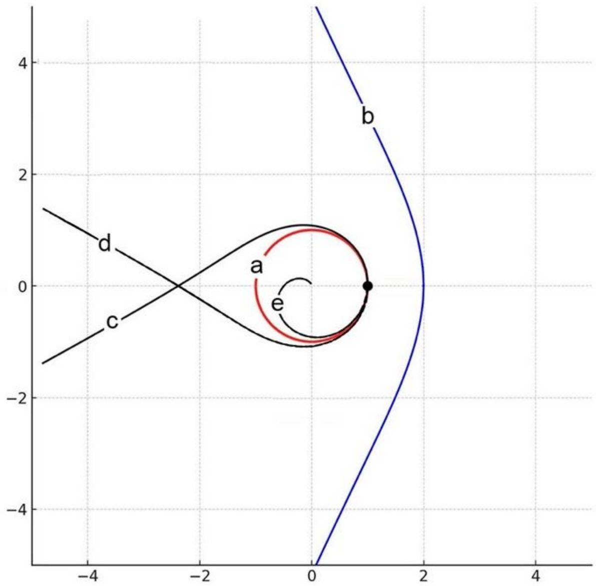
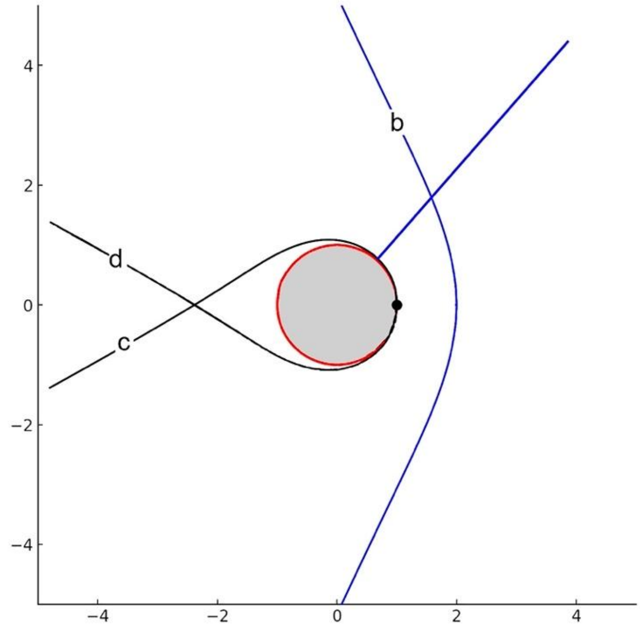


Fig.3 : Solution curves .
(b) : (r > α ; h < α). (c): (r > α ; h = α) . (e) : (r < α ; h < α)

This choice of Lagrangian amounts to choosing an object whose topology is homotopic to $\mathbb{R}^2$.If we define the Lagrangian as:

(36) $$L = \sqrt{G} = \sqrt{\frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2\dot{\varphi}^2}$$

Then we have two possible topologies, since the drawing of curves inside the circle is now forbidden.



$Fig. 5 : \ The\ object\ cannot\ be\ homotopic\ to\ \ \mathbb{R}^2$

Either we consider that the geometry is that of a manifold with boundary. It turns out that the object can be embedded in $\mathbb{R}^3$. By adding a third dimension z, and setting:

(37)
$$dr^2 + dz^2 = \frac{dr^2}{1 - \frac{\alpha}{r}}$$

This gives the meridian curve :

(38)
$$z = \pm 2\sqrt{\alpha(r - \alpha)}$$

But we have a third option: that of the two-sheeted covering of this manifold with boundary. This covering is then regular, and the meridian becomes:

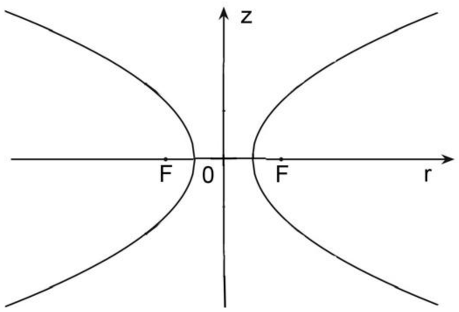


Fig.6 : Meridian of the object viewed as a two-sheeted covering of a manifold with boundary.

Since this object has the particularity of being embeddable in $\mathbb{R}^3$, we can provide a perspective view of it in this three-dimensional representation space:

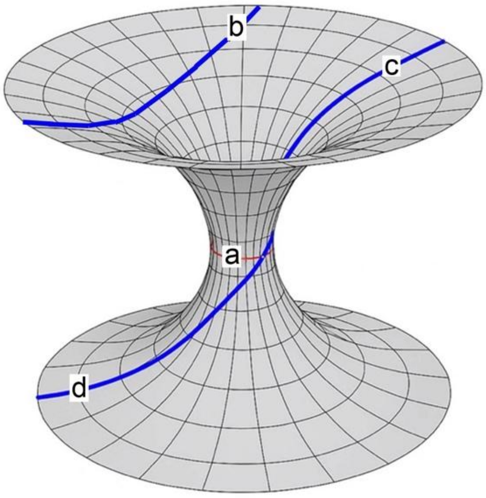


Fig.6 : Perspective view of the Flamm surface, embedded in $\mathbb{R}^3$

It is the surface generated by the rotation of a lying parabola around its axis—a Flamm surface. This allows us to consider that curves (c) and (d) each lie on one of the two sheets joined along a throat circle.

It is immediately evident that this reasoning can be extended to the bilinear form corresponding to the spatial 3D part of the modern form of the Schwarzschild solution.

$$G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2\dot{\theta}^2 + sin^2\theta\, \dot{\varphi}^2 \tag{39}$$

When Schwarzschild presents his solution, the object is then the union of two manifolds joined along their common boundary, which is not the Schwarzschild sphere but the surface of the star. Flamm presents its meridian [3]:

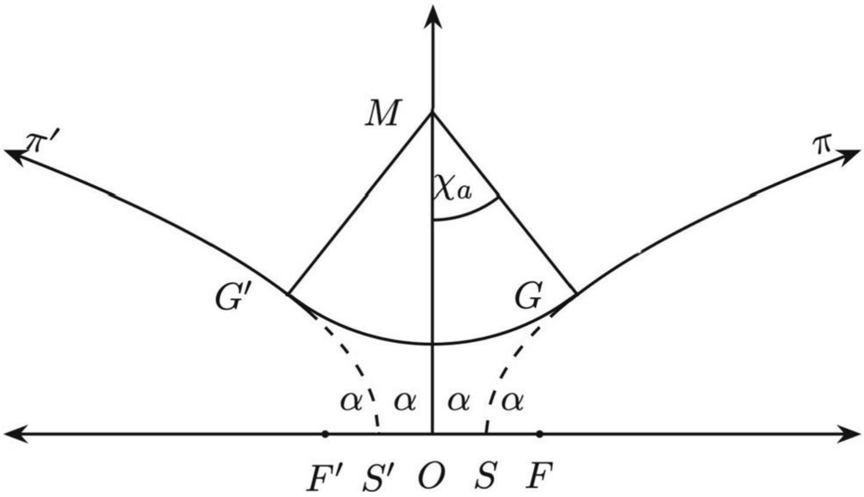


Fig.7 : Meridian of the Schwarzschild hypersurface [3].

Flamm then uses only part of the lying parabolas, which connect via a circular arc. The interior geometry is that of a portion of S3 spher, with metric:

(40) $$d\sigma^2 = \hat{R}^2(d\chi^2 + sin^2\chi \, d\theta^2 + sin^2\chi \, sin^2\theta \, d\varphi^2)$$

With:

(41) $$\hat{R} = \sqrt{\frac{3\,c^2}{8\,\pi\,G\,\rho}}$$

The star's surface corresponds to $\chi = \chi_a$. Schwarzschild and Flamm obviously had no problem with topology. The 3D hypersurface is contractile and unfolds by translation in the time dimension.

The black hole model consists in considering, as a full-fledged geometric object, the one associated with the bilinear form:

(42) $$\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{2M}{r}} - r^2\dot{\theta}^2 - r^2 sin^2\theta \, \dot{\varphi}^2$$

with the a priori hypothesis of contractibility, that is, of topology $\mathbb{R}_+ \times \mathbb{R}^3$, which amounts to asserting that what corresponds to values $0 \leq r \leq \alpha$ is part of physics and gives rise to a true singularity at r = 0.

8 – **The Birkhoff theorem** [14].

When a theoretical physicist constructs a mathematical solution to an equation and wants that solution to correspond to a phenomenon situated within the domain of physics, it then appears essential to him that this solution be unique. The Birkhoff theorem is therefore, above all, a uniqueness theorem which immediately prohibits the presence of a cross term in drdt. Put differently, it requires that the solution be not only invariant under time translation—that is, stationary—but also invariant under the change t→−t , that is, static.

In 1934, in his book [15], the mathematician Richard Tolman was the first to note that, mathematically, the general solution to Einstein's equation without source term, in spherical symmetry, includes a cross term in drdt. What is the consequence of this term? It is easy to reveal it by applying the time coordinate change proposed by A. Eddington in 1924:

(43) $$t = t_E - \frac{\varepsilon\alpha}{c} Ln\left| \frac{r}{\alpha} - 1 \right| \quad with \quad \varepsilon = \pm 1$$

Then the bilinear form becomes :

(44) $$G = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E{}^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\,\varepsilon\alpha c}{r} dr\, dt_E - r^2 (\, d\theta^2 + sin^2\theta \, d\varphi^2)$$

We note in passing that this transformation eliminates the coordinate singularity, which was Eddington's goal. Focusing on radial geodesics, P. Koiran [17] analyzed free-fall and escape times. Without a cross term, the null-length geodesics lead to the same velocity evolution as perceived by a distant observer:

(45) $$V_\varphi = \pm c \left(1 - \frac{\alpha}{r}\right)$$

The value is identical for emerging and plunging light rays. This velocity cancels out on the Schwarzschild sphere. In the form resulting from the Eddington transformation, this velocity depends on the direction and sign of ε.

First case: $\varepsilon = -1$. n the case of photons in free fall, they have a constant radial velocity$V_\varphi = -$ c.

Escaping photons have radial velocity :

(46) $$V_\varphi = c \, \frac{r - \alpha}{r + \alpha}$$

Then this time becomes infinite.

If we choose $\varepsilon = +1$ he values are reversed. This time it is the photons on the escape trajectory that have a constant velocity c. If we refer to the Kerr metric, which also has a cross term in $dr\, d\varphi$ it also has two different speeds, depending on whether or not the azimuthal limine ray accompanies the rotational movement. This is known as "frame dragging", which is reminiscent of Mach's principle, as opposed to the covariance principle. In this way, the solution mentioned above could be associated with "radial frame-dragging".

Free-fall time calculation($\nu = -1$), or escape time ($\nu = +1$), of a mass with zero velocity at infinity gives :

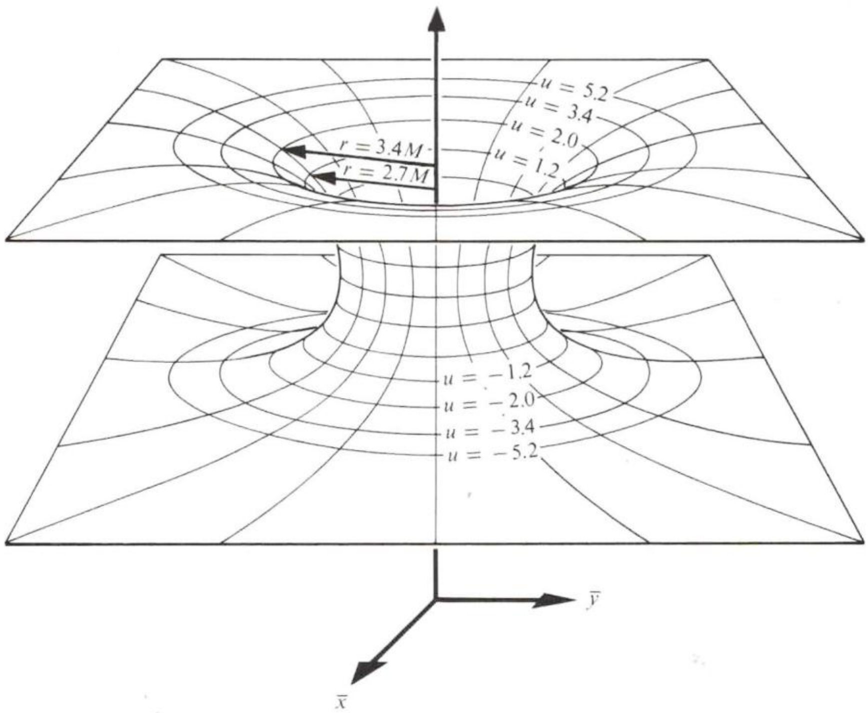
$$dt_E = \nu \frac{r + \alpha \varepsilon \nu \sqrt{\frac{\alpha}{r}}}{c(r - \alpha)} \sqrt{\frac{\alpha}{r}}\, dr \qquad (47)$$

Whether this time is finite or infinite depends on how the equation behaves in the vicinity of $r = \alpha$:

$$dt_E \approx \nu \frac{r + \alpha \varepsilon \nu}{c(r - \alpha)}\, dr \qquad (48)$$

We see that if $\varepsilon = +1$ this time is finite for plunging radial trajectories, infinite for escape trajectories. Inverse conclusion if $\varepsilon = -1$

To manage this non-uniqueness of the solution, we are forced to abandon the topology $\mathbb{R}_+ \times \mathbb{R}^3$ to that of a two-sheet covering of a manifold with a (spherical) edge. The first to do so were Einstein and Rosen in 1935 [18], taking up the project of a geometric modeling of masses as a singular topological structure describing masses, a point connecting two similar spacetimes, an idea initially suggested by H.Weyl in 1917 [19]. This concept was also the basis of the wormhole object, supposed to represent a bridge, either between two universes, or between two different regions of space-time. In this case, the two identical metrics represent an abstraction. Free-fall times, expressed as proper times, are finite and brief. It has been deduced that these unstable objects must reform immediately, but this adverb is only relative, since to a distant observer this time would be infinite.



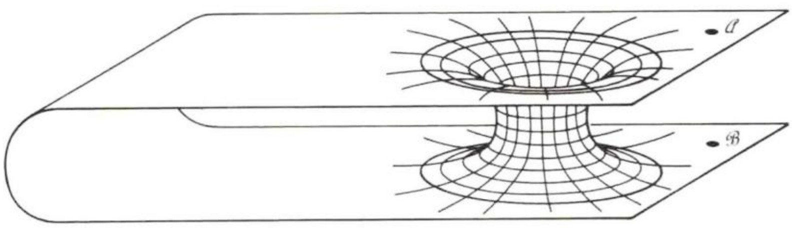


Fig. 8 : Wormholes [20]

If we try to interpret the reformulation of the solution with a cross term and impose continuity in speed, we are led to opt for $\varepsilon = -1$ in our own fold and for ur $\varepsilon = +1$ in the adjacent fold.
If we opt for an “old-fashioned” definition of length and proper time, we have in our fold:
(49)

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E{}^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\,\alpha c}{r} dr\, dt_E - r^2(\, d\theta^2 + sin^2\theta\, d\varphi^2)$$

In the adjacent fold :
(50)

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E{}^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 + \frac{2\alpha c}{r} dr\, dt_E - r^2(\, d\theta^2 + sin^2\theta\, d\varphi^2)$$

This results in a very short one-way transit time [17], which would invalidate Oppenheimer and Snyder's hypothesis [1]. Another view [21] is to attribute the inversion of the sign of the cross term to the inversion of the time coordinate in the second sheet, which becomes T-symmetrical to ours. With the help of an extremely simple diagram, it is also shown in [21] that the passage of masses through the throat sphere also generates a PT-symmetry.

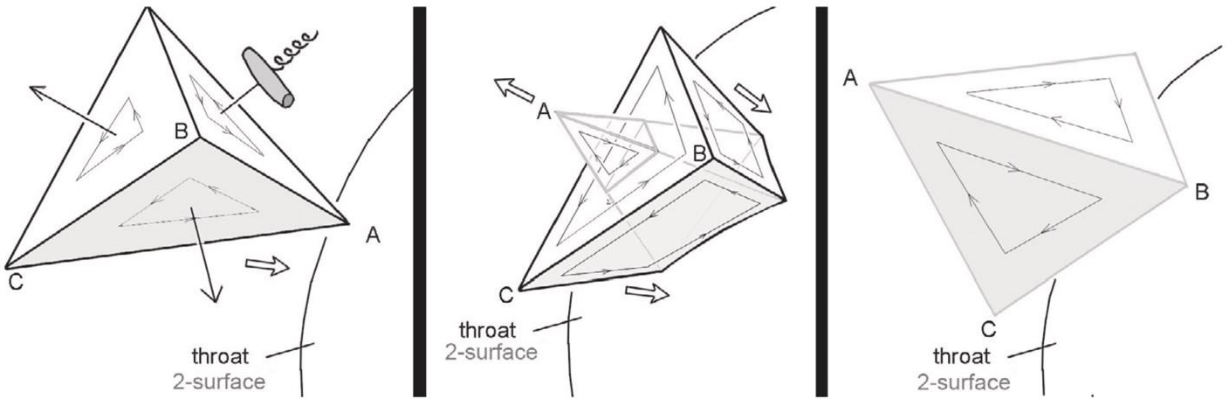


Fig. 9 : P-symmetry when crossing the throat sphere [21]

This PT-symmetry refers to the extension of the Poincaré group made in [22] and to the idea that this inversion of the time coordinate then leads to the inversion of the mass [23], leading to a reformulation of the pair of metrics, with:

(51) $$\alpha = \frac{2\,G\,M}{c^2}$$

In our own old (attractive field):
(52)

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E{}^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\,\alpha c}{r} dr\, dt_E - r^2(\, d\theta^2 + sin^2\theta\, d\varphi^2)$$

In the adjacent fold (repulsive field):

(53)

$$ds^2 = \left(1 + \frac{\alpha}{r}\right) c^2 dt_E{}^2 - \left(1 \ \frac{\alpha}{r}\right) dr^2 - \frac{2\,\alpha c}{r} dr\, dt_E - r^2(\, d\theta^2 + \sin^2\theta\, d\varphi^2)$$

9 – **Taking into account Schwarzschild's interior solution** [24]

In 1916, shortly before his death, Karl Schwarzschild published—after his exterior solution [2]—a second article, which became available in English translation only in 1999 [27], describing the geometry inside the mass [24]. His metric solution is then:

(54)

$$ds^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 c^2 dt^2 - \hat{R}^2(d\chi^2 + \sin^2\chi\, d\theta^2 + \sin^2\chi\, \sin^2\theta\, d\varphi^2\,)$$

Following his approach, the exterior metric ceases to be valid when its ds^2 coefficient becomes negative. Let us call this situation a *geometric criticality*. The angle χ then reaches the value $\chi_a = \pi/2$. The Flamm meridian then takes the following form:

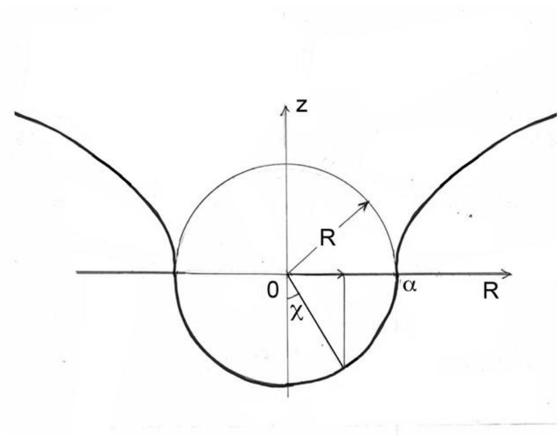


Fig.10 : Flamm meridian under geometric criticality.

The g_{tt} term vanishes at the center of the object ($\chi = 0$) when χ_a = 1/3 , i.e., for an angle close to 71°. The pressure at the center of the object then blows up to infinity. We can then draw the corresponding meridian curve. The central point becomes singular.

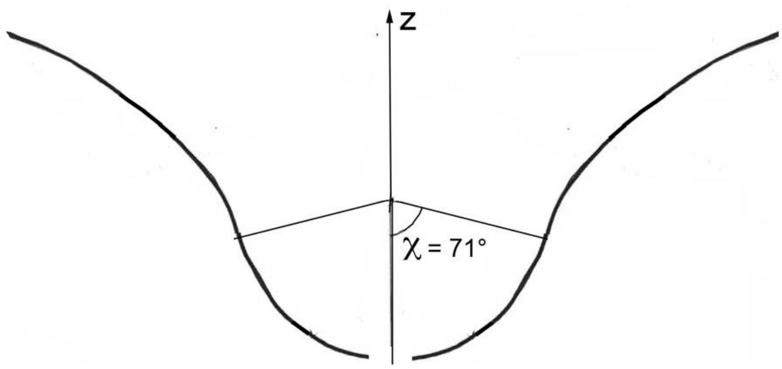


Fig.11 : Flamm meridian under physical criticality

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Pressure is a volumetric energy density. Faced with this observation, two attitudes are possible. The first, illustrated by reference [20], consists in discarding this exact solution of Einstein's equation with source term as physically irrelevant.

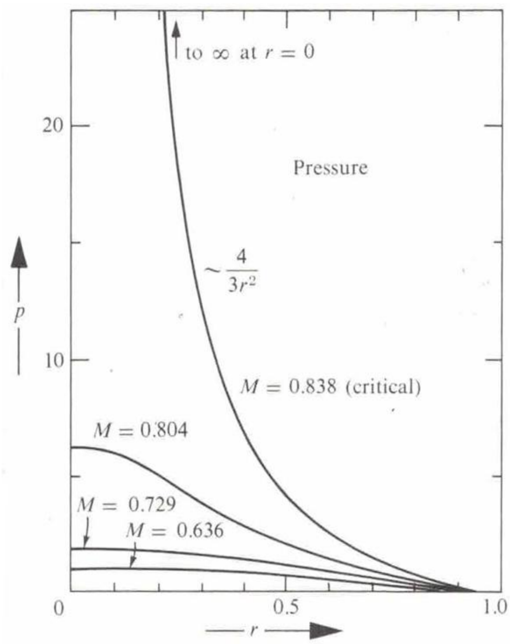

Fig.12 : Physical criticality, pressure [20]

The second approach is to try to understand what this equation—and its solution—may be trying to tell us. In a neutral gas, assimilated to a perfect gas of constant density, pressure is the kinetic energy density associated with the thermal agitation of the components:

(55) $$p_m = \frac{\rho < V^2 >}{3}$$

A hyperdense medium such as that in neutron stars is above all a plasma, where pressure is radiation pressure:

(56) $$p_r = \frac{\rho\, c^2}{3}$$

If this pressure diverges to infinity, it means that the speed of light ceases to be invariant. But this invariance is only required in a vacuum. Nothing compels us to assert absolute invariance inside matter under extreme density conditions, when the interparticle distances approach the Compton wavelength of the components. Schwarzschild was thus the first to contemplate a possible variability of ccc, later followed by other attempts [25].

10 – **The plugstar mechanism.**

If we decide to heed the "message" conveyed by this solution, one could argue that the explosion of volumetric energy density induces a *topological surgery*, opening a window toward another universe—namely, in the context of [23], toward the adjacent PT-symmetric sheet. To explore what may happen, we can go further in constructing the time factor:

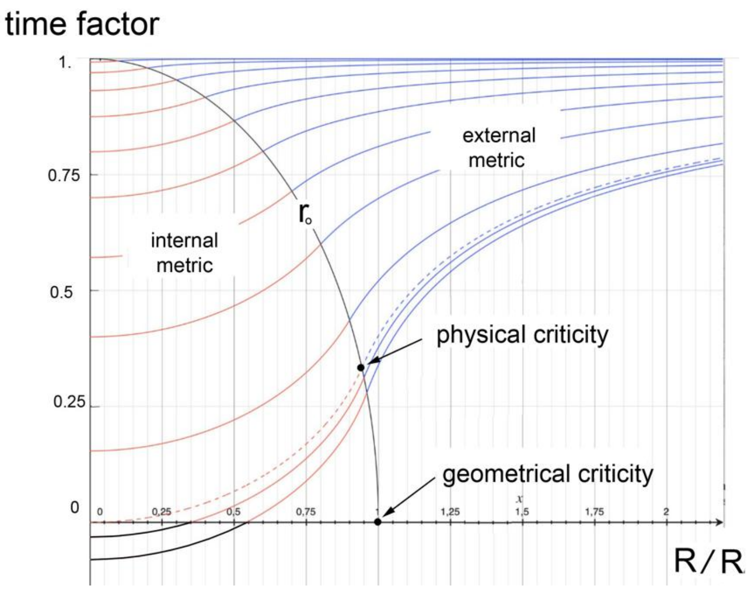

Fig.12 : Time factor

Beyond physical criticality, the time factor f becomes negative:

(57) $$d\tau = \frac{ds}{c} \, f \, dt < 0$$

Then this means that dt becomes negative. This mass then moves through a region of spacetime where the time coordinate is reversed, which, according to [22], implies that its energy and mass are also reversed.

We can even observe that, as this penetration progresses beyond the physical criticality, the diameter of this region grows parabolically until it reaches a maximum, at $r = \alpha$ which point the entire mass has been reversed:

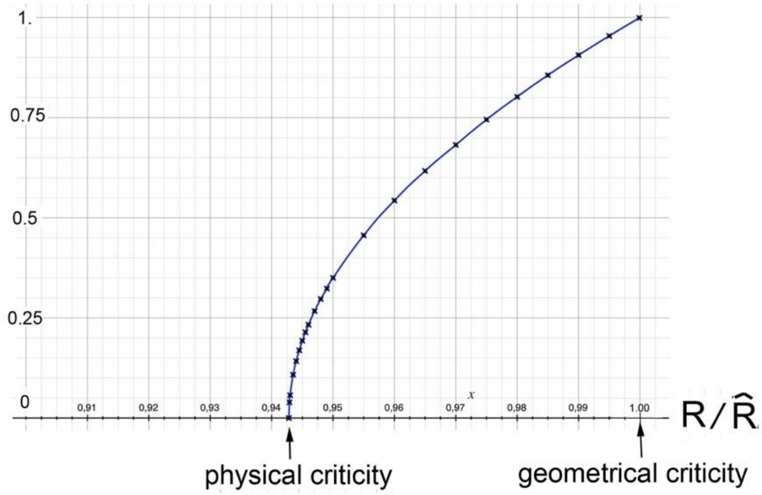

Fig.13 : Growth of the region where mass is reversed.

When we consider this rise to physical criticality, in the context of the Janus cosmological model, we must read this evolution by considering the two conjugated metrics. In the sheet of positive masses, the inner metric is:
(58)

$$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{8\pi G \rho {r_a}^2}{3c^2}} - \frac{1}{2}\sqrt{1 - \frac{8\pi G \rho r^2}{3c^2}}\right]^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{8\pi G \rho r^2}{3c^2}} - r^2(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2)$$

It connects with the external metric:

(50) $$ds^2 = \left(1 - \frac{8\pi G \rho}{3c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{8\pi G \rho}{3c^2 r}} - r^2(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2)$$

In the negative mass fold, the inner metric is:
(60)

$$ds^2 = \left[\frac{3}{2}\sqrt{1 + \frac{8\pi G \rho {r_a}^2}{3c^2}} - \frac{1}{2}\sqrt{1 + \frac{8\pi G \rho r^2}{3c^2}}\right]^2 c^2 dt^2 - \frac{dr^2}{1 + \frac{8\pi G \rho r^2}{3c^2}} - r^2(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2)$$

And the external metric:

(61) $$ds^2 = \left(1 + \frac{8\pi G \rho}{3c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 + \frac{8\pi G \rho}{3c^2 r}} - r^2(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2)$$

When this object receives additional mass, this suggests that a window opens at the center, and an equivalent quantity of matter is inverted, its mass being transferred to the negative fold and, under the effect of the gravitational field, expelled from the object. This would create a feedback effect that would ensure that geometric criticality would never be reached. If this scheme is valid, it would limit the mass of neutron stars to just over 2.5 solar masses instead of three. We suggest to call such objects plugstars.

This phenomenon of mass inversion would also occur when two neutron stars orbiting their common center of gravity merge. If both have these sub-critical masses of 2.5 solar masses, their merger would cause the inversion and dispersion of 2.5 solar masses, resulting in the production of a very strong gravitational wave. Such a powerful signal, interpreted according to the standard scheme, would lead to a significant overestimation of the masses involved

12 – **About observational features.**

Recently, the combination of powerful observing resources has enabled us to obtain images of the hypermassive objects M87*[28] and SgrA*[29].

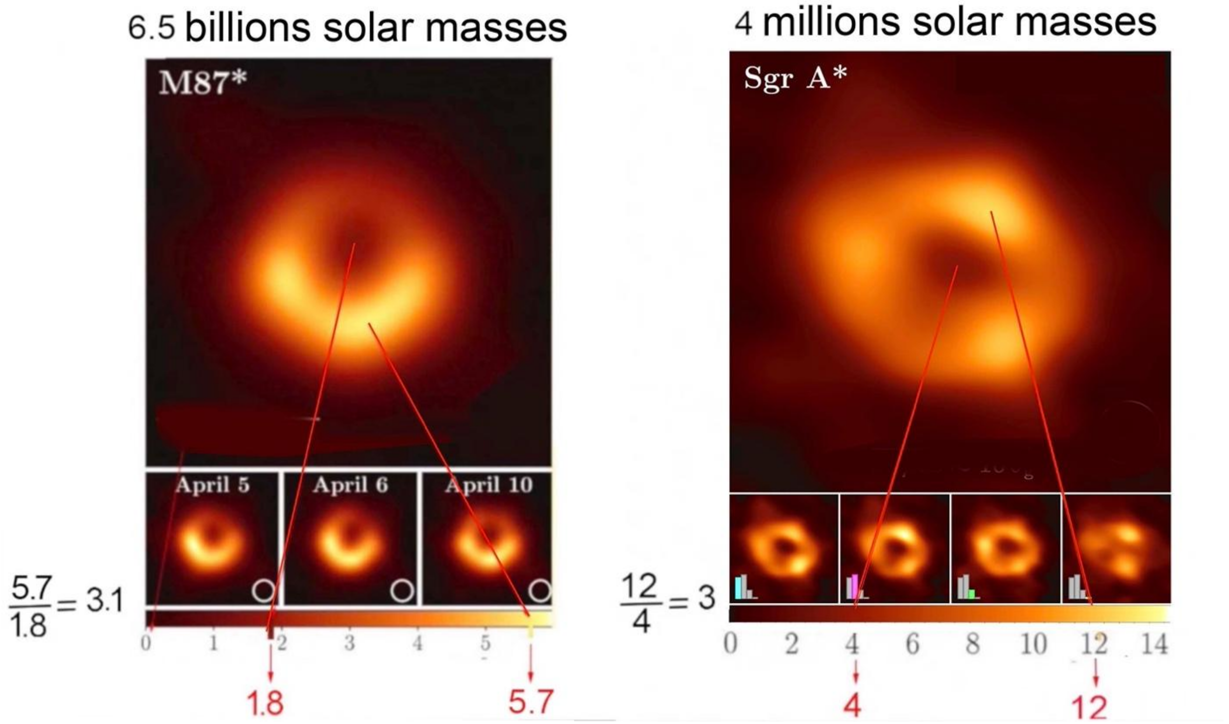

Fig.14 : mages of hypermassive objects M87*and SgrA*

The chromaticity bar, providing “temperature equivalents”, can be used to evaluate the maximum and minimum wavelength ratios in the two images. The figures obtained are surprisingly close to 3 . Strictly speaking, therefore, we can't immediately identify them as giant black holes, for which the gravitational redshift effect would then lead to infinite values, unless the luminosity of the central part can be attributed to a mass of hot gas in the foreground. However, given the vast differences in the masses and temperature equivalents of these objects, this hypothesis is not very credible. There's another aspect to the formation of such giant black holes, by successive accretions. If this were the case, the axis of symmetry of these objects would have a direction uncorrelated with that of the galaxy that hosts it. However, observers agree with the generalization of this correlation between the two axes.

Let's examine the plugstars hypothesis. Under subcritical conditions, the time factor becomes :

$$f = \frac{3}{2}\sqrt{1-\frac{8}{9}} - \frac{1}{2}\sqrt{1-\frac{8}{9}} = \frac{1}{3} \qquad (62)$$

We have a gravitational redshift:

$$1 + z = \frac{\lambda(observer)}{\lambda(emitter)} = \frac{\sqrt{g_{tt(observer)}}}{\sqrt{g_{tt(emitter)}}} = \frac{1}{f} = 3 \qquad (62)$$

This gives a wavelength ratio (observer)/(emitter) of 3, which is consistent with these observational data. Can these be considered significant and systematic? Naturally, we'll be keeping a close eye on the figures emanating from future observations. But these cannot be obtained with ground-based radio telescopes. Indeed, the two sources had special status. The one at the center of the Milky Way was obviously the closest, and the one at the center of the M87 galaxy was exceptionally powerful. For further data, we'll have to wait for new radio telescopes to be put into orbit.

12 – **About the origin of such hypermassive objects and the nature of quasars.**

Seyfert galaxies, modulo the difficulty of evaluating the distance, seem to produce a Hubble constant significantly lower than the standard value. If we use the figures for the Hoag galaxy with D = 56 MPc and z = 0.004 we obtain a value of Ho of 21.4 km/s/Mpc. We conjecture that very irregular galaxies should give a value of Ho higher than the standard value. The idea is to consider a phenomenon of expansion turbulence, resulting from joint fluctuations of the metrics, which would have the effect of varying the confinement of the galaxies. In the case of weakening of this field this could go as far as the dislocation of the structure. In the opposite case where there is a strengthening of the gravitational field, this could cause the start of a centripetal density wave. This, destabilizing the gas, would generate the birth of new stars which, during their early youth, would reveal their presence, as in the spiral structure, by illuminating the gas through fluorescence. This is the interpretation that we propose to give to the circular formation present in the Hoag galaxy, as a density wave.

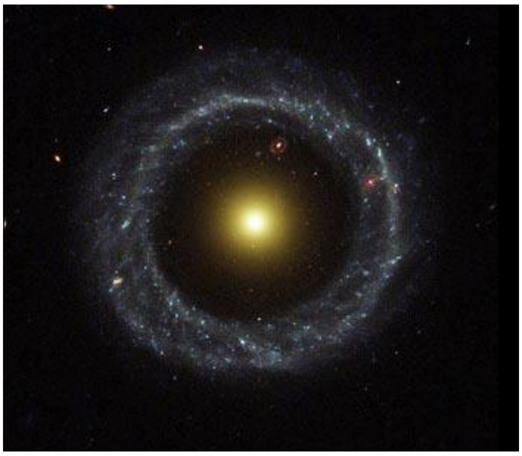

Fig.15 : Galaxie de Hoag ([30], [31])

Young stars are emissive for times between 5 and 20 million years. Measurements relating to the year give a rotation speed of 250 km/s, but no radial speed. If the ring is a density wave, this is normal, because it is the speed of the wave (comparable to a tsunami with spherical symmetry) and not that of the wave of matter that reacts to its passage. The width of the ring being Hoag's being 7.4 kpc, this allows an evaluation of the radial speed of the wave between 730 and 1470 km/s. It would then reach the center of the galaxy in a time between thirteen and fifty million years. The magnetic Reynolds number being large compared to unity, the wave then brings together the lines of force of the pre-existing magnetic field in the galaxy, which is evaluated at one microgauss. By bringing these lines together, an intense dipole magnetic field is formed in a direction that coincides with the axis of symmetry of the galaxy. The object that forms when the wave is focused at the galactic center is small. Its angular rotation speed, as a solid body, is that which animated the galactic center before the impact, therefore moderate. There is a sudden excursion of density and temperature such that the Lawson conditions are reached in a very large mass. Hence a powerful plasma emission according to the two poles of the magnetic field. This is our vision of the quasar phenomenon. Moreover, this system automatically behaves like a natural accelerator of charged particles, capable of giving them considerable energies and we see there the origin of cosmic rays. From this angle, the object located at the center of our galaxy could be considered as an extinct quasar. This mechanism of joint fluctuations of metrics, which we are currently trying to model, could periodically restart the emission, stimulated by the convergence of new density waves, of very low amplitude, but each time causing an emissive burst. Hence the dashed structure of the M87 jet. We conjecture that these joint fluctuations of metrics would have been very important at the very moment of the formation of galaxies. This is the reason why we observe the presence of these hyperdense objects in the oldest galaxies.

13 – **Conclusion**.

We've given a broad overview of all aspects of black hole theory, highlighting the fact that this theory is based on considering the Schwarzschild outer solution as self-sufficient, describing an object on its own. The first hypothesis also relies on the absence of a cross term, linked to Birkhoff's uniqueness theorem, which allows the vision of a freeze frame, where the time variable becomes the proper time of a distant observer. The second idea, of a topological nature, is to impose the hypothesis of a local contractibility of space-time. Finally, the construction of trajectory curves depends on the choice of Lagrangian, immediately pointing out that using a Lagrangian or its square gives the same Lagrangian equations. In fact, solutions to Einstein's equation are not automatically metrics, but bilinear forms, as noted by David Hilbert. The nature of the object then depends on how the solution is interpreted. We have given the example of a two-dimensional bilinear form whose interpretation depends on the topological hypothesis. If we impose the idea of a contractile space, then we build curves that spiral towards the center, but with an altered signature. The same applies to a 3D interpretation. If we wish to avoid altering the signature, then the 2D object becomes the Flamm surface and the 3D object a bridge connecting two Euclidean spaces, through a gorge sphere. A very disturbing idea then emerges that the black hole model may have arisen from the refusal to take into account a different topology. We then return to the two geometric solutions established by Schwarzschild, pointing out that there is a physical criticality that has not been taken into account by theorists. We show that beyond this criticality the time factor reverses, which, based on dynamical group theory, suggests a time inversion and enantiomorphism at the passage of the gorge sphere. With this topology, uniqueness is no longer required, allowing the presence of a term in dr dt, which then drastically modifies transit times, making a sphere of throat traversable in one direction, but not in the other. An alternative model is then proposed, in which geometric criticality is never reached, but the mass inversion phenomenon keeps the object, known as a plugstar, in a subcritical configuration with respect to physical criticality. We calculate the gravitational redshift effect that such objects would present: 3 and find that this is exactly what emerges from observational data from hypermassive objects M87*¨and SgrA*.

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