

**The Janus Cosmological Model predicted a law of expansion
with variable acceleration as early as 2014.**

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Abstract : It is worth recalling that the exact solution of the Janus Cosmological Model's coupled field equations has, since 2014, yielded an expansion law consistent with the most recent DESI observations, that is, with a greater acceleration in the past. We retain the model's major achievements, which ensure the very early formation of a large-scale, incomplete structure, as well as that of first-generation galaxies and stars. The model interprets the spiral structure as a perennial mechanism for the exchange of momentum between a non-collisional system and a negative-mass environment. The negative lensing effect accounts for all observations of light ray deflection. Thus, negative masses fulfill all the functions previously attributed to the dark matter-dark energy system.

1 – Introduction :

In 2011, the Nobel Prize was awarded to Perlmutter, Riess, and Schmidt [1,2,3], confirming the idea that cosmic expansion, instead of decelerating as predicted by Friedmann's models, is actually accelerating. The initial assumption was a constant acceleration, and to account for this, the cosmological constant Λ was reintroduced into Einstein's equation. Thus, the model of general relativity, which had initially been modified by adding cold dark matter—that is, matter composed of elements whose speed was small compared to the speed of light—to the observable visible matter, became the Λ CDM model. This term corresponds to a contribution to the gravitational field of a repulsive component, called the "repulsive power of the vacuum," which is invariant over time. Dynamically, this gives the expansion law of the scale factor $a(t)$ a form of exponential evolution. As the years passed, this development became contested. For a more general description of cosmic dynamics, the concept of a barotropic fluid was introduced, associated with an equation of state:

(1)

$$p = w \rho c^2$$

Posing $c = 1$:

(2)

$$p = w \rho$$

This, more generally, gave rise to the law of evolution:

(3)

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$$\rho = a^{-3(1+w)}$$

- Dust fluid : $w = 0$, which gives $\rho \approx a^{-3}$

- Radiation fluid : $w = 1/3$, which gives $\rho \approx a^{-4}$

- For the equivalent of the cosmological constant : $w = -1$, which gives $\rho \approx \text{constant}$

But this latter description has proven to be erroneous over time, and we then turn to a purely phenomenological model, introduced by Chevallier, Polarski and Linder [4,5] (CPL representation) using two parameters w_0 and w_a , according to:

(4)

$$w(a) = w_0 + w_a(1 - a)$$

Or, equivalently:

(5)

$$w(a) = w_0 + w_a \frac{z}{1 + z}$$

If the cosmological constant governs the expansion, this corresponds to the following values:

(6)

$$w_0 = -1 \quad w_a = 0$$

The exploitation of data collected by the DESI collaboration [6] tends to distance it from such a description with:

(7)

$$w_0 > -1 \quad w_a < 0$$

This translates to saying that "dark energy would have been more important in the past".

2 – The Janus Cosmological Model.

The motivation behind the work that led to this model did not originate in cosmic dynamics. In 1967, the Russian Andrei Sakharov [7,8,9] proposed a model intended to account for the absence of observations of primordial antimatter. He started from the principle that baryons are formed from three quarks and antibaryons from three antiquarks. For the sake of symmetry, he assumed that the universe is composed of two twin cosmic entities, linked by the "Big Bang singularity." He endowed these two universes with opposing arrows of time:

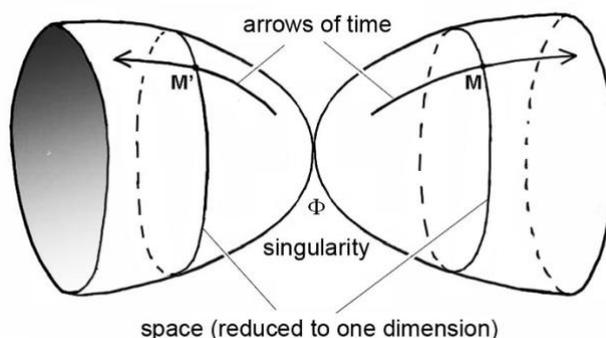


Fig.1 : Didactic model of the universe of A. Sakharov.

Sakharov suggests that the synthesis of baryons from quarks would have been faster than that of antiquarks from antibaryons in our universe sheet. The opposite is true in the twin universe sheet. Thus, our own universe would be composed of matter and a small fraction of free antibaryons, in a 3:1 ratio. He envisions that these two universes are linked by a PT symmetry. This idea has recently been taken up by Boyle, Finn, and Turok [10]. From this idea, the construction of the Janus model is a long story involving dynamical group theory [11] and topology [12]. In short, the dynamical group of relativity is initially the restricted Poincaré group, limited to its orthochronous components.

In 1970, the mathematician J.M. Souriau [13], using the tools of symplectic geometry (the action of the group on the dual of his Lie algebra), revealed the physical quantities of energy, momentum, and spin as the components of objects moving along the geodesics of non-zero length (matter) and the geodesics of zero length (photons) of spacetime. The dynamical group associated with Sakharov spacetime is then the complete Poincaré group, including its antichronous components (T-symmetry). This allows us to conclude that the components of the twin sheet are photons of negative energy and zero mass, and particles of non-zero mass and negative energy $m c^2 < 0$.

In 1994 [14], the author envisaged a geometric configuration where the two sheets introduced by Sakharov are "folded", by considering the involvement of the masses of the adjacent portions:

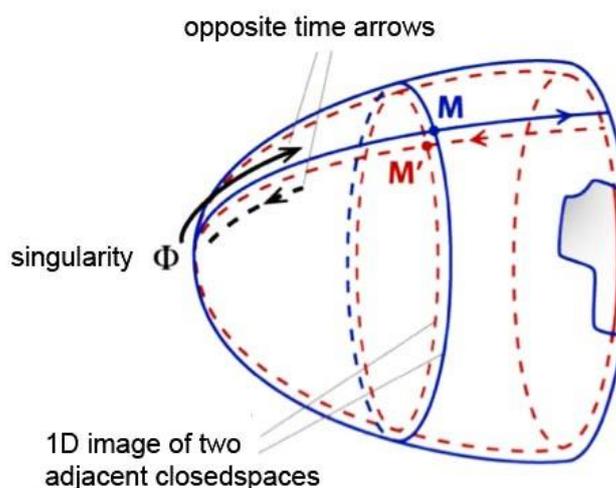


Fig.2 : Sakharov's universe "folded up".

We are considering, in passing, associating the closure of space with a closure of time. The didactic 2D image is then that of a spherical spacetime where the north pole represents the Big Bang, and the south pole the Big Crunch. Space is then represented by the circles forming the parallels of the sphere, whose perimeter reaches a state of maximum expansion (equator) before decreasing again to zero (Big Crunch).

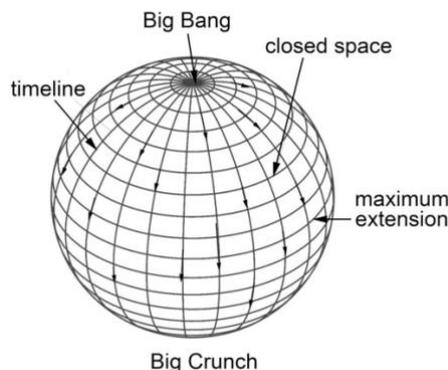


Fig3 : Spherical 2D spacetime.

We then configure this spacetime by bringing the pairs of antipodal points into coincidence, which has the effect of configuring it according to a projective P^2 . For the sphere S^2 , it is then configured according to the two-sheet covering of a projective P^2 , so the image is the Boy surface (d) (further explanations can be found in reference [12]).

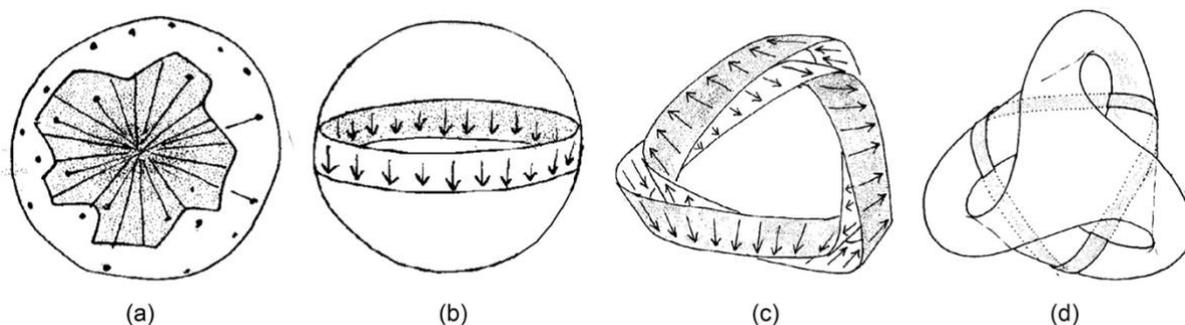


Fig.4 : Coincidence of pairs of antipodal points[12].

The operation can be repeated with spheres of higher dimensions. The S^4 sphere is thus configured according to a P^4 projective. The Big Bang and the Big Crunch are thereby brought into coincidence. When the dimension is even, this operation generates a PT -symmetry of adjacent sheets (even-dimensional projective spaces are non-orientable; while odd-dimensional ones are orientable). But we know that if we attempt, in the context of general relativity, to make masses of opposite signs interact, we encounter the unmanageable runaway phenomenon [15].

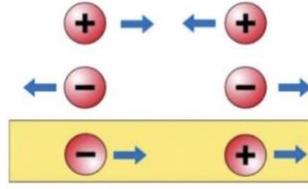


Fig.5 : Interaction laws (after General Relativity)

In general relativity, all particles follow geodesics stemming from the same metric $g_{\mu\nu}$. Masses, in particular, follow geodesics of non-zero length. In 2002, the first attempt [16] of a bimetric configuration appeared, without specifying the geometric context. It then only considered particles "(following geodesics of a metric $g_{\mu\nu}^R$ right" and "left" (following geodesics of a metric $g_{\mu\nu}^L$), assumed to belong to branes. Starting from the action:

(8)

$$S = \int (R^L - \chi L^L) \sqrt{|g^L|} d^4x + \int (R^R - \chi L^R) \sqrt{|g^R|} d^4x - \mu^4 \int V(g^R, g^L) (g^R g^L)^{1/4} d^4x$$

Where R^L and R^R are the Ricci scalars, L^L and L^R are the Lagrangians of the two materials, μ is a coupling constant and $V(g^R, g^L)$ is a potential accounting for the interactions between the two populations, they obtain the system of coupled field equations:

(9)

$$R_{\mu\nu}^L - \frac{1}{2} R^L g_{\mu\nu}^L = \chi (T_{\mu\nu}^L + t_{\mu\nu}^L)$$

(10)

$$R_{\mu\nu}^R - \frac{1}{2} R^R g_{\mu\nu}^R = \chi (T_{\mu\nu}^R + t_{\mu\nu}^R)$$

$T_{\mu\nu}^L$ and $T_{\mu\nu}^R$ are source tensors of the populations with masses m_L and m_R . $t_{\mu\nu}^L$ and $t_{\mu\nu}^R$ are tensors describing the interaction. Considering that these two populations have positive masses $m_L = m > 0$ and negative masses $m_R = -m < 0$, and if these populations can be approximated as dust particles, these tensors can be reduced to the following terms:

(11)

$$T_{00}^L > 0 \quad , \quad t_{00}^L < 0 \quad , \quad T_{00}^R < 0 \quad , \quad t_{00}^R > 0$$

The direction of the forces can then be easily determined:

(12)

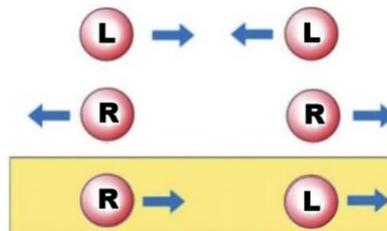


Fig.6 : Interaction forces with « left-positives masses» - « right-negatives masses»

Thus, we see that if we try to make these masses, left and right, play the role of positive masses and negative masses respectively, we encounter the unmanageable runaway effect. However, there is a simple and elegant way to resolve this dilemma by noting that in action (8) there is an unexploited degree of freedom, which consists of taking equal and opposite Einstein constants: (13)

$$\chi_R = -\chi_L = -\chi$$

The action is modified by a simple change of sign: (14)

$$S = \int (R^L - \chi L^L) \sqrt{|g^L|} d^4x + \int (R^R + \chi L^R) \sqrt{|g^R|} d^4x - \mu^4 \int V(g^R, g^L) (g^R g^L)^{1/4} d^4x$$

And the system becomes, by setting $g_{\mu\nu}^L = g_{\mu\nu}$ (positive masses) and $g_{\mu\nu}^R = \bar{g}_{\mu\nu}$ (negative masses):

(15)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi (T_{\mu\nu} + t_{\mu\nu})$$

(16)

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = -\chi (\bar{T}_{\mu\nu} + \bar{t}_{\mu\nu})$$

Under the same conditions as in (11):

(17)

$$T_{00} > 0 \quad , \quad t_{00} < 0 \quad , \quad \bar{T}_{00} < 0 \quad , \quad \bar{t}_{00} > 0$$

This gives us the general shape of the system of coupled field equations of the Janus cosmological model. Under these conditions, we obtain the direction of the forces corresponding to the Janus model [12].

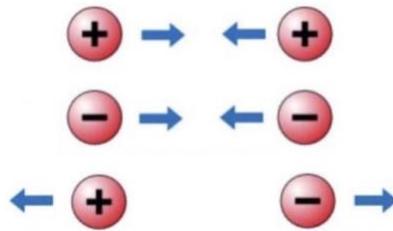


Fig.7 : Direction of forces in the Janus Cosmological Model [12].

However, in order to exploit such a system of coupled field equations, they must satisfy the Bianchi conditions:

(18)

$$\nabla^\nu T_{\mu\nu} = 0 \quad \bar{\nabla}^\nu \bar{T}_{\mu\nu} = 0$$

(19)

$$\nabla^\nu t_{\mu\nu} = 0 \quad \bar{\nabla}^\nu \bar{t}_{\mu\nu} = 0$$

Conditions (18) are achieved by giving the tensors the classical forms:

$$(20) \quad T_{\mu}^{\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \quad \bar{T}_{\mu}^{\nu} = \begin{pmatrix} \bar{\rho} \bar{c}^2 & 0 & 0 & 0 \\ 0 & -\bar{p} & 0 & 0 \\ 0 & 0 & -\bar{p} & 0 \\ 0 & 0 & 0 & -\bar{p} \end{pmatrix}$$

In reference [12], we provided forms of the interaction tensors such that the Bianchi conditions are satisfied in Newtonian approximation. However, in the present article, we are interested in the unsteady solution under homogeneity and isotropy conditions. Another form of Lagrangian derivation is proposed in [12] and leads to forms of the field equations in which the ratios of the square roots of the determinants of the two metrics appear. This method is then similar to that of reference [17]. In this second approach, it is again the identification of the two Einstein constants with a common value that generates a system of forces incompatible with physics.

3 – The unsteady solution of 2014 [18].

Applying Bianchi's condition to Einstein's equation yields conservation equations. In unsteady, homogeneous, and isotropic conditions, and in matter-dominated environments, we obtain the conservation of mass:

$$(21) \quad \rho a^3 = Cst$$

This should actually be treated as a conservation of energy:

$$(22) \quad \rho c^2 a^3 = Cst$$

In 2014 [18] we started from the system of equations (15) and (16) by giving the interaction tensors the following forms:

$$(23) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi (T_{\mu\nu} + \varphi \bar{T}_{\mu\nu})$$

$$(24) \quad \bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = -\chi (\bar{T}_{\mu\nu} + \phi T_{\mu\nu})$$

Where φ and ϕ are functions of time. Then, with the common coordinates $\{x^o, u, \theta, \vartheta\}$, giving the metrics the FRLW forms:

$$(25) \quad ds^2 = dx^{o2} - a^2 \left(\frac{du^2}{1 - k u^2} + u^2 d\Omega^2 \right)$$

$$(26) \quad d\bar{s}^2 = dx^{o2} - \bar{a}^2 \left(\frac{du^2}{1 - \bar{k} u^2} + u^2 d\Omega^2 \right)$$

We then find [18] that the compatibility of the equations was ensured by the condition:

(27)

$$E = \rho c^2 a^3 + \bar{\rho} \bar{c}^2 \bar{a}^3 = Cst$$

That is to say, a condition for the overall conservation of energy. Then, if we assume that the energy is predominantly negative, we obtain the two exact solutions, both having the curvature index $k = -1$, satisfying the differential equations, with $\chi < 0$:

(28)

$$a^2 \ddot{a} = \frac{\chi}{2} E$$

(29)

$$\bar{a}^2 \ddot{\bar{a}} = -\frac{\chi}{2} E$$

The fact that the acceleration \ddot{a} is positive indicates that $E < 0$. The Janus Cosmological model is therefore essentially deeply asymmetric, that is, with $\bar{\rho} < 0$ that:

(30)

$$|\bar{\rho} \bar{c}^2 \bar{a}^3| \gg \rho c^2 a^3$$

The dynamics are driven by the negative energy content. The solution to equation (28) is that given by W. Bonnor, in parametric form in 1989 [19]:

(31)

$$a = A c h^2 \eta$$

(32)

$$t = A \left(1 + \frac{sh2\eta}{2} + \eta \right)$$

The corresponding curve $a(t)$ is :

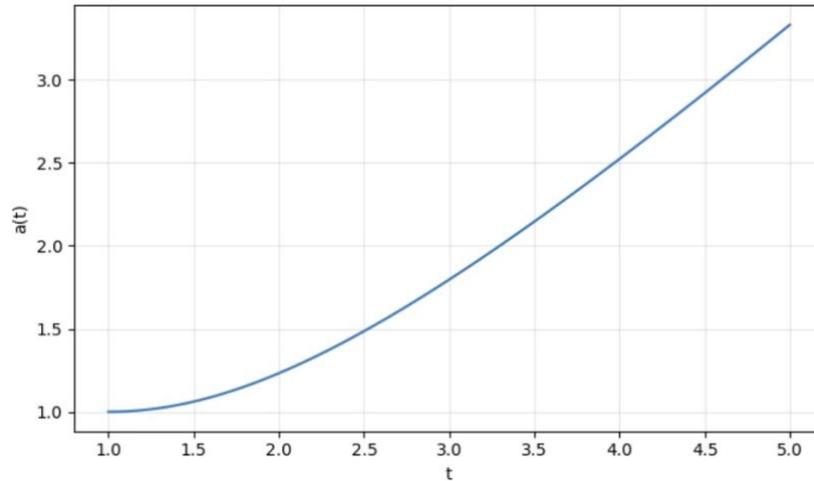


Fig.8 : Evolution of the scale factor a of positive masses.

Evolution of the scale factor has positive masses.

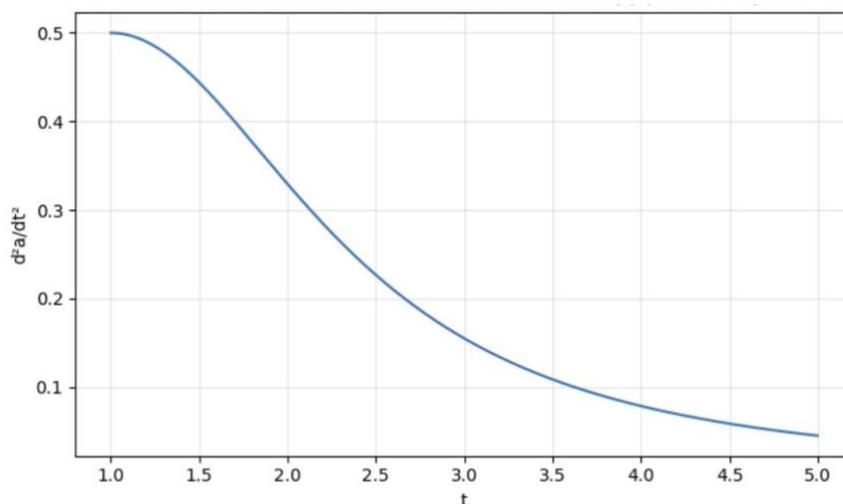


Fig.9 : Evolution of acceleration over time.

Unlike the Λ CDM model, where the curve $a(t)$ is an exponential, the acceleration tends towards zero at infinity, and the expansion $a(t)$ tends towards a linear law, which fits with a value of the curvature index $k = -1$.

What drives the expansion of the positive sector is not a fluid with the equation of state $p = w\rho$. It is the matter of the negative sector, with a density $\bar{\rho} < 0$ and negligible pressure. The description of the evolution of the two sectors corresponds to the matter-dominated era. These are therefore two interacting universes of dust, with negligible pressures. As we will see later, this negative mass content of the Janus Cosmological Model replaces both dark energy and dark matter, which can be illustrated by the image below:

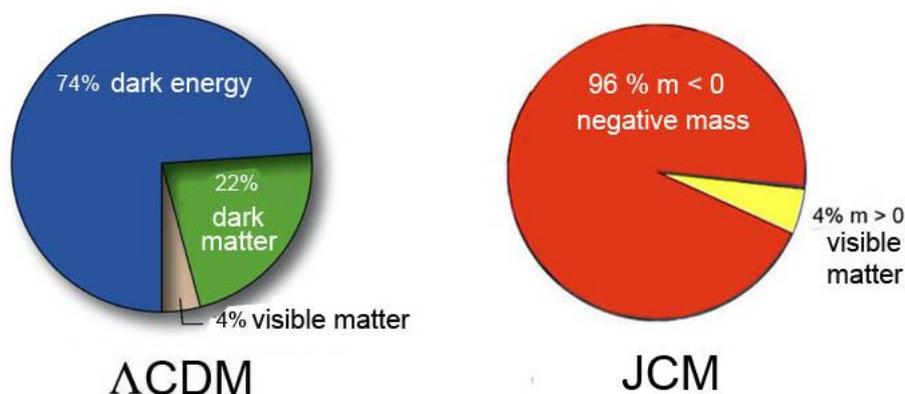


Fig.10 : Comparative proportions for the two models [12] .

In both models, only 4% of ordinary matter is observable. Negative mass, emitting negative-energy photons, is fundamentally unobservable by optical means. It reveals its presence only through the negative lensing effect it exerts on positive-energy photons. It is therefore predicted that all attempts to demonstrate the existence of positive-mass dark matter, essential components of the Standard Model, will be futile. The profound asymmetry of the model has an origin, which will be presented in a subsequent article, and which stems from the instability of the field equations in the early phase of evolution.

4 – The agreements of JCM with observation.

We will begin with the elements that correspond to recent observational data. As early as 1995 [20], the model based on the direction of forces in figure (7), presented at that time as a heuristic hypothesis, and on the basis of a preeminence of the density of negative masses had produced the outline of a lacunar structure of the distribution of positive mass.

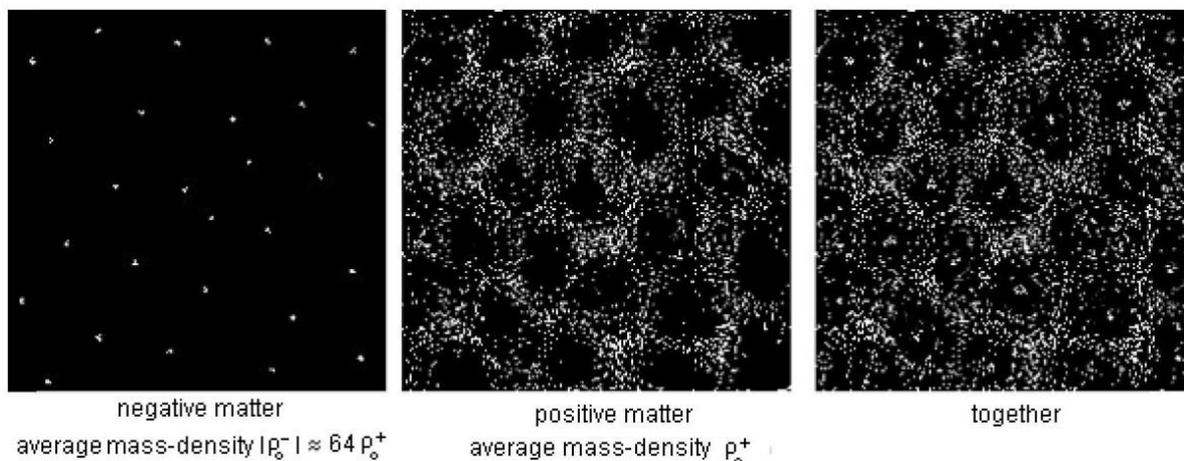


Fig11 : Large-scale, lacunar structure [20].

The existence of vast voids was confirmed in 2017 with the discovery of the Dipole Repeller [21]. Numerous other voids of this kind have since been identified. We propose, in passing, a possible observational confirmation of the Janus Model based on the negative lensing that would result from the presence of a conglomerate of negative mass at the center of the Dipole Repeller.

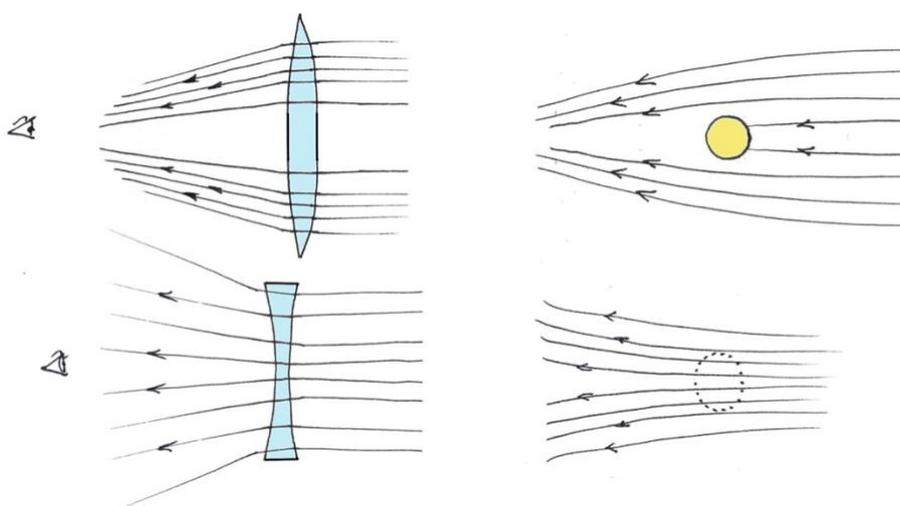


Fig.12 : Reduction of brightness by negative lensing.

Light passes freely through any negative mass. Image reconstruction will therefore be achieved by combining the luminosity attenuation corresponding to background sources, linked to both the metric $\bar{g}_{\mu\nu}$ of the negative sector, both external and internal. By identifying this region of attenuated luminosity, we should be able to estimate the diameter of the negative mass conglomerate, as schematically illustrated in the image below.

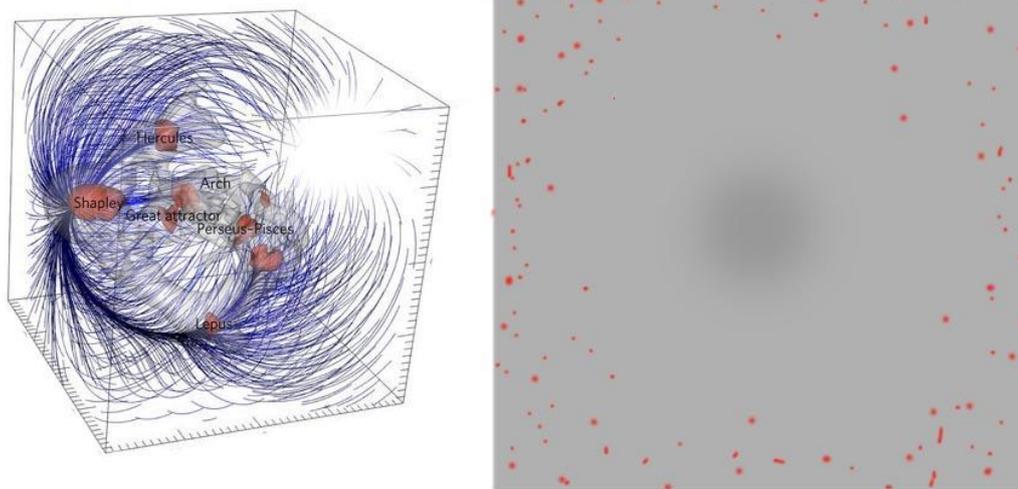


Fig.13 : Prediction of the reduction in schematic brightness attenuation due to the negative mass conglomerate located at the center of the Dipole Repeller.

The very large-scale structure forms very rapidly, immediately after decoupling. Negative conglomerates form first, due to the relative brevity of their Jeans time:

(33)

$$\bar{t}_J = \frac{1}{\sqrt{4\pi G |\bar{\rho}|}} \ll t_J = \frac{1}{\sqrt{4\pi G \rho}}$$

The schematic distribution of masses can be done on the basis of two coupled Vlasov equations, as had been outlined in 1994 [14]:

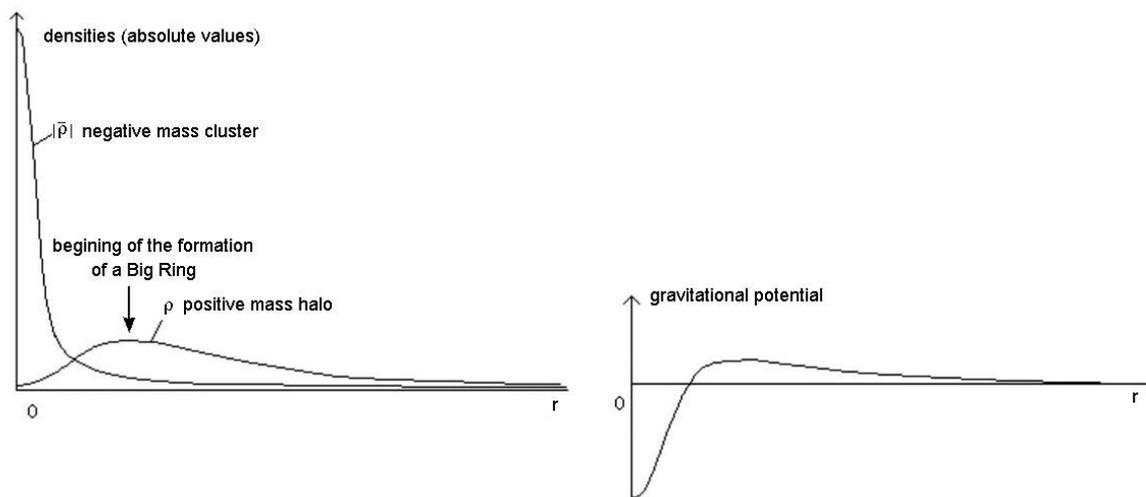


Fig.14 : Big Ring formation (schematic) [14].

In a numerical simulation performed with the appropriate tools, it should be possible to reveal the formation of an annular overdensity of positive mass, thus explaining the origin of the Big Bang [21]. As early as 1995, in this model of the rapid and abrupt formation of the very large-scale structure of the universe, the formation of negative mass clusters is accompanied by the

back-compression of positive mass, forced to arrange itself in contiguous bubbles within the remaining available space. Thus repelled by adjacent negative masses, it becomes compressed into plates and is thereby heated. However, unlike quasi-spheroidal structures, this mass is optimally suited to rapid cooling through radiative losses. The positive mass, thus destabilized, immediately gives rise to galaxies and first-generation stars within the first hundred million years. This model is therefore completely different from the standard model and explains why the JWST found massive galaxies fully formed at such a remote epoch.

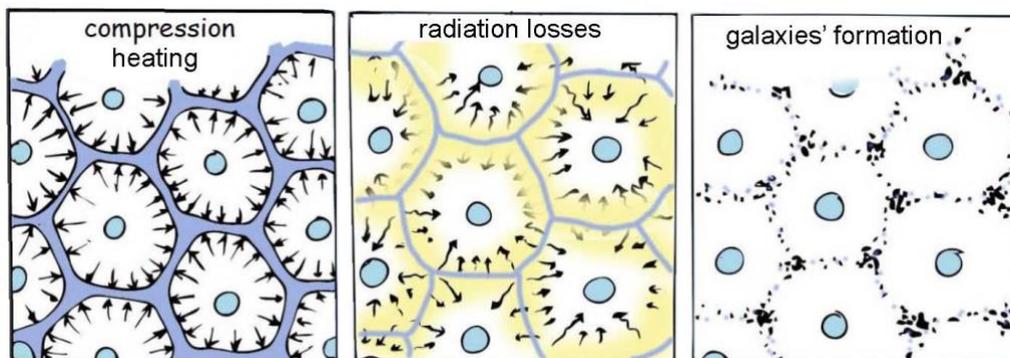


Fig.15 : Diagram of galaxy formation [12].

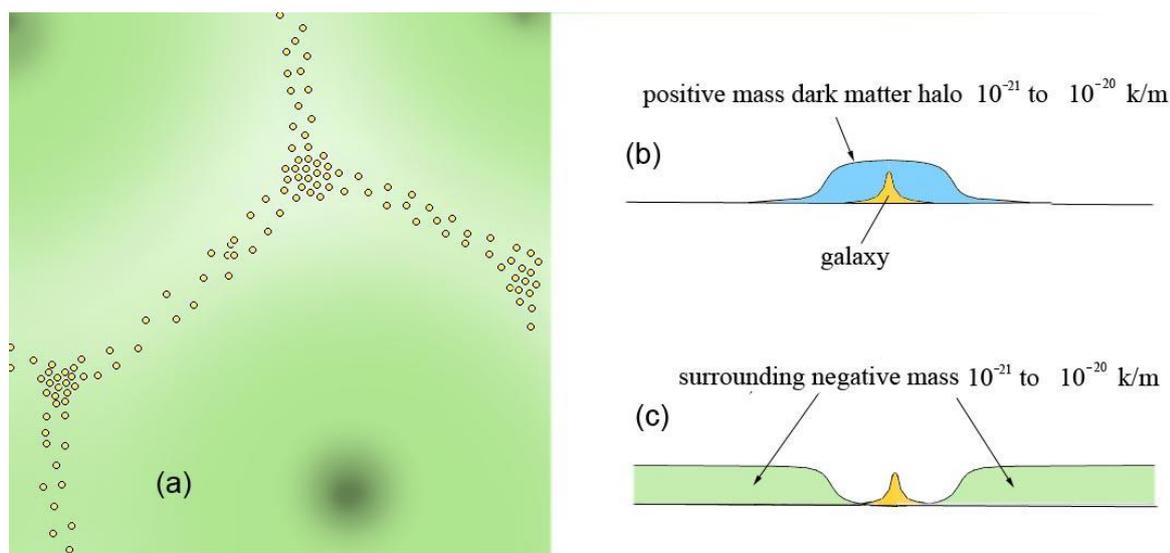


Fig.16 : Confinement of galaxies by surrounding negative mass.

This negative mass environment is then gravitationally stable. As early as 2001 [22], numerical simulations showed that barred spiral structures appeared from the very first rotations of the galaxy. These simulations also showed that this barred spiral structure was the way in which non-collisional systems could transmit momentum and energy to their negative mass environment, via density waves. Such structures are thus stable and long-lasting, destined to persist in galaxies as long as they possess gas to give rise to them.

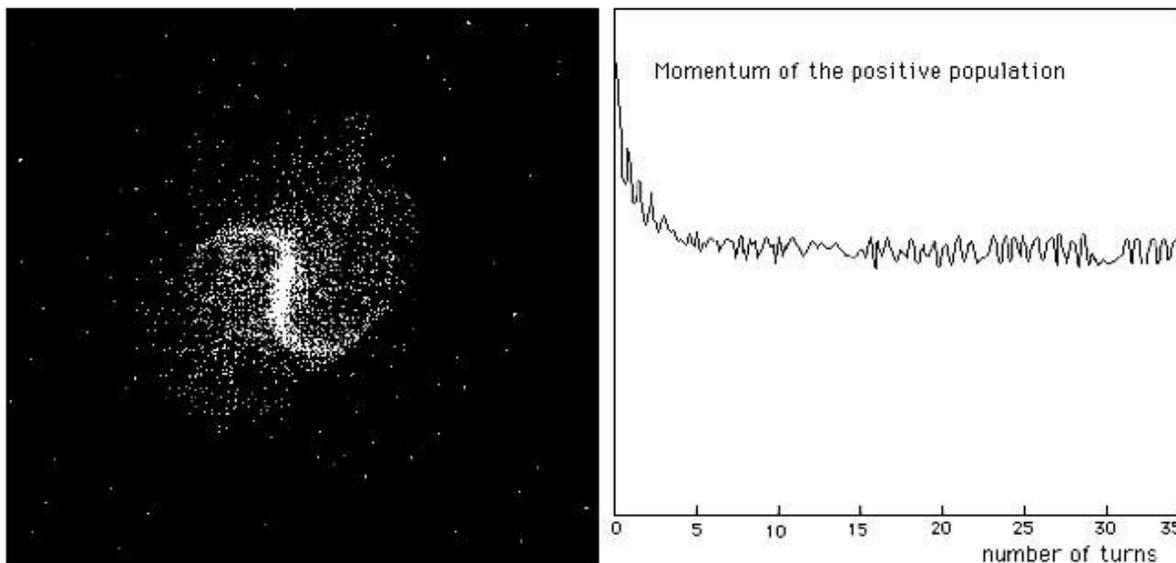


Fig.17 : Formation of barred spiral structure and loss of momentum [22].

The repulsion from the negative mass environment accounts for both the high velocities of galaxies in clusters and the flatness of their velocity curves at the periphery. Below is the recently constructed velocity curve [23] as an exact stationary solution of the Vlasov equation in an axisymmetric configuration:

(34)

$$v = \frac{\omega_0 \rho}{\sqrt{\frac{m}{kT_0} + 2(a - \alpha)\rho^2}}$$

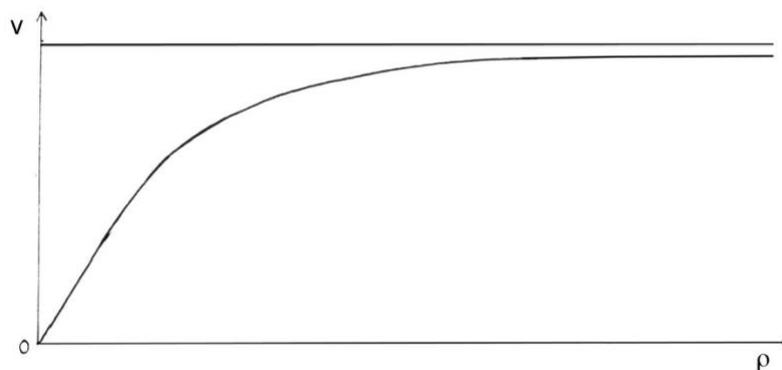


Fig.18 : Theoretical rotation curve of the galaxy [23].

The Janus Cosmological Model also offers an alternative to the black hole model, the plugstar [24].

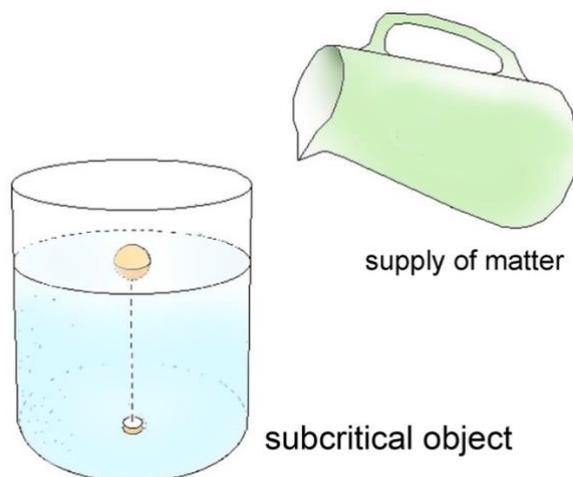


Fig.19 : Didactic image of the operation of a plugstar [24].

This model, based on the pairs of interior metric solutions $g_{\mu\nu}^{int}$ and $\bar{g}_{\mu\nu}^{int}$ agrees with the available observational data, in that it predicts a maximum/minimum brightness temperature ratio of 3, i.e. a gravitational redshift $z = 2$. And this, even though the two objects have masses differing by more than three orders of magnitude.

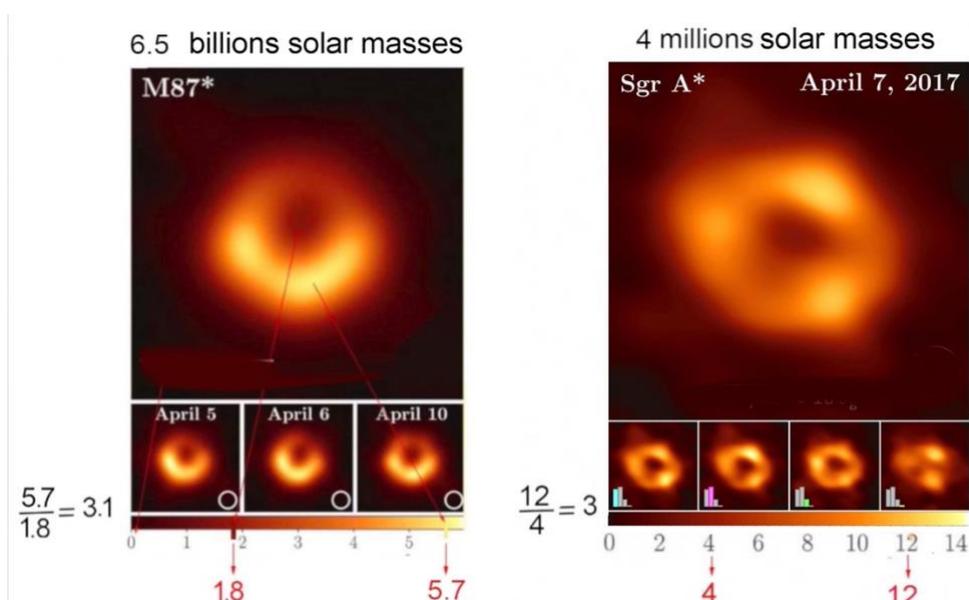


Fig.20 : Images of the supermassive objects M87* and SgrA* [24]

We will eagerly await the future images of four supermassive objects selected for a future observation campaign, which should in principle start in 2027.

5 – An alternative to the inflation model.

The Janus cosmological model has long offered a different interpretation [20] of the extreme homogeneity of the CMB, attributing it to the variation of the constants of physics in the early

phase of cosmic evolution. This model circumvents the conceptual difficulty of converting inflations into ordinary matter. The advantage of the Janus version is that the components of the universe are then devoid of mystery. The joint variations of the constants are shown below. Note that these joint variations preserve all the equations of physics as well as the Lorentz invariance.

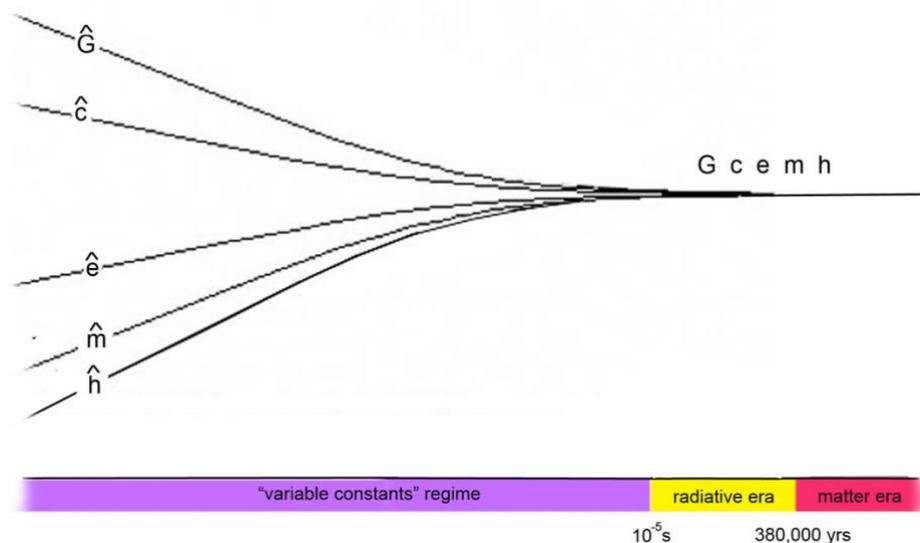


Fig.21 : Conjugate variations of the set of constants of physics in the primitive phase of evolution [20].

Although this theme has not resonated within the scientific community, it adds to all the elements that constitute JCM

5 – Nature of the invisible components of the universe.

The article based on group theory [11] provides a starting point for an answer: the content of this other layer of the universe consists of antimatter, negative mass, and photons with negative energy. The complexity of the world of positive mass arises from the fact that gravitational instability gives rise, very early on, to galaxies and stars, within which nucleosynthesis enriches the medium with heavy elements up to iron, the phenomenon of supernovae completing the periodic table of elements. In galaxies, planets form, some of which then harbor life. Such a sequence stops, in the negative world, after the nucleosynthesis of light elements transforms a fraction of the primordial antihydrogen into helium. But it stops there. When the very large-scale structure is formed, the negative world is then populated by immense spheroidal conglomerates made up of these light elements. The cooling time of such structures then exceeds the age of the universe. If these spheroidal masses continue to radiate outside the red and infrared spectrum, they practically cease to evolve. Thus, this negative world creates neither galaxies, nor stars, nor planets. Life is absent from it.

6 – Regarding the fluctuations of the CMB.

We intend to offer an alternative interpretation of these fluctuations, attributing them to a completely different origin. The Janus model operates with a set of physical constants and

spatial and temporal scale factors different from our own, even though the relationships between all these parameters mean that the underlying physics remains the same. This leads us to consider cosmic fluctuations resulting from the gravitational instability prevalent during the era when the cosmos was entirely composed of photons. This problem has never interested the community for the simple reason that the wavelength of such fluctuations is then on the order of the cosmological horizon, and would therefore escape observation. But things appear differently when we consider fluctuations occurring in the negative world. Representing these two entities as the front and back of the same surface is a convenient way to suggest this idea. In the drawing below, a "mirror" provides the image of this back side of the surface.

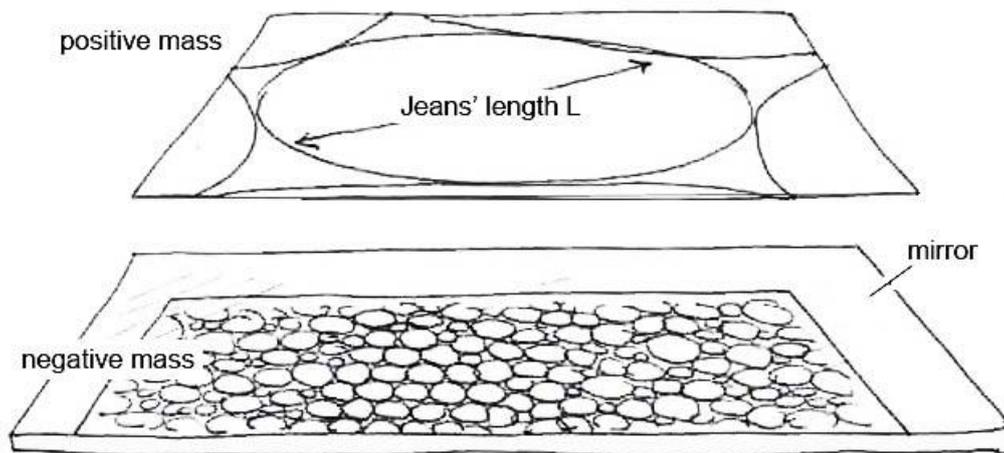


Fig.22 : Different scales of Jeans distances, in the primordial era of the two populations.

The phenomenon generates fluctuations that then interact with the positive world, creating fluctuations there as well. This is the basic idea, which will require significant development work. At this stage, the ratios of these wavelengths, and also the ratio of the scale factors, are on the order of 1 to 100. Thus, distances are a hundred times shorter in the negative layer.

7 – Conclusion.

The Janus Cosmological Model represents a profound shift in the geometric paradigm, but is ultimately only an extension of general relativity, which it in no way contradicts, within its observational range, that of local observations. Indeed, since masses of opposite signs are mutually exclusive, they are practically absent in the vicinity of the solar system, and the first equation of the system is then identical to Einstein's equation, without a cosmological constant.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \quad (35)$$

The model therefore automatically verifies classic local confirmations: the advance of Mercury's perihelion, the deflection of light rays by the Sun, and the alteration of time in the vicinity of massive objects. It is often said that every extraordinary theory requires extraordinary confirmations. For the Janus model, these are numerous. It accounts for an expansion with decreasing acceleration over time. It provides the key to the rapid formation of a very large-scale, sparse structure, suggesting in the process an explanation for the presence of

the Big Ring. It accounts for the very early formation of barred galaxies with high mass. It explains the reason for such formations, reflecting the interaction of galaxies with their negative-mass environment in the form of density waves. It accounts for the flatness of the rotation curves of distant galaxies and the high velocities of galaxies in clusters, attributing them to the confining action of their repulsive, negative-mass environments. It provides a precise identity for the invisible components of the universe while confirming Andrei Sakharov's idea regarding the absence of observed primordial antimatter. It accounts for all gravitational lensing effects in the vicinity of galaxies and clusters. But the model also overturns many established ideas. It challenges objects that seemed to have secured their place in the observational landscape: black holes of all sizes, proposing instead the plugstar model. This theory immediately proves falsifiable, based on a systematic property of such objects: the fact that the ratio of their maximum/minimum brightness temperatures is significantly close to 3, regardless of their mass. The need remains to account for CMB fluctuations using an alternative theory, outlined in the article.

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