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Experimental test to verify whether quantum states of conjugated mass and energy exist

Benoît Guay
Manaty Research Group, Canada
https://manaty.net/research-group/en
benoit.guay@manaty.net

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Based on the fact that the electromagnetic potential can interact with charged particle and using the Dirac field of conjugated mass and energy introduced in a previous paper, we show, in the framework of quantum field theory (QFT), that it would be possible experimentally to highlight significant differences in the probabilities of occurrence of electron and positron states depending on whether electrons and positrons states of conjugated mass and energy exist or not.

Keywords: Dirac fields; quantum field theory; Dirac and Feynman antimatter; negative mass and energy; enantiomorphous twin states.

1. Introduction

The Dirac equation with conjugated mass (i.e. negative mass) has made it possible to make sense of the existence of the right-handed antineutrino.[1] Positive mass for the left-handed neutrino and negative mass for the right-handed antineutrino but the absolute value of the mass (at rest) is the same. We can say that this is possible by the fact that the energy equation contains the squared mass m

$$E = \sqrt{(cp)^2 + (mc^2)^2}, \tag{1}$$

but in this case the energy is kept positive. On the other hand, the quadratic form for the energy equation

$$E^2 = (cp)^2 + (mc^2)^2, \tag{2}$$

gives us the possibility to look at quantum states with the same absolute value of mass and energy so new quantum states have conjugated mass and also conjugated energy.

Physics rejects negative energy because it is assumed that it violates causality and leads to physically undesirable effect such as the runaway effect.[2] But this one disappears in a bi-metric approach[3] and if causality is not a fundamental law but the simple fact of the second law of thermodynamics it no longer constitutes an obstacle to negative energy at rest[4] at quantum scale. Furthermore, the Dirac theory does not prohibit negative masses and energy[5,6] as well as the Janus cosmological model (JCM)[3,7,8] in which negative mass antimatter takes the place of unidentified dark matter and dark energy well known in cosmology. In quantum mechanics the exclusion of conjugated energies through the use of an antiunitary and antilinear time reversal operator T is a simple hypothesis.[9] Nothing forbids to use a unitary and linear T operator.

According to Ref. 9, there are two types of antimatter states. The first one, Dirac's antimatter state, is associated with an antiunitary PT transformation. The other, the Feynman's one, is the apparently missing cosmological antimatter state, which actually belongs to Sakharov's twin universe.[10–12] It is associated with a unitary PT transformation and consists of the same antiparticles states but of conjugated mass and energy. It has been established that the reversal of time in Sakharov's twin universe must be accompanied by a sign inversion of mass and energy.[13,14]

Therefore, it is desirable to find a test to check whether or not fermions and antifermions of conjugated mass and conjugated energy exist. In this work, we propose a test based on the fact that an electromagnetic potential (not field) transforms a fermion (antifermion) state of positive mass and energy into a conjugated mass and energy antifermion (fermion) state. By this we will show significant differences in the probabilities of occurrence of electron and positron states submitted to the action of an electromagnetic potential depending on whether electrons and positrons states of conjugated mass and energy exist or not.

2. Dirac Field of Positive and Negative Energy

Dirac fields operators in flat spacetime with mass $+\mu$ and $-\mu$ (i.e. $\mu > 0$) are given, respectively, by[15]

$$\Psi_+(\mathbf{r},\mathrm{t}) = \int d^3\mathbf{p} \sum_S \Big(\Phi_{+,S,\mathbf{p}}(\mathbf{r})\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) + \Phi_{-,S,\mathbf{p}}(\mathbf{r})\ \mathrm{a}^\dagger_{-,S,-\mathbf{p}}(\mathrm{t})\Big), \tag{3}$$

and

$$\Psi_-(\mathbf{r},\mathrm{t}) = \int d^3\mathbf{p} \sum_S \Big(\Theta_{-,S,\mathbf{p}}(\mathbf{r})\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}) + \Theta_{+,S,\mathbf{p}}(\mathbf{r})\ \mathrm{b}^\dagger_{+,S,-\mathbf{p}}(\mathrm{t})\Big), \tag{4}$$

where $\mathrm{a}_{j,S,\mathbf{p}}$ ($\mathrm{b}_{j,S,\mathbf{p}}$) and $\mathrm{a}^\dagger_{j,S,\mathbf{p}}$ ($\mathrm{b}^\dagger_{j,S,\mathbf{p}}$) are the operators of annihilation and creation of fermions of positive (negative) mass and energy in the space of the number of occupations of the one-particle state characterized by quantum numbers j, S and $\mathbf{p}$. They obey the usual anti-commutation relations. The index $j = (+,-)$ characterizes positive and negative frequencies. $S = (R, L)$ is the state of helicity along momentum vector $\mathbf{p}$. The vector $\mathbf{p}$ is the classical canonical variable of momentum conjugated to the vector of position $\mathbf{r}$. $\Phi_{j,S,\mathbf{p}}(\mathbf{r})$ and $\Theta_{j,S,\mathbf{p}}(\mathbf{r})$ are the stationary states of

$$H_{\mathrm{o}}^{(+)}\,\Phi_{j,S,\mathbf{p}}(\mathbf{r}) = j\varepsilon(\mathbf{p})\;\Phi_{j,S,\mathbf{p}}(\mathbf{r}), \tag{5}$$

and

$$H_{\mathrm{o}}^{(-)}\,\Theta_{j,S,\mathbf{p}}(\mathbf{r}) = j\varepsilon(\mathbf{p})\;\Theta_{j,S,\mathbf{p}}(\mathbf{r}). \tag{6}$$

The one-particle hamiltonian operator $H_{\mathrm{o}}^{(\pm)}$ of Dirac for a free particle of spin 1/2 and mass $\pm\mu$ is given by

$$H_{\mathrm{o}}^{(\pm)} = \mathrm{c}\alpha\cdot\mathbf{P} \pm \beta\mu\mathrm{c}^{\mathbf{2}}. \tag{7}$$

β and α_u $(u = x, y, z)$ are the Dirac matrices. c is the speed of the light in vacuum. $\varepsilon(\mathbf{p})$ is the eigenvalue of energy

$$\varepsilon(\mathbf{p}) = (\mathrm{c}^2\mathbf{p}^2 + \mu^2\mathrm{c}^4)^{1/2}. \tag{8}$$

States $\Phi_{j,S,\mathbf{p}}(\mathbf{r})$ and $\Theta_{j,S,\mathbf{p}}(\mathbf{r})$ can be expressed as follows:

$$\Phi_{j,S,\mathbf{p}}(\mathbf{r}) = \phi_{j,S,\mathbf{p}}\frac{\mathrm{e}^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}}, \quad \Theta_{j,S,\mathbf{p}}(\mathbf{r}) = \theta_{j,S,\mathbf{p}}\frac{\mathrm{e}^{i(\mathbf{p}\cdot\mathbf{r}/\hbar)}}{\sqrt{(2\pi\hbar)^3}}, \tag{9}$$

where $\phi_{j,S,\mathbf{p}}$ and $\theta_{j,S,\mathbf{p}}$ are the Dirac's spinors and they are related by[15]

$$\begin{pmatrix}\theta_{+,S,\mathbf{p}}\\ \theta_{-,S,\mathbf{p}}\end{pmatrix} = \begin{pmatrix}\eta\sin(\varphi) & \cos(\varphi)\\ \cos(\varphi) & -\eta\sin(\varphi)\end{pmatrix}\begin{pmatrix}\phi_{+,S,\mathbf{p}}\\ \phi_{-,S,\mathbf{p}}\end{pmatrix}, \tag{10}$$

and the operators by

$$\begin{pmatrix}\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}\\ \mathrm{b}_{-,S,\mathbf{p}}\end{pmatrix} = \begin{pmatrix}\cos(\varphi) & \eta\sin(\varphi)\\ -\eta\sin(\varphi) & \cos(\varphi)\end{pmatrix}\begin{pmatrix}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}\\ \mathrm{a}_{+,S,\mathbf{p}}\end{pmatrix}, \tag{11}$$

where $\eta = +1$ for $S = R$ and -1 for $S = L$. The angle φ is defined by relations

$$\cos(\varphi) = \frac{\mu\mathrm{c}^2}{\varepsilon(\mathbf{p})}, \quad \sin(\varphi) = \frac{\mathrm{c}|\mathbf{p}|}{\varepsilon(\mathbf{p})}. \tag{12}$$

Transformations (10) and (11) are the results of a reversal of mass and energy and according to Ref. 9 are equivalent to a *unitary* time reversal operation. Transformation (11) is different from that given in Ref. 16 because the authors uses an *antiunitary* time reversal operation.

In (3) matter and antimatter have masses and energies that are positive. In (4) matter and antimatter have exactly opposite masses and energies, i.e. negative. In addition, the electric charges in (3) are exactly opposite to those in (4). So, positive mass matter in (3) becomes negative mass antimatter in (4) and positive mass antimatter in (3) becomes negative mass matter in (4). Note that the anti-commutations relations between operators "a" and "b" are not zero and depend on the energy and the momentum. A similar effect occurs in the case of a curved space-time.[16,17] So, the zero particle state for one observer is not zero for another. In our case, the reason for this is that the vacua $|0\rangle_{\mathrm{a}}$ and $|0\rangle_{\mathrm{b}}$ are not invariant under an operation of *unitary* time reversal $\mathrm{T_u}$ (that is to say, energy inversion)

$$|0\rangle_{\mathrm{a}} \xleftrightarrow{\mathrm{T_u}} |0\rangle_{\mathrm{b}}. \tag{13}$$

However, note that each vacuum is CPT invariant (for an *antiunitary* time reversal operator T). Finally, we can show with (10) and (11) and the anti-commutation rules of operators that

$$\Psi_{+}(\mathbf{r},\mathrm{t}) = \Psi_{-}(\mathbf{r},\mathrm{t}), \tag{14}$$

it is the same Dirac field but with two different representations or two basis states of fermions–antifermions. Actually, the Dirac's spinors $\phi_{j,S,\mathbf{p}}$ and $\theta_{j,S,\mathbf{p}}$ characterize enantiomorphous twin states with opposite charges and masses.

3. Axial Chirality

The concept of axial chirality in the framework of quantum mechanics gives a geometric meaning to the strange notions of negative energy (or mass) and unitary time reversal. It allows to show the enantiomorphous (chiral) character of the two kinds of fermions–antifermions states: those (i.e. $\phi_{j,S,\mathbf{p}}$) associated with Dirac fields of positive masses and energies and those (i.e. $\theta_{j,S,\mathbf{p}}$) with Dirac fields of negative masses and energies.

Consider the one-particle Schrödinger equation with negative and positive mass

$$i\hbar\frac{\partial}{\partial t}\Theta_{j,S,\mathbf{p}}(\mathbf{r},t) = H_{\mathrm{o}}^{(-)}\,\Theta_{j,S,\mathbf{p}}(\mathbf{r},t), \quad i\hbar\frac{\partial}{\partial t}\Phi_{j,S,\mathbf{p}}(\mathbf{r},t) = H_{\mathrm{o}}^{(+)}\,\Phi_{j,S,\mathbf{p}}(\mathbf{r},t), \tag{15}$$

whose time-independent solutions are

$$\Theta_{j,S,\mathbf{p}}(\mathbf{r},t) = \Theta_{j,S,\mathbf{p}}(\mathbf{r})\mathrm{e}^{-\mathrm{i}[\frac{j\varepsilon(\mathbf{p})\mathrm{t}}{\hbar}]}, \quad \Phi_{j,S,\mathbf{p}}(\mathbf{r},t) = \Phi_{j,S,\mathbf{p}}(\mathbf{r})\mathrm{e}^{-\mathrm{i}[\frac{j\varepsilon(\mathbf{p})\mathrm{t}}{\hbar}]}. \tag{16}$$

Let us look at the time-related part of these solutions. Figure 1 shows a schematic illustration of this time-related part with a series of straight (black) segments that rotate around a vertical axis representing the time axis t. To increase readability, these line segments are drawn within circles centered on this axis. Different times t are shown as successive events on the left side of the figure. The angle between two adjacent line segments in a series along a time axis has the absolute value $\varepsilon(\mathbf{p})/\hbar$. The vertical line M in the middle of the figure separates two sets of temporal segments and acts as a mirror for one another. Hence, the sequence on the right is the mirror image of the sequence on the left side, it is its chiral representation. The direction of rotation of the segment on the right-hand side is opposite to that of the left-hand side. On the left sequence, the phase is $\varepsilon(\mathbf{p})\mathrm{t}/\hbar$ while on the right sequence it is $-\varepsilon(\mathbf{p})\mathrm{t}/\hbar$. This is equivalent to changing the sign of $\varepsilon(\mathbf{p})$ or t.

Actually, such inversion of the sign of time or energy mathematically conveys a more fundamental geometric property, namely that the two sequences on either side of M are enantiomers with axial chirality. The one in the first column is dextrorotatory while the other is levorotatory. It is a helicity along the time axis (mirror or chirality axis M) described by the Dirac's spinor. For this reason, the two sets of Dirac's spinors (i.e. $\phi_{j,S,\mathbf{p}}$ and $\theta_{j,S,\mathbf{p}}$) are called enantiomorphs.

On the other hand, an antiunitary time reversal operation is characterized by[15]

$$t \to -t \quad \text{and} \quad \mathrm{i} \to -\mathrm{i}, \tag{17}$$

and a unitary one by

$$t \to -t \quad \text{and} \quad \mathrm{i} \to \mathrm{i}. \tag{18}$$

It can be seen in Fig. 1 that along the first series (i.e. dextrorotatory) a shift towards $-$t goes with an electrical charge +e, whereas towards +t there is a charge $-$e. In the second series (i.e. levorotatory) the exact opposite holds. It is worth noting that the exponential term at the bottom left in Fig. 1 behaves as if, in order to obtain it, one had made an *antiunitary* time reversal in the exponential term at the top left.

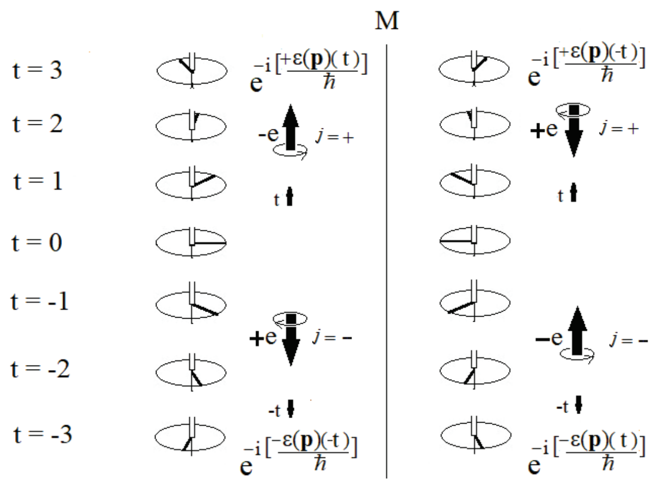

Fig. 1. On the left side of the vertical mirror M, the first series of segments (black lines within circles) rotate around a vertical axis in a complex plane: it illustrates the exponential term $\mathrm{e}^{-\mathrm{i}\left[\frac{j\varepsilon(\mathbf{p})\mathrm{t}}{\hbar}\right]}$. This sequence is related to the matter ($j = +$) and antimatter ($j = -$) spinor contained in $\phi_{\pm,S,\mathbf{p}}$. The two matter–antimatter spinor states are represented by thick vertical up (i.e. $j = +$) and down (i.e. $j = -$) arrows, whose axis is circled using conventional (counterclockwise) direction. The associated electrical charge is $-j$e. On the right side, the second series is related to the matter ($j = -$) and antimatter ($j = +$) spinor contained in $\theta_{\mp,S,\mathbf{p}}$ and the associated charge is je. On the left, a time scale t has been represented which is valid for both series. The variable t (on the right of exponential terms) is inverted with respect to that on the left. The small vertical black arrows are the arrows of time, they point toward the flow direction of events which accompanies the spinor. For example, if one moves along the left-hand (right-hand) sequence according to the $-$t direction, the spinor is pointing downwards (upwards). It describes the sense of rotation of the black segments in the series. On the contrary, if one moves in the +t direction then the spinor is pointing upwards (downwards). In both cases the phase is $+\varepsilon(\mathbf{p})\mathrm{t}/\hbar(-\varepsilon(\mathbf{p})\mathrm{t}/\hbar)$ or dextrorotatory (levorotatory).

Note also that the exponential terms at the top and bottom right of Fig. 1 behave as if, to obtain them, one had made a *unitary* time reversal in the exponential term at the top left.

4. Coupling to Electromagnetic Potential

Taking into account the Heisenberg representation, Dirac's Hamiltonian for free particles of positive mass and energy $\mathscr{H}_o^{(+)}(\mathrm{t})$ is simply given by[18]

$$\begin{aligned}\mathscr{H}_{\mathrm{o}}^{(+)}(\mathrm{t}) &\equiv \int d^3\mathbf{r}\,\Psi_+^\dagger(\mathbf{r},\mathrm{t})H_{\mathrm{o}}^{(+)}\Psi_+(\mathbf{r},\mathrm{t}) + \mathrm{C}\\ &= \int d^3\mathbf{p}\sum_{j,S}\varepsilon(\mathbf{p})\mathrm{a}_{j,S,\mathbf{p}}^\dagger(\mathrm{t})\mathrm{a}_{j,S,\mathbf{p}}(\mathrm{t}),\end{aligned} \tag{19}$$

where

$$\mathrm{C} \equiv 2\delta(0)\int d^3\mathbf{p}\,\varepsilon(\mathbf{p}). \tag{20}$$

and the coupling to the classical electromagnetic potentials, Φ and $\mathbf{A}$ imposed from the outside by a macroscopic source is[18]

$$\mathscr{H}_1^{(+)}(\mathrm{t}) \equiv \int d^3\mathbf{r}\Big(\Phi(\mathbf{r},\mathrm{t})\rho^{(+)}(\mathbf{r},\mathrm{t}) - \mathbf{A}(\mathbf{r},\mathrm{t})\cdot\mathbf{J}^{(+)}(\mathbf{r},\mathrm{t})\Big). \tag{21}$$

Charge and current density operators of Dirac's field Ψ_+ at $\mathbf{r}$ and t are given, respectively, by

$$\rho^{(+)}(\mathbf{r},\mathrm{t}) \equiv -\frac{\mathrm{e}}{2}\Big(\Psi_+^\dagger(\mathbf{r},\mathrm{t})\Psi_+(\mathbf{r},\mathrm{t}) - \tilde{\Psi}_+(\mathbf{r},\mathrm{t})\tilde{\Psi}_+^\dagger(\mathbf{r},\mathrm{t})\Big), \tag{22}$$

$$\mathbf{J}^{(+)}(\mathbf{r},\mathrm{t}) \equiv -\frac{\mathrm{e}}{2}\Big(\Psi_+^\dagger(\mathbf{r},\mathrm{t})\ \mathrm{c}\alpha\ \Psi_+(\mathbf{r},\mathrm{t}) - \tilde{\Psi}_+(\mathbf{r},\mathrm{t})\ \mathrm{c}\tilde{\alpha}\ \tilde{\Psi}_+^\dagger(\mathbf{r},\mathrm{t})\Big), \tag{23}$$

where α are Dirac matrices α_ν ($\nu = x, y, z$)

$$\alpha_\nu = \begin{pmatrix} 0 & \sigma_\nu \\ \sigma_\nu & 0 \end{pmatrix}, \tag{24}$$

and σ_ν are the Pauli spin matrices. Symbol $\sim$ indicates matrix transposition. $\mathrm{e} > 0$ is the electric charge. The Dirac hamiltonian $\mathscr{H}(\mathrm{t})$ of the fermions–antifermions' systems submitted to classical electromagnetic potentials is

$$\mathscr{H}(\mathrm{t}) = \mathscr{H}_o^{(+)}(\mathrm{t}) + \mathscr{H}_1^{(+)}(\mathrm{t}). \tag{25}$$

5. Dynamic Equations

We place ourselves in the situation where the electromagnetic potential is uniform over the entire wave packet of the particle. It is assumed that the wave packet is of short extension in space with respect to the instrumentation with essentially only one dominant linear momentum vector $\mathbf{p}$. We neglect the edge effects of the wave packet (i.e. the presence of other secondary wave vectors). In this case, the potential

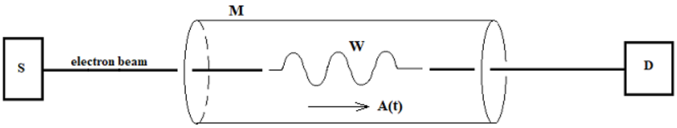

Fig. 2. Schematic experiment with time-dependent vector potential A(t) along the electron beam and no scalar potential, i.e. $\Phi(\mathrm{t}) = 0$. W: wave packet, M: cylindrical superconductive tube, S: source of electrons, D: detector.

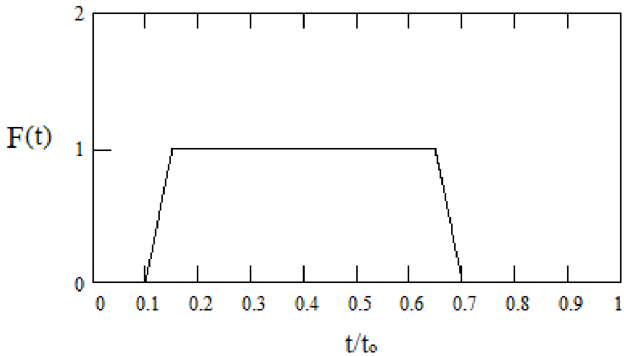

Fig. 3. Schematic time-dependence of the vector potential: $\mathrm{A(t)} = A\mathrm{F(t)}$, where A is a constant. $\mathrm{t_o} \equiv \hbar/\mu \mathrm{c}^2$.

becomes independent of the position: $\Phi(\mathbf{r}, \mathrm{t}) \to \Phi(\mathrm{t})$ and $\mathbf{A}(\mathbf{r}, \mathrm{t}) \to \mathbf{A}(\mathrm{t})$. Finally, the vector potential $\mathbf{A}(\mathrm{t})$ is oriented parallel to the linear momentum vector $\mathbf{p}$ of the particle then $\mathbf{A}(\mathrm{t}) = A(\mathrm{t})\mathbf{n}$, where $\mathbf{n} = \frac{\mathbf{p}}{|\mathbf{p}|}$ and $A(\mathrm{t})$ is a scalar. It is assumed that the particle is not affected by any electromagnetic field.

Figure 2 gives the schematic experiment. The potential is nonzero only while the electron is well inside the tube M. When the particle is everywhere outside M there is no potential. The arrangement must ensure that particles are in a time-varying potential without ever being in a field. Note that a real action of electromagnetic potential on charged particle in regions of space where electromagnetic field is zero has already been demonstrated.[19,20] Figure 3 shows the schematic time-dependence of the vector potential. Under these conditions we will consider two scenarios.

5.1. *Scenario 1: Negative mass and energy do not exist*

In this scenario, the twin state $\theta_{j,S,\mathbf{p}}$ cannot be realized, only state $\phi_{j,S,\mathbf{p}}$ can exist. Therefore, the evolution of the operators "a" of positive mass and energy over time in the Heisenberg representation is given by

$$i\hbar \frac{\partial}{\partial t} \mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) = \left[\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}), \mathscr{H}(\mathrm{t})\right] = \omega_{\mathrm{o}}^{-}(\mathrm{t})\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) + \omega_1(\mathrm{t})\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}), \tag{26}$$

$$i\hbar \frac{\partial}{\partial t} \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) = \left[\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}), \mathscr{H}(\mathrm{t})\right] = -\omega_{\mathrm{o}}^{+}(\mathrm{t})\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) + \omega_1(\mathrm{t})\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}), \tag{27}$$

where

$$\omega_{\rm o}^{\pm}(\mathrm{t}) \equiv \varepsilon(\mathbf{p}) \pm \mathrm{e}\Phi(\mathrm{t}) + \mathrm{ce}A(\mathrm{t})\sin(\varphi), \tag{28}$$

$$\omega_1(\mathrm{t}) \equiv \mathrm{ce}\eta A(\mathrm{t})\cos(\varphi). \tag{29}$$

Here, the operators "b" are simply particular linear combinations of the operators "a". By finding the "a" we find the "b" via Eq. (11). General solution is given by

$$\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) = \mathrm{f}^*(t)\mathrm{a}_{+,S,\mathbf{p}} + \mathrm{g}^*(t)\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}, \tag{30}$$

$$\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) = \mathrm{f}(t)\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}} - \mathrm{g}(t)\mathrm{a}_{+,S,\mathbf{p}}, \tag{31}$$

with the initial conditions $\mathrm{f}(0) = \mathrm{f}^*(0) = 1$ and $\mathrm{g}(0) = \mathrm{g}^*(0) = 0$. By introducing (30) and (31) in (26) and (27), we find two sets of coupled differential equations

$$i\hbar\frac{\partial}{\partial t}\mathrm{f}^*(t) = \omega_{\rm o}^{-}(\mathrm{t})\mathrm{f}^*(t) - \omega_1(\mathrm{t})\mathrm{g}(t), \tag{32}$$

$$i\hbar\frac{\partial}{\partial t}\mathrm{g}^*(t) = \omega_{\rm o}^{-}(\mathrm{t})\mathrm{g}^*(t) + \omega_1(\mathrm{t})\mathrm{f}(t), \tag{33}$$

$$i\hbar\frac{\partial}{\partial t}\mathrm{f}(t) = -\omega_{\rm o}^{+}(\mathrm{t})\mathrm{f}(t) + \omega_1(\mathrm{t})\mathrm{g}^*(t), \tag{34}$$

$$i\hbar\frac{\partial}{\partial t}\mathrm{g}(t) = -\omega_{\rm o}^{+}(\mathrm{t})\mathrm{g}(t) - \omega_1(\mathrm{t})\mathrm{f}^*(t). \tag{35}$$

Note that if $\Phi = 0$ this system of four equations reduces to two because $\omega_{\rm o}^{+} = \omega_{\rm o}^{-}$ and in that case $\mathrm{f}^*(t)$ and $\mathrm{g}^*(t)$ are, respectively, the conjugate complexes of $\mathrm{f}(t)$ and $\mathrm{g}(\mathrm{t})$.

5.2. *Scenario 2: Negative mass and energy do exist*

The twin state $\theta_{j,S,\mathbf{p}}$ can also be realized in this scenario. The evolution of the operators "a" of positive mass and energy over time in (26) and (27) can be rewritten as follows:

$$i\hbar\frac{\partial}{\partial t}\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) = (\varepsilon(\mathbf{p}) - \mathrm{e}\Phi(\mathrm{t}))\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) + (\mathrm{ce}\eta A(\mathrm{t}))\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t}), \tag{36}$$

$$i\hbar\frac{\partial}{\partial t}\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) = -(\varepsilon(\mathbf{p}) + \mathrm{e}\Phi(\mathrm{t}))\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) + (\mathrm{ce}\eta A(\mathrm{t}))\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}). \tag{37}$$

Operators of fermion $\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})$ and antifermion $\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})$ of positive mass and energy are coupled, respectively, to the operators of antifermion $\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t})$ and fermion $\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t})$ of negative mass and energy. This is the consequence that the $\mathbf{n}\cdot\alpha$ operation on states with positive mass and energy transforms them into states of negative mass and energy with opposite charge but same helicity

$$(\mathbf{n}\cdot\alpha)\phi_{j,S,\mathbf{p}} = \theta_{j,S,\mathbf{p}}. \tag{38}$$

α is in the coupling term, see Eqs. (21) and (23). Note that if the vector potential $\mathbf{A}(\mathrm{t})$ is oriented perpendicular to the linear momentum $\mathbf{p}$ of the particle, the operators "a" are coupled to each other and not to the operators "b". This is because the $\mathbf{u}\cdot\alpha$ operation (i.e. $\mathbf{u}$ is a unit vector perpendicular to $\mathbf{n}$) on states with positive mass and energy transforms them into states of positive mass and energy with opposite charge and helicity (i.e. $S' \neq S$)

$$(\mathbf{u}\cdot\alpha)\phi_{j,S,\mathbf{p}} = \phi_{-j,S',\mathbf{p}}. \tag{39}$$

The evolution over time of the operators "b" of negative mass and energy is given by

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t}) &= \left[\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t}), \mathscr{H}(\mathrm{t})\right] \\ &= (\varepsilon(\mathbf{p}) - \mathrm{e}\Phi(\mathrm{t}))\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t}) + (\mathrm{c}\eta\,\mathrm{e}A(\mathrm{t}))\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) \\ &\quad - (2\mu\mathrm{c}^2)\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}), \end{aligned} \tag{40}$$

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}) &= \left[\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}), \mathscr{H}(\mathrm{t})\right] \\ &= -(\varepsilon(\mathbf{p}) + \mathrm{e}\Phi(\mathrm{t}))\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}) + (\mathrm{c}\eta\,\mathrm{e}A(\mathrm{t}))\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) \\ &\quad + (2\mu\mathrm{c}^2)\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}). \end{aligned} \tag{41}$$

These equations can also be obtained by linearly combining the two equations (36) and (37) in agreement with (11). The factor in front of the operators "b" in the right-hand side is the associated eigenenergy. Note that the eigenenergies (natural frequencies) of the operators $\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})$ and $\mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t})$ are identical (i.e. compare the first terms of Eqs. (36) and (40)) which can lead to a phenomenon of quantum resonance. Same remark for $\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})$ and $\mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t})$. The last term in the right-hand side of (40) and (41) comes from the fact that the operators "b" are not operators of eigenstates or stationary states of $\mathscr{H}_o^{(+)}(\mathrm{t})$. They are therefore subject to quantum fluctuations which are generated through that term.

The general solution can be expressed as follows:

$$\begin{pmatrix} \mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t}) \\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t}) \\ \mathrm{b}_{-,S,\mathbf{p}}(\mathrm{t}) \\ \mathrm{b}^{\dagger}_{+,S,-\mathbf{p}}(\mathrm{t}) \end{pmatrix} = \begin{pmatrix} f_{11}(\mathrm{t}) & f_{12}(\mathrm{t}) & f_{13}(\mathrm{t}) & f_{14}(\mathrm{t}) \\ f_{21}(\mathrm{t}) & f_{22}(\mathrm{t}) & f_{23}(\mathrm{t}) & f_{24}(\mathrm{t}) \\ f_{31}(\mathrm{t}) & f_{32}(\mathrm{t}) & f_{33}(\mathrm{t}) & f_{34}(\mathrm{t}) \\ f_{41}(\mathrm{t}) & f_{42}(\mathrm{t}) & f_{43}(\mathrm{t}) & f_{44}(\mathrm{t}) \end{pmatrix} \begin{pmatrix} \mathrm{a}_{+,S,\mathbf{p}} \\ \mathrm{a}^{\dagger}_{-,S,-\mathbf{p}} \\ \mathrm{b}_{-,S,\mathbf{p}} \\ \mathrm{b}^{\dagger}_{+,S,-\mathbf{p}} \end{pmatrix}, \tag{42}$$

where the $f_{lk}(\mathrm{t})$ are scalar functions which depend on time with initial conditions

$$\begin{aligned} f_{lk}(0) &= 1, \quad \text{if} \quad k = l, \\ f_{lk}(0) &= 0, \quad \text{if} \quad k \neq l. \end{aligned} \tag{43}$$

When we introduce (42) in Eqs. (36), (37), (40) and (41), we obtain four sets of four coupled differential equations

$$i\hbar\frac{\partial}{\partial t}f_{1k}(\mathrm{t}) = (\varepsilon(\mathbf{p}) - \mathrm{e}\Phi(\mathrm{t}))f_{1k}(\mathrm{t}) + (\mathrm{ce}\eta A(\mathrm{t}))f_{4k}(\mathrm{t}), \tag{44}$$

$$i\hbar\frac{\partial}{\partial t}f_{2k}(\mathrm{t}) = -(\varepsilon(\mathbf{p}) + \mathrm{e}\Phi(\mathrm{t}))f_{2k}(\mathrm{t}) + (\mathrm{ce}\eta A(\mathrm{t}))f_{3k}(\mathrm{t}), \tag{45}$$

$$i\hbar\frac{\partial}{\partial t}f_{3k}(\mathrm{t}) = -(\varepsilon(\mathbf{p}) + \mathrm{e}\Phi(\mathrm{t}))f_{3k}(\mathrm{t}) + (\mathrm{c}\eta\mathrm{e}A(\mathrm{t}))f_{2k}(\mathrm{t}) + (2\mu\mathrm{c}^2)f_{1k}(\mathrm{t}), \tag{46}$$

$$i\hbar\frac{\partial}{\partial t}f_{4k}(\mathrm{t}) = (\varepsilon(\mathbf{p}) - \mathrm{e}\Phi(\mathrm{t}))f_{4k}(\mathrm{t}) + (\mathrm{c}\eta\mathrm{e}A(\mathrm{t}))f_{1k}(\mathrm{t}) - (2\mu\mathrm{c}^2)f_{2k}(\mathrm{t}), \tag{47}$$

where $k = 1$–4 which corresponds to the number of each column of the matrix of f_{lk} in (42).

6. Transition Probabilities

Let us consider the case of a particle of mass $+\mu$ and electric charge $-\mathrm{e}$ in the vacuum $|0\rangle_{\mathrm{a}}$. It could be an electron.

6.1. *Scenario 1*

Using (30) and (31) and the fact that

$$\begin{aligned} {}_{\mathrm{a}}\langle 0|\mathrm{a}_{+,S,\mathbf{p}}(0)\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}} &= 1, \\ {}_{\mathrm{a}}\langle 0|\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(0)\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}} &= 0, \end{aligned} \tag{48}$$

the probability that having created this electron in the state $(S, \mathbf{p})$ at $\mathrm{t} = 0$ we can recover it in the same state at $\mathrm{t} >, 0$ is equal to

$$\mathrm{P}_f(\mathrm{t}) \equiv \mathscr{B}^{-1}(\mathrm{t})|{}_{\mathrm{a}}\langle 0|\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}}|^2 = \mathscr{B}^{-1}(\mathrm{t})|f^*(\mathrm{t})|^2, \tag{49}$$

and the probability we can recover a positron of mass $+\mu$ in state $(S, -\mathbf{p})$ at $\mathrm{t} > 0$ is

$$\mathrm{P}_g(\mathrm{t}) \equiv \mathscr{B}^{-1}(\mathrm{t})|{}_{\mathrm{a}}\langle 0|\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}}|^2 = \mathscr{B}^{-1}(\mathrm{t})|g(\mathrm{t})|^2, \tag{50}$$

where

$$\mathscr{B}^{-1}(\mathrm{t}) \equiv |f^*(\mathrm{t})|^2 + |g(\mathrm{t})|^2, \tag{51}$$

is the normalization factor.

6.2. *Scenario 2*

The measurable probability that having created this electron in the state $(S, \mathbf{p})$ at $\mathrm{t} = 0$ we can recover it in the same state at $\mathrm{t} > 0$ is equal to

$$\mathrm{P}_{m11}(\mathrm{t}) \equiv \mathscr{A}_m(\mathrm{t})^{-1}\left|{}_{\mathrm{a}}\langle 0|\mathrm{a}_{+,S,\mathbf{p}}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}}\right|^2 = \mathscr{A}_m(\mathrm{t})^{-1}|f_{11}(\mathrm{t})|^2, \tag{52}$$

and the measurable probability we can recover a positron of mass $+\mu$ in state $(S, -\mathbf{p})$ at $\mathrm{t} > 0$ is

$$\mathrm{P}_{m21}(\mathrm{t}) \equiv \mathscr{A}_m(\mathrm{t})^{-1}\left|{}_{\mathrm{a}}\langle 0|\mathrm{a}^{\dagger}_{-,S,-\mathbf{p}}(\mathrm{t})\mathrm{a}^{\dagger}_{+,S,\mathbf{p}}(0)|0\rangle_{\mathrm{a}}\right|^2 = \mathscr{A}_m(\mathrm{t})^{-1}|f_{21}(\mathrm{t})|^2, \tag{53}$$

where the normalization factor $\mathscr{A}_m(\mathrm{t})$ is

$$\mathscr{A}_m(\mathrm{t}) \equiv |f_{11}(\mathrm{t})|^2 + |f_{21}(\mathrm{t})|^2. \tag{54}$$

Note that according to JCM[3,7,8] usual measuring devices cannot detect quantum particles of negative energy (i.e. the "b" operators) unless these detectors have a negative temperature,[4] detectors in a metastable state with relatively high positive energy.

7. Results and Discussion

The next step is to compare $\mathrm{P}_f(\mathrm{t})$ to $\mathrm{P}_{m11}(\mathrm{t})$ and $\mathrm{P}_g(\mathrm{t})$ to $\mathrm{P}_{m21}(\mathrm{t})$ which are measurable with conventional detectors. When A increases or the time interval $\Delta\,\mathrm{t}$ (during which F(t) in Fig. 3 is different from zero) increases the probabilities can display several oscillations in $\Delta\,\mathrm{t}$. In the following examples, the value of A and the time interval $\Delta\,\mathrm{t}$ chosen limited the number of oscillations to zero.

Figures 4–7 show the evolution of the probabilities over time. The solid lines are those of the electron (of positive mass and energy) and the broken lines are those of the positron (of positive mass and energy). At $\mathrm{t} = 0$ the probability of detecting a positron is zero since the vector potential is applied only between $\mathrm{t} = 0.1t_{\mathrm{o}}$ and $\mathrm{t} = 0.7t_{\mathrm{o}}$ (i.e. Fig. 3).

In scenario 1 (i.e. Figs. 5–7) we see that the probabilities depend a lot on the modulus of the momentum $\mathbf{p}$ and the sign of A which is not the case in scenario 2 (i.e. Fig. 4). The temporal behavior of the probability of occurrence of the electron in detector D (Fig. 2) is therefore significantly different depending on whether quantum states with conjugated mass and energy (i.e. negative) exist (scenario 2) or not (scenario 1).

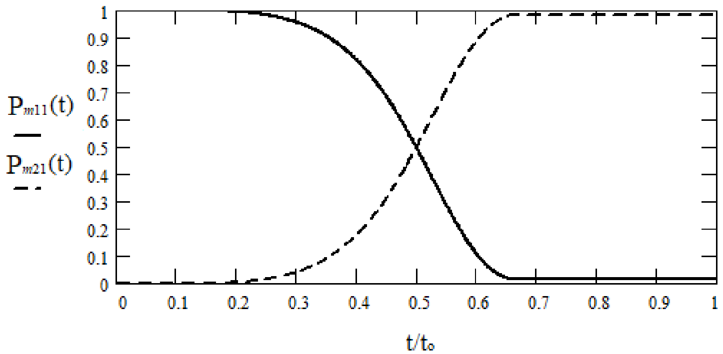

Fig. 4. Scenario 2. $\frac{\mathrm{c}eA}{\mu\mathrm{c}^2} = \pm 2.64$, $\frac{\mathrm{e}\Phi}{\mu\mathrm{c}^2} = 0$, $\frac{\mathrm{c}|\mathbf{p}|}{\mu\mathrm{c}^2} = 0.0048$ and 0.48.

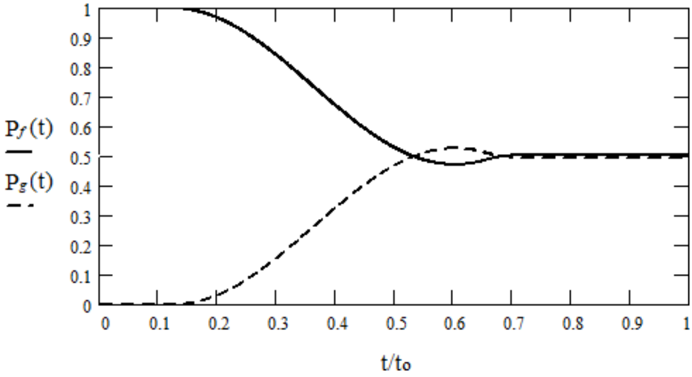

Fig. 5. Scenario 1. $\frac{ceA}{\mu c^2} = 2.64$, $\frac{e\Phi}{\mu c^2} = 0$, $\frac{c|\mathbf{p}|}{\mu c^2} = 0.48$.

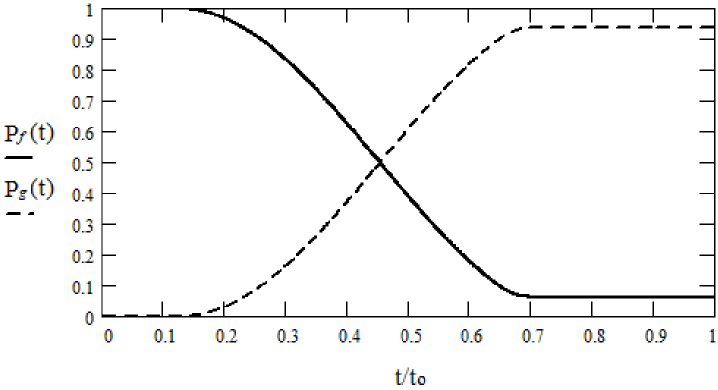

Fig. 6. Scenario 1. $\frac{ceA}{\mu c^2} = -2.64$, $\frac{e\Phi}{\mu c^2} = 0$, $\frac{c|\mathbf{p}|}{\mu c^2} = 0.48$.

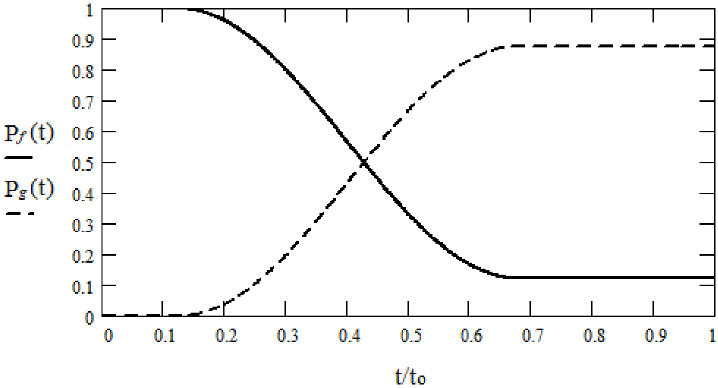

Fig. 7. Scenario 1. $\frac{ceA}{\mu c^2} = \pm 2.64$, $\frac{e\Phi}{\mu c^2} = 0$, $\frac{c|\mathbf{p}|}{\mu c^2} = 0.0048$.

8. Conclusion

In this work within the framework of the quantum field theory as a result of applying a time-dependent vector potential, oriented in the direction of the trajectory of the incident electron screened from electromagnetic fields by a superconducting tube, we have shown that it would be possible experimentally to highlight significant differences in the probabilities of occurrence of electron (of positive mass and energy) at the exit of the tube in the detector depending on whether quantum states of electron and positron of negative mass and energy exist or not.

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