# The missing mass problem

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**Abstract**

A new field equation is proposed, associated to a S3 x R1 topology. We introduce a differential involutive maping A which links any point of space **σ** to the antipodal region A(**σ**). According to this equation the geometry of the manifold depends both on the energy-momentum tensor **T** and on the antipodal tensor A(**T**). Considering time-independent metric with low fields and small velocities, we derive the associated Poisson equation, which provides cluster-like structures interacting with halo-like antipodal structures. The second structure helps the confinement of the first. It is suggested that this model could explain the missing mass effect and the large scale structure of the universe.

## 1- Introduction

The equilibrium of a galaxy is studied through a certain set of non-relativistic equations, as for example, Vlasov equation coupled to Poisson equation, which comes from the general Einstein field equation

(1)

$$\mathbf{S} = \chi \, \mathbf{T}$$

plus a steady-state hypothesis in which we take weak fields and small velocities. It is well known that the gravitational field due to the visible mass of our galaxy cannot balance the centrifugal and the pressure forces. Some people assume that some invisible mass, dark matter, may contribute to the field and balance the centrifugal force. In the following we are going to propose another model, based on a new field equation.

## 2- A new field equation

We assume that the universe has the topology of S3 x R1 .
The Gaussian coordinates are

(2)

$$\mathbf{x} = (x^\circ , \boldsymbol{\sigma})$$

where x° is a time-marker and the vector **σ** represents the spatial markers. Space-time is oriented. It is possible to define a differential involutive maping linking a given point **σ** to the antipodal point **σ***

(3)

$$\boldsymbol{\sigma}^* = A ( \boldsymbol{\sigma})$$

Consider two tensor fields **S** and **T**, defined on the manifold. Suppose that they are linked in the following field equation

(4)

$$\mathbf{S} = \chi ( \mathbf{T} - A(\mathbf{T}))$$

with

(5)

$$A(\mathbf{T}) = \mathbf{T}^* = \mathbf{T}(x^\circ, \boldsymbol{\sigma}^*)$$

We assume that the light follows the geodesics of space-time. **g** is the metric tensor. **R** is the Ricci tensor, so that

(6)

$$\mathbf{g}^* = \mathbf{g} (x^\circ, \boldsymbol{\sigma}^*)$$

$$\mathbf{R}^* = \mathbf{R}(x^\circ, \boldsymbol{\sigma}^*)$$

We can write the field equation in the more explicit form

(7)

$$\mathbf{R} - \frac{1}{2}\mathbf{g} R = \chi ( \mathbf{T} - \frac{1}{2}\mathbf{g} T - \mathbf{T}^* + \frac{1}{2}\mathbf{g}^* T^* )$$

Let us write the tensors **T** and **T*** as

(8)

$$\mathbf{T} = \begin{vmatrix} \rho & 0 & 0 & 0 \\ 0 & -\frac{p}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p}{c^2} \end{vmatrix}$$

(9)

$$\mathbf{T}^* = \begin{vmatrix} \rho^* & 0 & 0 & 0 \\ 0 & -\frac{p^*}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p^*}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p^*}{c^2} \end{vmatrix}$$

with

$$\rho^* = \rho\ (x^\circ, \boldsymbol{\sigma}^*)$$

$$p^* = p\ (x^\circ, \boldsymbol{\sigma}^*)$$

If we take the zero-divergence condition, the fluid obeys the following conservation equations

(10)

$$\partial\, \mathbf{T} = 0$$

## <u>3- Time independent conditions with weak fields and small velocities. The Poisson equation</u>

We can apply the classical method, taking a quasi-Lorentzian metric
(11)

$$\mathbf{g} = \boldsymbol{\eta} + \varepsilon\, \boldsymbol{\gamma}$$

where $\boldsymbol{\eta}$ is the Lorentzian metric and $\varepsilon$ is a small parameter.

In three-dimensional notations
(12)

$$\frac{d^2\mathbf{x}}{dt^2} = -\frac{c^2}{2}\gamma_{oo}|_i = -\frac{c^2}{2}\nabla\gamma_{oo}$$

The newtonian law applies over all space. In addition the gravitational potential is defined as the following:

(13)

$$\Psi = - \frac{c^2}{2} \varepsilon \gamma_{oo}$$

Conversely, given the gravitational potential ψ , the motion of a particle will be along a four-dimensional geodesic if the goo terms of the metric tensors has the form
(14)

$$g_{oo} = 1 + \frac{2\Psi}{c^2}$$

we get
(15)

$$\varepsilon \sum_{\beta = 1}^{3} \gamma_{oo|\beta|\beta} = - \chi ( \rho - \rho^* )$$

By identification we get the following Poisson equation

(16)

$$\Delta\psi = 4 \pi G ( \rho - \rho^*)$$

If we consider a spherically symmetric system

(17)

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4 \pi G ( \rho - \rho^* )$$

where

(18)

$$\rho^* = \rho(\boldsymbol{\sigma}^*)$$

From (17)

(19)

$$\psi^* = - \psi$$

## 4- Spherically symmetric solution

In 1916 Eddington derived a spherically symmetric steady-state solution, combining the Vlasov and the Poisson equations. He assumed that the ellipsoid of the velocities was spherically symmetric and pointed towards the center of the system.

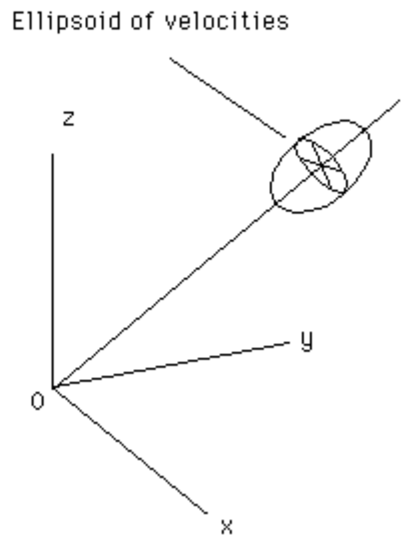


**Figure 1 (ga3114): Ellipsoid of velocities corresponding to an Eddington-type solution.**

Eddington derived the following relation between the mass density and the gravitational potential

(20)

$$\rho = \rho_o \frac{e^{-\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}}$$

which represents a steady-state distribution of matter in a collision-free gas, in a gravitational potential Y, in which the gravitational force balances the pressure force. Let us take the same kind of a solution for the antipodal region

(21)

$$\rho^* = \rho_o \frac{e^{-\frac{m\Psi^*}{kT}}}{1 + \frac{r^2}{r_o^2}} = \rho_o \frac{e^{\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}}$$

So that we have to solve the following equation

(22)

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4\pi G \rho_o \left( \frac{e^{-\frac{m\Psi}{kT}} - e^{\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}} \right)$$

Take

(23)

$$r_o = \lambda \sqrt{\frac{kT}{4 \pi G \rho_o m}}$$

Introduce the following adimensional quantities:

(24)

$$r = \sqrt{\frac{kT_0}{4 \pi G \rho_o m}} \; \xi \qquad \Psi = \frac{kT}{m} \varphi$$

We get

(24)

$$\varphi'' + \frac{2}{\xi} \varphi' = \frac{e^{-\varphi} - e^{\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

which can be solved by numerical computation. We can take the following initial conditions

$\varphi'o = 0$

$\varphi''o = 10$

$\lambda = 10$

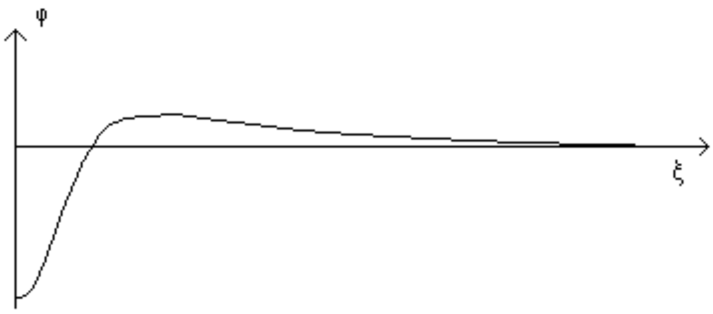


**Figure 2: Spherically symmetric Eddington-type solution. The gravitational potential**

$$\rho = \rho_0 \frac{e^{-\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

$$\rho^* = \rho_0 \frac{e^{\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

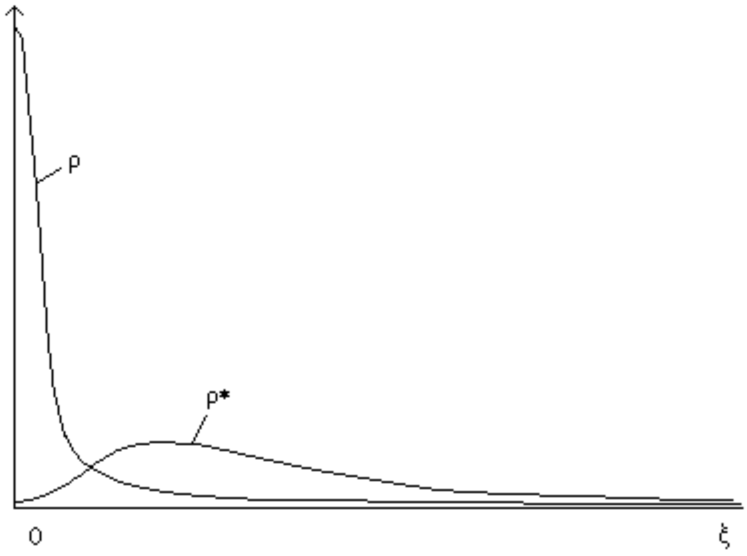


**Figure 3: Spherically symmetric Eddington-type solution. Mass densities. If a cluster exists in one fold, an associated diffuse halo exists in the conjugated region of the second fold.**

## 5- The large size structure of the universe

From the equation (24) we see that if a cluster exists in one fold, an associated diffuse halo structure exists in the conjugated region of the second fold. If this model is correct we should find halo-structures in our fold of the universe. With the help of Dr. Pierre Midy, from the university of Orsay, France, we have performed numerical simulations, using a Cray-1 computer. We consider two distributions of 350 points. The first is represented by little circles and the second by small crosses. At the begining the points are randomly distributed on the screen and are supposed to represent two uniform gazes. Each mass owns a random velocity corresponding to an isotropic Maxwellian distribution with an averaged thermal velocity <V> . Call m1 the elements of the first population and m2 the elements of the second population. We apply the Newton law with

- m1 attracts m1: gravitational effect
- m2 attracts m2: gravitational effect
- m1 and m2 repel each other: antigravitational effect

We consider this two-dimensional system as periodic over space. In other terms the upper boundary is linked to the lower one and the right to the left (Euclidean 2D torus). So that we can compute the sum of the mutual actions of the particles. For each interval of time?t we compute the acceleration of each particle and determine the trajectory by Taylor expansion. Each particle that comes out through the right boundary reappears through the left one, and same thing for the upper and lower boundaries. This makes possible to study the gravitational instability of these two coupled systems in a finite portion of space (with toroidal topology). The interval of time is determined in order to get significant computational results. In other terms we demand the trajectory of a particle to be approximatively regular. The following figures show the typical behaviour of the system after 4000 intervals of time. In figures 4 and 5 we find both clusters and cellular patterns. This is enhanced in the figures 7 and 8.

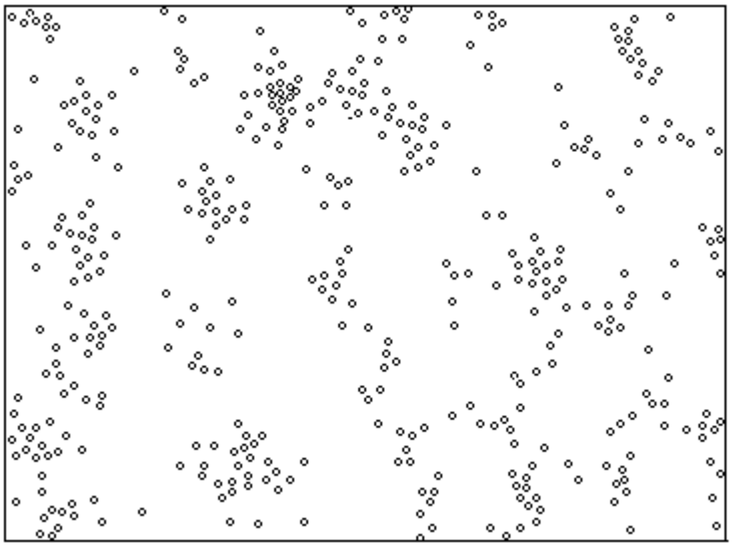

**Fig. 4: Effect of the gravitational instability on the system 1**

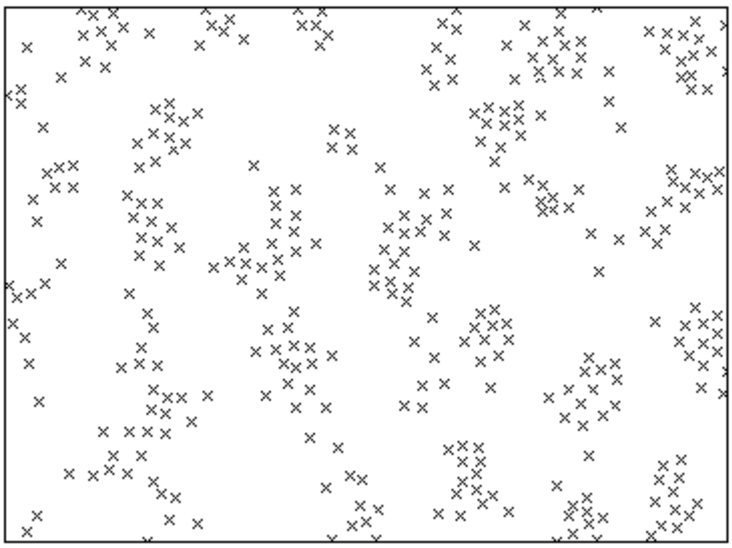

**Fig. 5: Effect of the gravitational instability on the system 2**

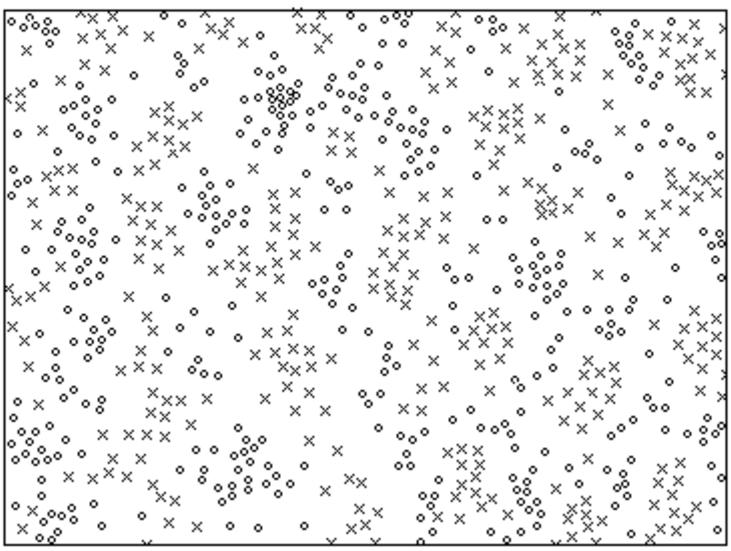

**Fig. 6: Superposition of the two.**

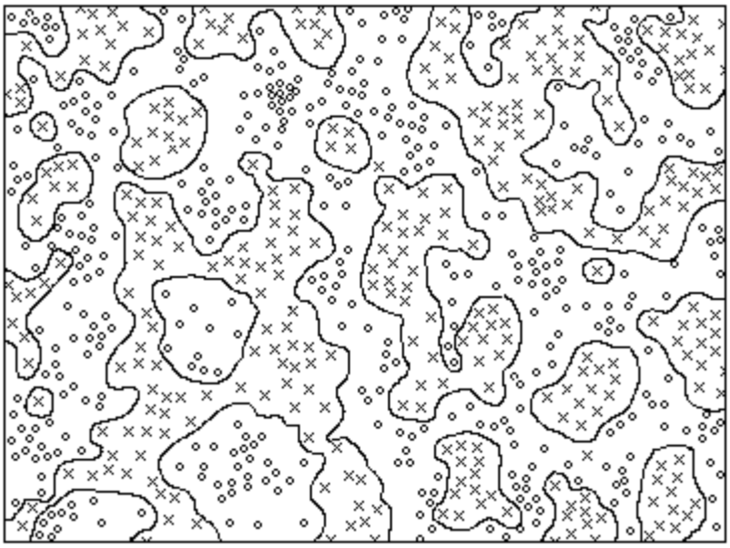

**Fig. 7: Enhanced spatial distribution of the two populations.**

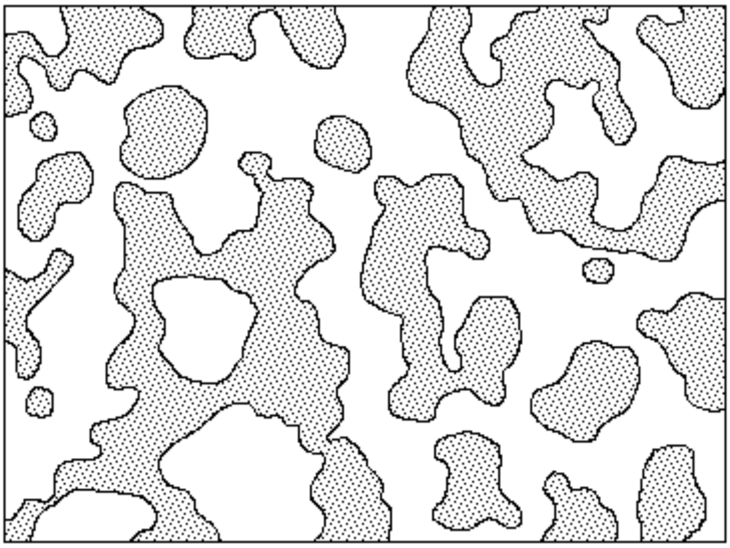

**Fig. 8: White: population 1. Grey: population 2**

We suggest that such a mechanism could explain the large scale structure of the universe and the observed distribution of galaxies. Suppose that our fold of the universe corresponds to the population 1. In the right lower part of the screen this matter is arranged around large "empty" bubbles. These bubbles correspond to a cluster arrangement in the population 2, supposed to be located in the second fold of the universe (in fact the antipodal region), according to our theory. But, as seen on the figure 8, for a given population, in some places the matter can be arranged as a Swiss "gruyère" (emmental) cheese and in other places as an emulsion.

These first crude numerical simulations have to be developped with a larger number of points and in a three-dimensional representation. We know that the three-dimensional's behaviour of a system can be somewhat different from the two-dimensional's one. But we expect the conclusions to be similar. We think that with a larger number of points we could get a fractal system, as suggested in the figure 14, but we precise that this peculiar computation has not yet been done, but are under study. According to this idea the galaxies should be located in the holes of the associated anti-matter cloud, which would ensure their confinement, as suggested earlier.

## 6- The interpretation of the solution

From the figure 2 we see that the potential ψ tends to a constant at the infinite. In the classical Eddington solution the potential owns a logarithmic growth. The figure 3 shows the association of a cluster of matter, located in the region **σ** , sourrounded by a smooth hollow located in the region **σ***.

In both regions matter attracts matter. But the negative sign, from the field equation and the Poisson equation, makes the matter and the "antipodal matter" to repel each other. This helps the confinement of the cluster. For a given thermal velocity the necessary quantity of matter to balance the pressure force is smaller. The smooth halo acts like a corset.

A field equation provides a macroscopic description of the universe. It does not take account of the corpuscular nature of matter. The model implies that particles and antipodal-particles live in very distant, antipodal portions of space. In fact their natures are identical. The physical meaning of the field equation is the following: the particules and antipodal-particles interact by gravitational effect, but not by electromagnetic effect. We assume that the antipodal particles, clusters, rings, are not observable with a telescope, or a radiotelescope. The observation of antipodal structures should require some sort of gravitational telescope.

From equation (22) clusters can be located in the antipodal region. Then, associated large halos, sourrounding wide rarefied regions, should exist in the observable universe too. In fact they do, for it corresponds, in our mind, to the observed large scale structure of the universe: the galaxies seem to be arranged around large rarefied bubbles. According to our model, large clouds of antipodal matter should exist in the corresponding associated antipodal regions.

The universe was assumed to have a S3 x R1 topology. The reader has probabily some difficulties to understand this strange three-dimensional geometry. In fact the sphere S3 is simply shaped as the double cover of a projective space P3. In such arrangement each point **σ** of the sphere is associated to its antipode A(**σ**). The situation is similar for a sphere S2 covering a projective space P2, which can be represented in our space R3 as the well known Boy surface.

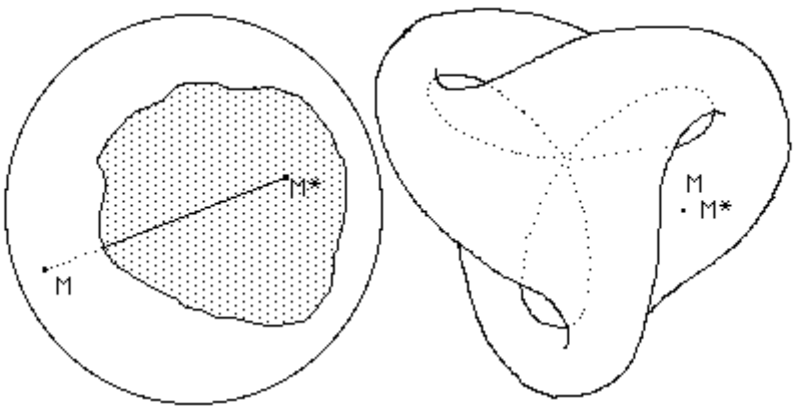


**Figure 9: A couple of antipodal points on a sphere S2 and the Boy surface, image of the projective space P2**

On the figure 10 we have figured the equator of a sphere and its location on the Boy Surface.

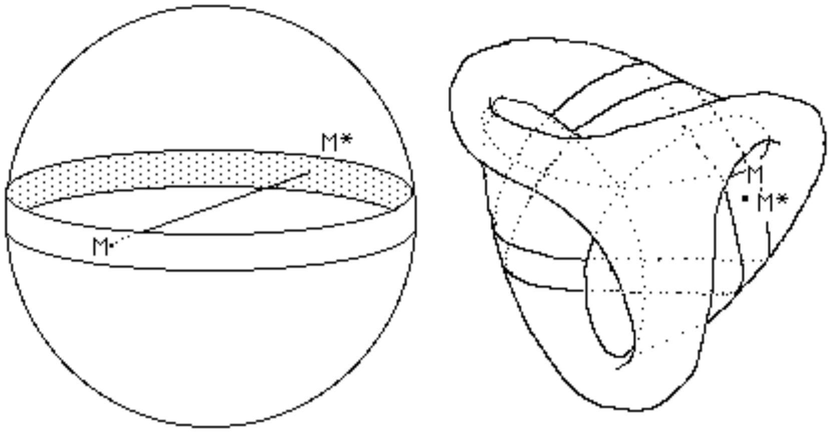


**Figure 10: The vicinity of the equator of a 2-sphere and its location on a Boy surface**.

The figure 11 shows how the equator of a S2 sphere can be glued on itself along a three half-turns Möbius belt. Locally the surface can be assimilated to a bundled manifold whose bundle owns two values + 1 and -1.

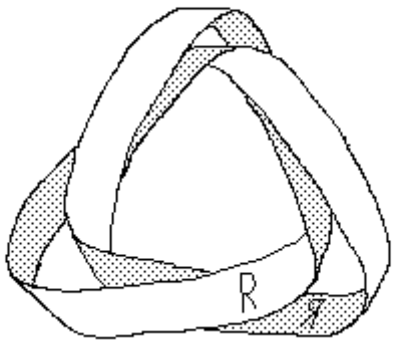


**Figure 11: Enantiomorphic image corresponding to the cover of a Möbius belt.**

In a 3-sphere S3, if one follows a geodesic, the antipodal point is at the half-way. If the 3-sphere is immersed in a four-dimensional space it is possible to make any point and its antipode to coincidate. These couples of points are associated through the antipodal diffential involutive maping A, but not identified.

As shown on the figure 12 we can proceed continously from a "gruyère" structure to a cluster structure. This peculiar feature was illustrated before, through 2d numerical simulations. When a region of space is put "in front" of the antipodal region, as suggested in the figure 12, the clusters nest in the holes.

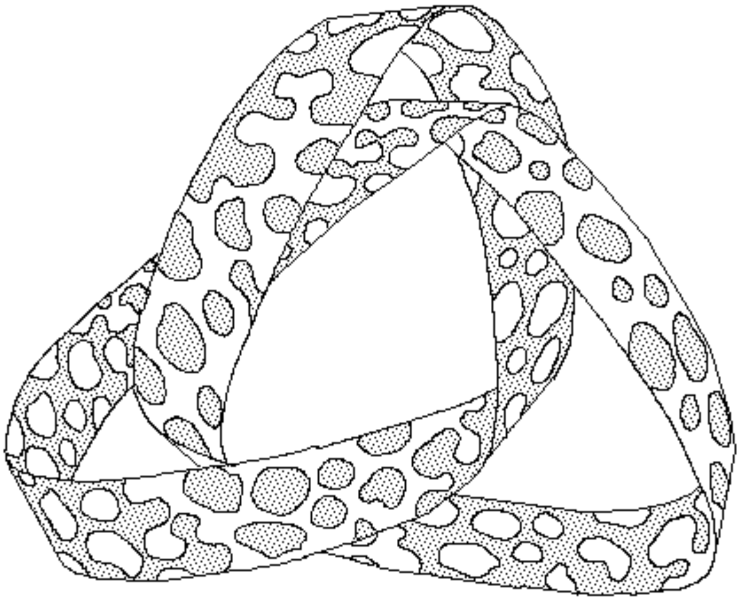

**Figure 12: Two-dimensional image of the global large structure of the universe**

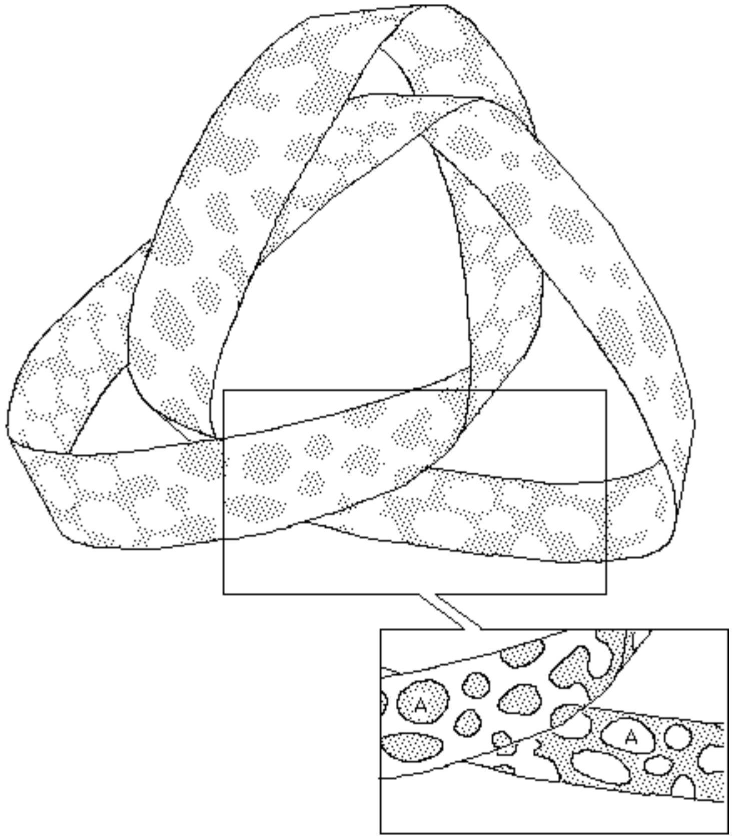


**Figure 13: The interaction between two antipodal regions**

This effect could act at the level of the galactic structure, as suggested in the figure 14, each galaxy nesting in a "hole" of the conjugated antipodal region.

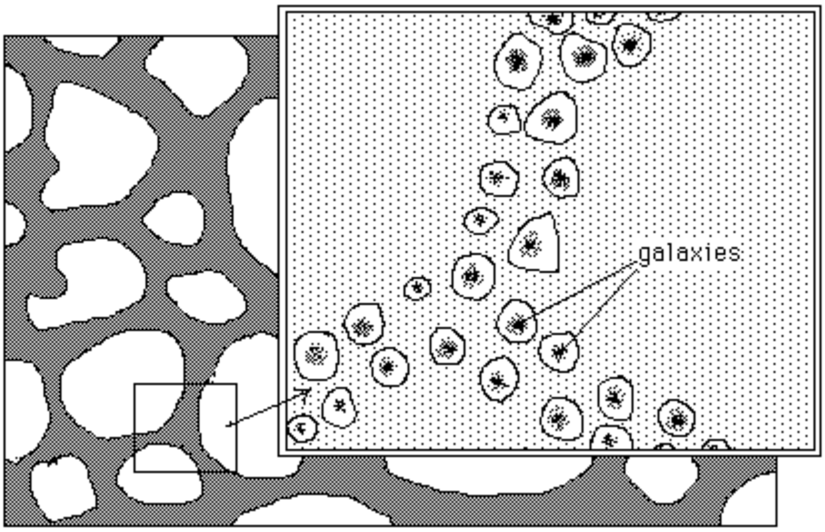


**Figure 14: Smaller size structure**

## 7- Some comments about the axioms

The classical General Relativity proposes a macroscopic description of the universe, shaped by the gravitational field. But, basically, the electromagnetic phenomena is not taken into account. In order to link this classical model to the observations, one has to bring the following additional axioms:

- The universe is filled by particles: neutral particles with a mass equal to m, and photons. Both contribute to the field.

- These particles move along geodesics of space-time

- A particle may send electromagnetic signal

- Another particle may receive this electromagnetic signal

- This electromagnetic signal, carried by photons, follows the null geodesics of space-time.

- A massive particle may send a gravitational signal, which is supposed to follow a null geodesic.

- A massive particle may receive this gravitational signal.

So that, for an observer composed by matter, the universe becomes optically perceptible, according to these axioms. The photons are the go-between bringing an optical message from a massive particle to another one.

In the present model the universe is be considered as a cover of a S3 sphere, locally we have a structure similar to a bundled manifold, whose bundle should be limited to two values: + 1 and -1.

Then we introduce the new following axioms.

- The universe is filled by particles: neutral particles whose mass is equal to m, and by photons. Both contribute to the field.

- The massive particles and the photon move along the geodesic of space time and cannot cross from a region to the conjugated antipodal region
of S3.

- A massive particle may send electromagnetic and gravitational signals, which can be received by another massive particle.

- The gravitational signal travels along the geodesics of space-time, but also along the geodesics of the "adjacent folds of the universe", "through the bundle structure" so that the gravitational signal owns some sort of ubiquity, because it acts both in a region of the manifold
(or in other terms in the "adjacent region", if we choose the bundled manifold image ).

- The structure of the new field equation brings the following features.

If a gravitational signal is emitted and received by two particles which "belong to the same fold" the phenomenon identifies with the classical description.

But a gravitational signal emitted by a massive particle can be received by another particle located is the adjacent region (the antipodal region) , in other terms "through the bundle structure", the negative sign in the second member of the field equation changing the nature of the signal, as if it was emitted by a "negative mass".

- The electromagnetic signal follows the ordinary null geodesics of the manifold, but does not own this property of ubiquity. It cannot cross from a fold to the "adjacent fold through the bundle structure". To travel from a region of the manifold to the antipodal region, light has to do a complete half-turn of the S3 sphere.

We confess that this proposed geometric description remains primitive and somewhat unclear. A correct description should imply a more refined model, including the gravitational and electromagnetic phenomena, i.e. an unified theory, which does not exist presently.

The bundled manifold local description is similar to a 5d Kaluza model, in which the fifth dimension would be limited to two values + 1 and - 1, as suggested earlier by Alain Connes.

## 8- Estimation of the "missing mass effect"

Apply a perturbation method to the Euler equations:

(25)

$$\Psi = \Psi_o + \delta\Psi$$

(25')

$$\Psi^* = \Psi_o^* + \delta\Psi^*$$

with the first order solution:

(26)

$$\Delta\psi = \Delta\psi\, o = 0$$

The Poisson equation gives:

(27)

$$\Delta\delta\Psi = 4\pi G \rho_o \left( e^{-\frac{m\,\delta\Psi}{kT}} - e^{-\frac{m\,\delta\Psi^*}{kT}} \right)$$

(27')

$$\delta\rho = \rho_o\, e^{-\frac{m\delta\Psi}{kT}} \qquad \delta\rho^* = \rho_o\, e^{\frac{m\delta\Psi}{kT}}$$

$$\Delta\psi = -\,\Delta\psi^*$$

(28)

$$\Delta\delta\Psi + \frac{8\pi G \rho_o}{kT}\delta\Psi = 0$$

Lj is the classical Jeans length

(29)

$$L_j = \sqrt{\frac{kT}{4\pi G \rho_o}}$$

(30)

$$\Delta\delta\Psi + 2\,\frac{\delta\Psi}{L_j^2} = 0$$

This is the well known Helmoltz equation.

In classical steady-state approach we had

(31)

$$\Delta\delta\Psi + \frac{\delta\Psi}{L_j^2} = 0$$

The interaction with the antipodal region shortens the Jeans length by a factor 1.414 so that we have a confinement effect. If we have a positive concentration of matter $\delta\rho$ in our space-time fold, we will find a negative $\delta\rho^*$ in the associated antipodal region, and vive-versa. The confinement of the mass due to the action of the antipodal region should reduce the necessary mass to balance pressure or centrifugal force by a factor:

$$\frac{1}{2^{3/2}} = 0,353$$

## 9- Writing the equation into a complex form

Write
(32)

$\mathbf{S_1} = \mathbf{S}\ (x^\circ , \boldsymbol{\sigma})$
$\mathbf{S_2} = \mathbf{S}\ (x^\circ , \boldsymbol{\sigma}^*)$
$\mathbf{T_1} = \mathbf{T}\ (x^\circ , \boldsymbol{\sigma})$
$\mathbf{T_2} = \mathbf{T}\ (x^\circ , \boldsymbol{\sigma}^*)$
$\Sigma = \Sigma_1 + i\, \Sigma_2$
$\tau = \tau_1 + i\, \tau_2$
$\tau^c = \tau_1 - i\, \tau_2$

The equation (4) can be written
(33)

$$\Sigma = \omega\, ( \tau - i\, \tau^c )$$

As suggested previously by Penrose, the quantification of the gravitation could be due to the complex form of the field equation. The equation (33) could perhaps bring a new insight on the problem.

## 10- Conclusion

We propose a new field equation, from which, with the classical approximation: steady-state, weak fields, low velocities, we derive the associated Poisson equation. Coupled Eddington solutions give a set of clusters, associated to interacting ring-like clouds, located in the antipodal region. The antipodal halo-like structure repels the cluster and helps its confinment. The reduction factor is roughly evaluated. It is suggested, through 2d numerical simulations, that this model could explain the large structure of the universe. In addition, the interaction between a cluster and its associated antipodal structure could provide spiral structure. A collision of a cluster with an anti-cluster could also explain the very irreguliar galaxies.

## References

[1] A.D. Sakharov, *ZhETF Pis'ma* 5: 32 (1967) ; *JETP Lett.* 5: 24 (1967) ; tr. Preprint R2-4267, *JINR*, Dubna.
[2] D. Novikov, *ZhETF Pis'ma* 3: 223 (1966) ; *JETP Lett.* 3: 142 (1966) ; tr. *Astr. Zh.* 43: 911 (1966), *Sov. Astr.* 10: 731 (1967).
[3] J.P. Petit: "Univers énantionmorphes à temps propres opposés" (*Enantiomorphic universes with opposite time arrows*), CRAS, t.263, Serie A, pp. 1315-1318, 23 May 1977.
[4] J.P. Petit: "*Univers en interaction avec leur image dans le miroir du temps*" (*Universes interacting with their opposite time-arrow fold*), CRAS, t.284, Serie A, pp. 1313-1316, 6 June 1977.
[5] J.P. Petit: "*Topologicon*", Belin Ed., Paris, 1984.