

Resummation of Massive Gravity

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We construct four-dimensional covariant non-linear theories of massive gravity which are ghost-free in the decoupling limit to all orders. These theories resum explicitly all the nonlinear terms of an effective field theory of massive gravity. We show that away from the decoupling limit the Hamiltonian constraint is maintained at least up to and including quartic order in non-linearities, hence, excluding the possibility of the Boulware-Deser ghost up to this order. We also show that the same remains true to all orders in a similar toy-model.

Introduction: Whether there exist a consistent extension of General Relativity by a mass term is a basic question of a classical field theory. A small graviton mass could also be of a significant physical interest, notably for the cosmological constant problem.

A ghost-free linear theory of massive spin-2 – the Fierz-Pauli (FP) model [1] – had been notoriously hard to generalize to the nonlinear level [2]: the Hamiltonian constraint gets lost in general and, as a result, the sixth degree of freedom – the Boulware-Deser (BD) ghost – emerges as a mode propagating on otherwise physically meaningful local backgrounds (*e.g.*, on a background of a lump of matter). Part of this problem can be seen in the effective field theory (EFT) approach to massive gravity [3] in the decoupling limit [3, 4]. There, the problem manifests itself in the Lagrangian for the helicity-0 component of the massive graviton. This Lagrangian generically contains nonlinear terms with more than two time derivatives. The latter give rise to the sixth degree of freedom on local backgrounds, while in general, these terms lead to the loss of well-posedness of the Cauchy problem for the helicity-0 field theory [3, 4].

A step forward has been made recently in [5] where it was shown that: (a) the coefficients of the EFT can be chosen so that the decoupling limit Lagrangian is ghost-free; this involves choosing the “appropriate coefficients” order-by-order, and an algorithm was set for this procedure to an arbitrary order; (b) once the “appropriate coefficients” are chosen in the effective Lagrangian, in the decoupling limit only a few terms up to the quartic order survive, all the higher order terms vanish identically. Moreover, the surviving terms are unique as their structure is fixed by symmetries [5, 6].

In the present work we build on the above two points, and go far beyond them. In particular: (1) We construct Lagrangians that *automatically* produce the “appropriate coefficients” once expanded in powers of the fields; these give rise to theories that are ghost-free automatically to all orders in the decoupling limit. (2) Using the obtained Lagrangians we study the issue of the BD ghost away from the decoupling limit; we show that the Hamiltonian constraint is maintained at least up to and including

quartic order, hence excluding the possibility of the BD ghost up to this order. We also express the exact potential for gravity in a simplified (1+1)-dimensional model and show explicitly how the constraint is preserved to all orders.

The present framework provides explicit resummation of the nonlinear terms in the EFT Lagrangian of massive spin-2. Another way to resum these terms is to use an auxiliary extra dimension [7, 8]. The latter has so far been shown to give the ghost-free decoupling limit only up to the cubic order [9]. In [7, 8] the resummation is obtained via the second order partial non-linear differential equation. The present approach achieves this via an algebraic non-linear equation.

Formalism: Define the tensor $H_{\mu\nu}$ as the covariantization of the metric perturbation, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$, where the four Stückelberg fields ϕ^a transform as scalars, and $\eta_{ab} = (-1, 1, 1, 1)$, [3]. The helicity-0 mode π of the graviton can be extracted by expressing $\phi^a = (x^a - \eta^{a\mu}\partial_\mu\pi)$, such that

$$H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta}\Pi_{\mu\alpha}\Pi_{\beta\nu}, \quad \Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi. \quad (1)$$

We may therefore define the following quantity

$$\mathcal{K}_\nu^\mu(g, H) = \delta_\nu^\mu - \sqrt{\delta_\nu^\mu - H_\nu^\mu} = - \sum_{n=1}^{\infty} d_n (H^n)_\nu^\mu, \quad (2)$$

$$\text{with} \quad d_n = \frac{(2n)!}{(1-2n)(n!)^2 4^n}. \quad (3)$$

Here $H_\nu^\mu = g^{\mu\alpha}H_{\alpha\nu}$, and $(H^n)_\nu^\mu = H_{\alpha_1}^\mu H_{\alpha_2}^{\alpha_1} \dots H_{\alpha_{n-1}}^{\alpha_{n-2}} H_{\alpha_{n-1}}^\nu$ denotes the product of n tensors H_β^α . Below, unless stated otherwise, all the contractions are made using the metric $g_{\mu\nu}$. The tensor $\mathcal{K}_{\mu\nu} = g_{\mu\alpha}\mathcal{K}_\nu^\alpha$ is defined in such a way that

$$\mathcal{K}_{\mu\nu}(g, H) \Big|_{h_{\mu\nu}=0} \equiv \Pi_{\mu\nu}. \quad (4)$$

We use the same notation as in [4] where square brackets [...] represent the trace of a tensor contracted using the Minkowski metric, *e.g.* $[\Pi] = \eta^{\mu\nu}\Pi_{\mu\nu}$ and

$[\Pi^2] = \eta^{\alpha\beta}\eta^{\mu\nu}\Pi_{\alpha\mu}\Pi_{\beta\nu}$, while angle brackets $\langle \dots \rangle$ represent the trace with respect to the physical metric $g_{\mu\nu}$, so that $\langle H \rangle = g^{\mu\nu}H_{\mu\nu}$ and $\langle H^2 \rangle = g^{\alpha\beta}g^{\mu\nu}H_{\alpha\mu}H_{\beta\nu}$.

We are first interested in the decoupling limit. For that, let us define the canonically normalized variables, $\hat{\pi} = \Lambda_3^3\pi$ with $\Lambda_3^3 = m^2M_{\text{Pl}}$ and $\hat{h}_{\mu\nu} = M_{\text{Pl}}h_{\mu\nu}$. The limit is then obtained by taking $M_{\text{Pl}} \rightarrow \infty$ and $m \rightarrow 0$ while keeping $\hat{\pi}$, $\hat{h}_{\mu\nu}$, and the scale Λ_3 fixed. First, we construct an explicit example of a non-linear theory that bears no ghosts in the decoupling limit, and then give a general formulation and show the absence of the BD ghost beyond the decoupling limit in quartic order.

Massive Gravity: The consistency of the Fierz-Pauli combination relies on the fact that the Lagrangian

$$\mathcal{L}_{\text{der}}^{(2)} = [\Pi]^2 - [\Pi^2], \quad (5)$$

is a total derivative. To ensure that no ghost appears in the decoupling limit, it is sufficient to extend $\mathcal{L}_{\text{der}}^{(2)}$ covariantly away from $h_{\mu\nu} = 0$, *i.e.* replace $[\Pi]$ and $[\Pi^2]$ by $\langle \mathcal{K} \rangle$ and $\langle \mathcal{K}^2 \rangle$ respectively, so that the total Lagrangian reads as

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2}\sqrt{-g} \left(R - \frac{m^2}{4}\mathcal{U}(g, H) \right), \quad (6)$$

with the potential \mathcal{U} expressed as an expansion in H as

$$\begin{aligned} \mathcal{U}(g, H) &= -4(\langle \mathcal{K} \rangle^2 - \langle \mathcal{K}^2 \rangle) \\ &= -4\left(\sum_{n \geq 1} d_n \langle H^n \rangle\right)^2 - 8\sum_{n \geq 2} d_n \langle H^n \rangle. \end{aligned} \quad (7)$$

Expanding this expression to quintic order,

$$\begin{aligned} \mathcal{U}(g, H) &= (\langle H^2 \rangle - \langle H \rangle^2) - \frac{1}{2}(\langle H \rangle \langle H^2 \rangle - \langle H^3 \rangle) \\ &\quad - \frac{1}{16}(\langle H^2 \rangle^2 + 4\langle H \rangle \langle H^3 \rangle - 5\langle H^4 \rangle) \\ &\quad - \frac{1}{32}(2\langle H^2 \rangle \langle H^3 \rangle + 5\langle H \rangle \langle H^4 \rangle - 7\langle H^5 \rangle) + \dots, \end{aligned} \quad (8)$$

we recover the decoupling limit presented in [5] with the special indices $c_3 = d_5 = f_7 = 0$.

Note that the Lagrangian (6) with (7) can be obtained from the Lagrangian

$$\begin{aligned} \mathcal{L}_\lambda &= \frac{M_{\text{Pl}}^2}{2}\sqrt{-g} (R - m^2(\mathcal{K}_{\mu\nu}^2 - \mathcal{K}^2)) \\ &\quad + \sqrt{-g}\lambda^{\mu\nu}(g^{\alpha\beta}\mathcal{K}_{\mu\alpha}\mathcal{K}_{\beta\nu} - 2\mathcal{K}_{\mu\nu} + H_{\mu\nu}), \end{aligned} \quad (9)$$

where $\mathcal{K}_{\mu\nu}$ is an independent tensor field that gets related to $H_{\mu\nu}$ as in (2) due to the constraint enforced by the Lagrange multiplier λ_{ν}^{μ} . Note, the expression (2) can be rewritten as $\mathcal{K}_{\nu}^{\mu} = \delta_{\nu}^{\mu} - \sqrt{\partial^{\mu}\phi^a\partial_{\nu}\phi^b\eta_{ab}}$, that gives a square root structure in the full Lagrangian.

Decoupling limit: It is straightforward to notice that the leading contribution to the decoupling limit

$$\sqrt{-g}\mathcal{U}(g, H)\Big|_{h_{\mu\nu}=0} = -4((\square\pi)^2 - (\partial_{\alpha}\partial_{\beta}\pi)^2), \quad (10)$$

is a total derivative. The resulting interaction Lagrangian in the decoupling limit is then given by [5]

$$\mathcal{L}_{\text{int}} = \hat{h}_{\mu\nu}\bar{X}^{\mu\nu}, \quad (11)$$

with

$$\bar{X}^{\mu\nu} = -\frac{M_{\text{Pl}}^2m^2}{8}\frac{\delta}{\delta h_{\mu\nu}}(\sqrt{-g}\mathcal{U}(g, H))\Big|_{h_{\mu\nu}=0}. \quad (12)$$

Using the relations

$$\frac{\delta\mathcal{K}(g, H)}{\delta h_{\mu\nu}} = \frac{1}{2}(g^{\mu\nu} - \mathcal{K}^{\mu\nu}), \quad (13)$$

$$\frac{\delta\langle\mathcal{K}(g, H)^2\rangle}{\delta h_{\mu\nu}} = H^{\mu\nu} - \mathcal{K}^{\mu\nu}, \quad (14)$$

the expression for \bar{X} simplifies to

$$\begin{aligned} \bar{X}_{\mu\nu} &= \frac{1}{2}\Lambda_3^3\left[\Pi\eta_{\mu\nu} - \Pi_{\mu\nu} + \Pi_{\mu\nu}^2 - \Pi\Pi_{\mu\nu}\right. \\ &\quad \left. + \frac{1}{2}(\Pi^2 - \Pi_{\alpha\beta}^2)\eta_{\mu\nu}\right]. \end{aligned} \quad (15)$$

The tensor $\bar{X}_{\mu\nu}$ is conserved and gives rise to at most second order derivative terms in the equations of motion. This tensor can be expressed as the product of two epsilon tensors appropriately contracted with powers of $\Pi_{\mu\nu}$ [6]. For the potential (7), the Lagrangian in the decoupling limit is then given by, see Ref. [5]

$$\mathcal{L}_{\Lambda_3}^{\text{lim}} = -\frac{1}{4}\hat{h}^{\mu\nu}(\hat{\mathcal{E}}\hat{h})_{\mu\nu} + \hat{h}_{\mu\nu}\bar{X}^{\mu\nu}, \quad (16)$$

and this result is exact (*i.e.* no higher order corrections). Notice that this is also in agreement with the results of [5] up to quintic order, for the special case $c_3 = d_5 = f_7 = 0$, but we explicitly demonstrate here that this result remains valid to all orders.

General formulation: As mentioned in [5], at each order in the expansion there exists a total derivative contribution

$$\mathcal{L}_{\text{der}}^{(n)}(\Pi) = -\sum_{m=1}^n (-1)^m \frac{(n-1)!}{(n-m)!} [\Pi^m] \mathcal{L}_{\text{der}}^{(n-m)}(\Pi), \quad (17)$$

with $\mathcal{L}_{\text{der}}^{(0)}(\Pi) = 1$ and $\mathcal{L}_{\text{der}}^{(1)}(\Pi) = [\Pi]$. These total derivatives generalize the ‘‘Fierz-Pauli’’ structure used previously to all orders. More generally, the potential of any theory of massive gravity with no ghosts in the decoupling limit can be expressed non-linearly as

$$\mathcal{U}(g, H) = -4\sum_{n \geq 2} \alpha_n \mathcal{L}_{\text{der}}^{(n)}(\mathcal{K}), \quad (18)$$

where $[\Pi^m]$ in (17) should be replaced by $\langle \mathcal{K}^m \rangle$ and expressed in terms of g and H using (2).

Here again this specific structure ensures that the leading contribution to the decoupling limit is manifestly a total derivative by construction,

$$\sqrt{-g}\mathcal{U}(g, H)\Big|_{h_{\mu\nu}=0} = \text{total derivative}, \quad (19)$$

and the resulting interaction Lagrangian can be derived by noticing the general relation

$$\frac{\delta}{\delta h^{\mu\nu}}\langle\mathcal{K}^n\rangle\Big|_{h_{\mu\nu}=0} = \frac{n}{2}(\Pi_{\mu\nu}^{n-1} - \Pi_{\mu\nu}^n), \quad (20)$$

so that

$$\frac{\delta}{\delta h^{\mu\nu}}\left(\sqrt{-g}\mathcal{L}_{\text{der}}^{(n)}(\mathcal{K})\right)\Big|_{h_{\mu\nu}=0} = \sum_{m=0}^n \frac{(-1)^m n!}{2(n-m)!} (\Pi_{\mu\nu}^m - \Pi_{\mu\nu}^{m-1}) \mathcal{L}_{\text{der}}^{(n-m)}(\Pi), \quad (21)$$

using the notation $\Pi_{\mu\nu}^0 = \eta_{\mu\nu}$ and $\Pi_{\mu\nu}^{-1} = 0$. The decoupling limit Lagrangian is then given by (16) with the same definition (12) for the tensor $X_{\mu\nu}$, giving here

$$\bar{X}_{\mu\nu} = \frac{1}{2}\Lambda_3^3 \sum_{n \geq 2} \alpha_n \left(X_{\mu\nu}^{(n)} + nX_{\mu\nu}^{(n-1)}\right), \quad (22)$$

with

$$X_{\mu\nu}^{(n)} = \sum_{m=0}^n (-1)^m \frac{n!}{2(n-m)!} \Pi_{\mu\nu}^m \mathcal{L}_{\text{der}}^{(n-m)}(\Pi). \quad (23)$$

This is in complete agreement with the results obtained up to quintic order for $\alpha_2 = 1$, $\alpha_3 = -2c_3$, $\alpha_4 = -2^2 d_5$ and $\alpha_5 = -2^3 f_7$. However we emphasize that the results in this paper are now valid to all orders. The special theory found in [7, 8] corresponds to the specific choices of coefficients $\alpha_2 = 1$ and $\alpha_3 = -1/2$, see Ref. [10].

Furthermore, at each order the tensors $X_{\mu\nu}^{(n)}$ are given by the recursive relation

$$X_{\mu\nu}^{(n)} = -n\Pi_{\mu}^{\alpha} X_{\alpha\nu}^{(n-1)} + \Pi^{\alpha\beta} X_{\alpha\beta}^{(n-1)} \eta_{\mu\nu}. \quad (24)$$

with $X_{\mu\nu}^{(0)} = 1/2\eta_{\mu\nu}$. So since $X_{\mu\nu}^{(4)} \equiv 0$ all these tensors vanish beyond the quartic one, $X_{\mu\nu}^{(n)} \equiv 0$ for any $n \geq 4$, and the decoupling limit therefore stops at that order, as previously implied in [5].

Boulware-Deser ghost: The previous argument ensures the absence of ghost in the decoupling limit, but it is feasible that the ghost reappears beyond the decoupling limit, and is simply suppressed by a mass scale larger than Λ_3 . Certain arguments have hinted towards the existence of a BD ghost, [4]. We reanalyze the arguments here and show the absence of ghosts within the regime studied. To compute the Hamiltonian, we fix unitary gauge for which $\pi = 0$, such that

$$\langle H^n \rangle = \sum_{\ell \geq 0} (-1)^\ell C_\ell^{\ell+n-1} [h^{\ell+n}], \quad (25)$$

where the C_m^n are the Bernoulli coefficients. We also focus on the case where $\alpha_2 = 1$ and $\alpha_n = 0$ for $n \geq 3$. In what follows, we work in terms of the ADM variables [11],

$$g^{00} = -N^{-2}, \quad g_{0i} = N_i, \quad \text{and} \quad g_{ij} = \gamma_{ij}, \quad (26)$$

with the lapse $N = 1 + \delta N$, and the three-dimensional metric $\gamma_{ij} = \delta_{ij} + h_{ij}$. In terms of these variables, the potential is then of the form

$$\sqrt{-g}\mathcal{U} = \mathcal{A} + \delta N \mathcal{B} + N_i N_j \left[-2\delta^{ij} + \mathcal{C}^{ij} + \delta N (\delta^{ij} + \mathcal{D}^{ij}) - \frac{1}{2} \delta N^2 \delta^{ij} - \frac{1}{8} \delta^{ij} N_k^2 \right], \quad (27)$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}^{ij}$ and \mathcal{D}^{ij} are functions of h_{ij} , at least first order in perturbations, and $\mathcal{C}^{ij} + 2\mathcal{D}^{ij} = -\frac{1}{2}h^{ij} + \mathcal{O}(h_{ij}^2)$, and in this section we raise and lower the space-like indices using δ_{ij} . Notice that this is completely consistent with the analysis performed in [4], and corresponds to setting the coefficients in (43) of [4] to $A = B = D = E = 0$, while $C = -1/2$. We emphasize here that the presence of a term of the form $C N_i^2 N^2$ does not signal the presence of a ghost, since any quadratic terms in the lapse disappear after integration over the shift as we prove in what follows. Indeed, in terms of redefined shift n_i ,

$$N_j = \left(\delta_j^i + \frac{1}{2} \delta N \delta_j^i - \frac{1}{8} \delta N h_j^i \right) n_i \equiv L_j^i n_i, \quad (28)$$

the Hamiltonian is of the form

$$\mathcal{H} = \frac{M_{\text{Pl}}^2}{2} \sqrt{\gamma} (NR^0 + N_j R^j) + \frac{m^2 M_{\text{Pl}}^2}{8} (\mathcal{A} + \mathcal{B} \delta N) - \frac{m^2 M_{\text{Pl}}^2}{4} L^{ij} \left(n_i n_j - \frac{1}{2} \mathcal{C}_i^k n_j n_k + \frac{1}{16} n_k^2 n_i n_j \right), \quad (29)$$

up to quartic order in the metric perturbations. Then, it is straightforward to check that the variation of the Hamiltonian (29) w.r.t. the shift n_i gives an equation which is independent of N , and serves to determine n_j . Moreover, the lapse remains a Lagrange multiplier even after integration over the shift, hence giving rise to a Hamiltonian constraint on the physical variables. Whether this constraint gives rise to a secondary constraint, and whether the system should be quantized as a first- or second class system, is a separate interesting question. The mere existence of the Hamiltonian constraint is sufficient to claim the absence of the BD ghost to that order [16], yet without breaking Lorentz invariance, [12].

The Hamiltonian evaluated on the constraint surface is proportional to m^2 and whether or not it is positive semi-definite is determined by the explicit expressions for $\mathcal{A}, \mathcal{B}, \mathcal{C}^{ij}$ and \mathcal{D}^{ij} . Thus, in general certain backgrounds could have slow tachyon-like instabilities, however, this is a separate issue from that of the BD ghost that we clarified above.

(1 + 1)-d massive gravity: Proving the absence of the BD ghost in complete generality beyond the quartic order is a grand task, which we save for a separate study. However, we can analyze here a similar issue in a (1+1)-d toy-model, where we consider the Hamiltonian

$$\mathcal{H} = M_{\text{Pl}}^2 \sqrt{\gamma} \left[NR^0 + \gamma^{11} N_1 R_1 + \frac{m^2}{4} NU(g, H) \right], \quad (30)$$

with R^0 and R_1 arbitrary functions of the space-like metric γ_{11} and its conjugate momentum, and the potential \mathcal{U} is given in (7). In 1 + 1 dimensions, it is relatively easy to check that the Hamiltonian then takes the exact form

$$\mathcal{H} = M_{\text{Pl}}^2 \sqrt{\gamma} \left[NR^0 + \gamma^{11} N_1 R_1 - 2m^2 N \right] - 2m^2 \left(1 - \sqrt{(\sqrt{\gamma} + N)^2 - \gamma^{11} N_1^2} \right), \quad (31)$$

and seemingly includes terms quadratic in the lapse when working at quartic order and beyond,

$$\mathcal{H} \sim \mathcal{H}_0 + \mathcal{H}_1 N + m^2 N_1^2 N^2 + \dots \quad (32)$$

By stopping the analysis at this point one would infer that the lapse no longer enforces a constraint. However, this should be determined after integrating the shift. In other words, in terms of the redefined shift n_1

$$N_1 = n_1 (\gamma_{11} + N \sqrt{\gamma}), \quad (33)$$

the Hamiltonian takes the much more pleasant form

$$\mathcal{H} = \sqrt{\gamma} NR^0 - 2m^2 (1 + \sqrt{\gamma} N) + (\sqrt{\gamma} + N) \left(n_1 R_1 + 2m^2 \sqrt{1 - n_1^2} \right), \quad (34)$$

which remains linear in the lapse, even after integration over the shift. It is again straightforward to see that the lapse does enforce a constraint, and does so for an “arbitrary background”.

Outlook: We have given a covariant non-linear realization of massive gravity in 4D which: (1) is automatically free of ghosts in the decoupling limit, to all orders in non-linearities; (2) keeps the lapse as a Lagrange multiplier away from the decoupling limit, at least up to quartic order in non-linearities. These findings constitute what we believe is a very significant step forward, and strongly suggests the existence of an entirely ghost-free classical theory of massive gravity. However, to prove this statement in complete generality, two important ingredients are yet missing: (a) proving that the lapse remains a Lagrange multiplier to all orders; (b) checking whether the secondary constraint is generated or not, and

whether the theory could be canonically quantized as a first or second class system. For the consistency of the theory at the quantum loop level one would have to establish the existence of a symmetry which protects this theory against quantum corrections that could revive the ghost. These points will be explored in a further study.

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- [1] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A **173**, 211 (1939).
 - [2] D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
 - [3] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. **305**, 96 (2003).
 - [4] P. Creminelli, A. Nicolis, M. Papucci and E. Trincherini, JHEP **0509**, 003 (2005).
 - [5] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010) [arXiv:1007.0443 [hep-th]].
 - [6] C. de Rham, G. Gabadadze, L. Heisenberg and D. Pirtskhalava, arXiv:1010.1780 [hep-th].
 - [7] G. Gabadadze, Phys. Lett. B **681**, 89 (2009) [arXiv:0908.1112 [hep-th]].
 - [8] C. de Rham, Phys. Lett. B **688**, 137 (2010) [arXiv:0910.5474 [hep-th]].
 - [9] C. de Rham and G. Gabadadze, Phys. Lett. B **693**, 334 (2010) [arXiv:1006.4367 [hep-th]].
 - [10] C. de Rham and A. J. Tolley, JCAP **1005**, 015 (2010) [arXiv:1003.5917 [hep-th]].
 - [11] R. M. Wald, Chicago, Usa: Univ. Pr. (1984) 491p.
 - [12] V. A. Rubakov and P. G. Tinyakov, Phys. Usp. **51**, 759 (2008) [arXiv:0802.4379 [hep-th]].
 - [13] A. H. Chamseddine and V. Mukhanov, JHEP **1008**, 011 (2010).
 - [14] L. Berezhiani, M. Mirbabayi, arXiv:1010.3288 [hep-th].
 - [15] L. Alberte, A. H. Chamseddine and V. Mukhanov, arXiv:1011.0183 [hep-th].
 - [16] The approach of [13] is equivalent to the EFT approach of [3], as was shown in [14]. Hence, the claim of [15] on the presence of the BD ghost in the quartic order, if correct, would contradict our results. However, what has really been diagnosed in [15] is the issue already raised in [4], which we have just addressed. In particular, the apparent ghost-like nonlinear terms identified in [15], to the extent they were presented in [15], are in fact removable at that order by a nonlinear field redefinition, in complete consistency with our results above. This will be discussed in more detail elsewhere.