Questionable black holes.

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Keywords :black hole, M87*, SgrA*, Birkhoff theorem, plugstar, Hilbert, two folds cover manifold, bounded manifold, PT-symmetry, geometrical criticity, physical criticity, mass inversion, negative mass, time factor, throat sphere, Hoag galaxy, density wave, supermassive objects, neutron stars, Janus Cosmological Model

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Abstract : We show that the model of the black hole is a complete mathematical and geometrical chimera, born from wrong interpretations of the Schwarzschild external metric solution. The first misconception stems from Hilbert's confusion of the Schwarzschild intermediate magnitude R with a radial coordinate r. The second arises from the omission of the cross term in drdt, which, by yielding a finite free-fall time, allowed Oppenheimer and Snyder in 1939 to suggest using a stationary solution to describe a highly unsteady phenomenon. The third is a consequence of the assumption of space contractibility, which creates the myth of the existence of the interior of black holes and a central singularity, with real spiral trajectories endowed with an imaginary length, thus located outside the hypersurface. This translates in the literature into the surreal idea that space and time coordinates exchange roles. Starting from the only accessible observational quantities, the images of the objects M87* and SgrA*, that is, a ratio of maximum to minimum brightness temperatures of 3, we show that this gravitational redshift effect then fits within the Schwarzschild inner metric solution when physical criticality is approached. The criticism formulated by Einstein in 1939 regarding this solution must be reconsidered, as the physical conditions in such criticality are such that the distances between baryons become on the order of their Compton length. Thus, the precise physical description of such states, and the suggested mass inversion process in objects described as plugstars, will require the completion of the unification between general relativity and quantum mechanics. As it stands, this reinterpretation of the Schwarzschild outer and inner solutions then fits into the Janus Cosmological Model. It is predicted that all future objects of this type will exhibit this same ratio of 3 and that the masses of neutron stars are less than 2.5 solar masses, therefore, in general, black holes of any size simply do not exist. It is suggested that the supermassive objects M87* and Sgr A* result from the implosion of density waves similar to those in Hoag's Galaxy.

Foreword.

In 1902 [9] it became increasingly clear that time could no longer be treated as something entirely independent of the phenomena it was supposed to describe. In 1905, Einstein published [8] his special relativity, breaking this centuries-old independence. The very nature of this fourth coordinate remained to be defined. In 1902, the mathematician Henri Poincaré suggested that it could be purely imaginary, and thus came very close to anticipating Einstein's discovery. In 1909, the mathematician Minkowski [10] united the four dimensions in a single space, but in his initial

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formulation, he retained the purely imaginary nature of the time coordinate. It was only later that time ct began to be considered as measured in meters, modulo the factor c, which is the speed of light. After the invariance of the speed of light, the second problem Einstein tackled was the variation in the orbital period of the planet Mercury. The astronomer Le Verrier had attempted to interpret these variations as the consequence of the perturbation of the orbit by a new planet, located closer to the Sun, which he named Vulcan. But observation did not confirm his calculations. At the time, no one dared to call it a dark planet. At the beginning of the century, Einstein was far from isolated in this quest for a new paradigm. Thus, when he published the equation of what then became the key to the new geometric paradigm of general relativity, the mathematician and astronomer Karl Schwarzschild immediately produced exact solutions to his equation, in stationary and spherically symmetric motion, equations without a forcing term [2] and with a forcing term [24].

Before this, Einstein had numerous exchanges with the mathematician Hilbert. Prior to this meeting with Einstein, Hilbert was convinced that the world of high mathematics, to which he was making significant contributions, was unrelated to the world of physics. Having been persuaded by Einstein, Hilbert tackled the problem and, through variational calculus, constructed the first version of the field equation [5] and submitted his publication a few days before Einstein. After a moment of confusion, the two decided to prioritize their relationship of esteem and friendship, acknowledging that Einstein would be the true discoverer. To understand the problems faced by today's theorists, it is essential to delve into the writings of the time, and in particular Hilbert's two articles [5], [4], which are difficult and perplexing to read. Both articles are titled "The Foundations of Physics." In its time, it was the first "theory of everything." Indeed, in 1915 only two of the four forces were known: the electromagnetic force and the force of gravity. Hilbert's ambition was therefore to integrate both of them into his variational formalism. Another essential aspect: for Hilbert (as for Poincaré), the time coordinate, denoted by the letter *l*, is purely imaginary. The solutions to the field equation he envisioned are not fundamentally lengths, but bilinear forms.

A few months after the publication of his first paper by Hilbert, Schwarzschild published two articles in January and February 1916. The first [2] gave the solution to Einstein's equation in steady state and spherically symmetric, and the second [24] gave the solution to the equation with a right-hand side. He then perfectly described the geometry inside an object consisting of a sphere of constant density, and outside of it. For the solution in a vacuum, Schwarzschild did not express it in terms of the variables (t, x, y, z) he initially defined. He introduced a coordinate $r = \sqrt{x^2 + y^2 + z^2}$, but using an "intermediate variable R" defined as follows:

$$(0) \qquad ds^2 = \left(1 - \frac{R}{\alpha}\right) c^2 dt^2 - \frac{dR^2}{1 - \frac{R}{\alpha}} - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad\qquad R = (r^3 + \alpha^3)^{1/3}$$

Hilbert then made a mistake by deciding to identify this coordinate R with r, which he presented as a simplification. This is described in the following article, in sections 3 to 6, indicating its implications. The error was first pointed out in 2001, eighty-five years later (…) by the Canadian mathematician Abrams [7], in an article that went completely unnoticed. Today, scientific work is disseminated rapidly and efficiently. But at that time, the primary means of dissemination was the book, in English, which was becoming the standard scientific language. In works that immediately became references, Eddington in 1924 [41] and Tolman [16] in 1934 echoed "Hilbert's approximation," that is, the identification of the intermediate quantity R with a "radial coordinate" r. But no one paid any attention. In 1939, Einstein referred to Schwarzschild's second article, describing the

geometry inside a mass, where the pressure p and a density ρ are linked by an equation of state. Schwarzschild then highlighted the conditions for physical criticality, occurring before the star's radius is caught by its Schwarzschild radius, such that the pressure soars towards infinity.
In the few lines Einstein devotes to this solution, he concludes that under such conditions the speed of sound would exceed the speed of light and that, consequently, this solution is not physical. Difficult to read, the article [24] was not available in English until… 1999. In 1939, Oppenheimer and Snyder [1], based on what is now considered "the standard form of the Schwarzschild solution," pointed out that the free-fall and escape times of a reference mass, measured in the time frame of a distant observer, are infinite. This article then became the birth of the black hole model, which uses a stationary solution to describe an implosion process that lasts only a few days. However, as shown in 2021 by the mathematician Koiran [18], this stems from the absence of a cross term in drdt, also historically considered a "simplification." Its presence, depending on its sign, completely alters the scenario, one of the two times becoming finite. Thus, the foundational hypothesis of the black hole model disappears. Furthermore, while this geometry had been perfectly understood and described in 1916 by the mathematician Flamm [3], the authors repeated Hilbert's second error, which consisted of taking as the Lagrange function, in the action that determines the geodesics, not the length, that is, the square root of the bilinear form, but the form itself. It turns out that when the derivatives $\dot{x}^i$are calculated with respect to the length s, the Lagrange equations obtained are the same. They therefore do indeed provide the geodesics. But they then provide what can be called virtual geodesics, real curves, equipped with a purely imaginary length, which do not belong to the hypersurface. Thus, the curves "spiraling towards the central singularity, inside the black hole" exist only in the imagination of the theorists who study them. In a future article, I will show, as an example, how this error allows us to equip the torus with these "virtual geodesics," spiraling towards its center and towards infinity, and how we can demonstrate that the "virtual geodesics of the sphere" are ellipses!

As a conclusion the black hole model is a mathematical and geometrical chimera

So, what happens when a massive star collapses? What about the supermassive objects identified at the centers of galaxies? At the end of the article, we present another scenario, which has the advantage of aligning with observational data (brightness temperature ratio 3). Regarding Einstein's critique, it's worth remembering that it's based on a constant density. However, for example, in neutron stars, when the criticality he invokes is reached, the distances between baryons tend to become smaller than their Compton length. A purely geometric description thus proves inadequate. The physics of such a medium can only be described through a complete and real unification of gravitation and quantum mechanics.

1 – Introduction: On what basis did the black hole model emerge?

a – The founding article is that of Oppenheimer and Snyder [1], in 1939. It is based on the fact that the free-fall time of a test mass toward the Schwarzschild sphere, when measured with the coordinate ttt, assumed to represent the proper time experienced by a distant observer, is infinite.

b – Moreover, the vanishing of the g_{tt} term on the Schwarzschild sphere implies that any light emitted from this surface undergoes an infinite gravitational redshift and thus behaves like a cosmological horizon.

c – The hypothesis is that the local topology is $R_+ \times R^3$. Thus, the universe is supposed to be locally contractile.

d – It is then assumed that in the bilinear form—solution of Einstein's field equation without source term, invariant under time translation, known as the "Schwarzschild solution":

(1) $$G_{ext}(t, r, \theta, \varphi, \dot{t}, \dot{r}, \dot{\theta}, \dot{\varphi}) = \left(1 - \frac{r}{\alpha}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{r}{\alpha}} - r^2\left(\dot{\theta}^2 + \sin^2\theta \,, \dot{\varphi}^2\right)$$

the letter r is supposed to represent a radial variable.

e – It is assumed that the solution curves derive from the Lagrangian:

(2) $$L = G_{ext}(t, r, \theta, \varphi, \dot{t}, \dot{r}, \dot{\theta}, \dot{\varphi})$$

involved in the construction of the action:

(3) $$J = \int L \, dp$$

It is assumed that these curves describe the geodesics of the entire contractile spacetime hypersurface, with topology $R_+ \times R^3$.

2 – The so-called "Schwarzschild solution" is not the original solution.

In his first article [2] from 1916, Schwarzschild clearly defines his initial coordinates $\{t, x, y, z\}$ as well as a coordinate r defined by:

(4) $$r = \sqrt{x^2 + y^2 + z^2}$$

although he does not state it explicitly $\{t, x, y, z\} \in R_+ \times R^3$. Thus $r \geq 0$

By expressing that the solution is spherically symmetric, which introduces a constant of integration α (later called the "Schwarzschild radius"), he effectively chooses to express his solution not with the coordinates $\{t, r, \theta, \varphi\}$ but with the coordinates $\{t, R, \theta, \varphi\}$ where R is an intermediate variable (called "*Hilfsgrößе*") defined by:

(5) $$R = (r^3 + \alpha^3)^{1/3} \geq \alpha$$

Which gives :

(6) $$ds^2 = \left(1 - \frac{R}{\alpha}\right)c^2 dt^2 - \frac{dR^2}{1 - \frac{R}{\alpha}} - R^2(d\theta^2 + \sin^2\theta \, d\varphi^2) \qquad R = (r^3 + \alpha^3)^{1/3}$$

The true Schwarzschild solution, from January 1916, expressed using his coordinates $\{t, r, \theta, \varphi\}$, is therefore:

(7) $$ds^2 = \frac{(r^3+\alpha^3)^{1/3}-\alpha}{(r^3+\alpha^3)^{1/3}}c^2 dt^2 - \frac{r^4, dr^2}{(r^3+\alpha^3)[(r^3+\alpha^3)^{1/3}-\alpha]} - (r^3+\alpha^3)^{2/3}(d\theta^2+\sin^2\theta\, d\varphi^2)$$

If we take dt = dr = 0 what remains is the spatial part of the metric:

(8) $$d\sigma^2 = (r^3+\alpha^3)^{2/3}(d\theta^2+\sin^2\theta, d\varphi^2)$$

This is the metric of a family of spheres with radius: $(r^3+\alpha^3)^{1/3}$, with a minimal area value:

(9) $$A4\pi = (r^3+\alpha^3)^{2/3}$$

The geometric object is therefore non-contractile. In terms of the spatial coordinates $\{r, \theta, \varphi\}$ the topology of the geometric object is $R_+ \times R^3$ but, based on the system $\{R, \theta, \varphi\}$ it a manifold with boundary, the boundary being the Schwarzschild sphere.

What happens to the metric coefficients as r $\longrightarrow$ 0 ?

(10) $$g_{tt} = \frac{(r^3+\alpha^3)^{1/3}-\alpha}{(r^3+\alpha^3)^{1/3}}c^2 \longrightarrow +0$$

This vanishing implies that on the Schwarzschild sphere, one cannot define a volume form, and thus no orientation.

For the g_{rr}, we must perform a Taylor expansion:

(11) $$g_{rr} \approx -\frac{r^4}{\alpha^3\left[\left(1+\frac{r^3}{\alpha^3}\right)^{1/3}-1\right]} \approx -\frac{r^4}{\alpha^3} \longrightarrow -0$$

At r = 0 or R $=$ α, spacetime is locally non-orientable. The regions of space corresponding to values R $<$ α lie outside the realm of real numbers. In this space, Flamm [3] constructs the meridional section in the form of a lying half-parabola, with equation:

(12) $$z = \pm 2\sqrt{\alpha(r-\alpha)}$$

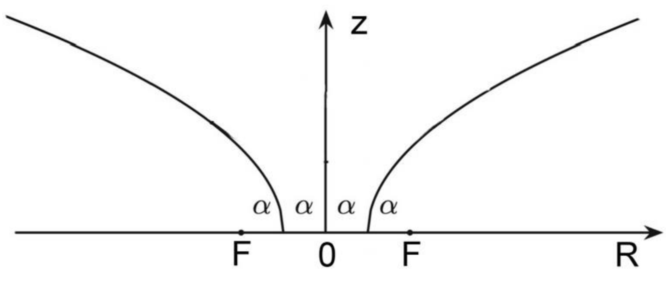

Fig.1 : Half-meridian of Flamm's surface [3]

The external metric being considered in isolation. Only a part of this meridian refers to the exterior of the star. The complete meridian therefore has no physical significance and its path is given only as an expression of a part (z > 0) of the solution to the differential equation (12), above.

Who initiated this choice? It was David Hilbert [4]. He treats the solutions to the field equation—of which he had published his own version [5] a few days before Einstein [6]—not as metrics but as bilinear differential forms:

$$G\left(x^i, \frac{dx^i}{dp}\right) \tag{13}$$

His field equation [5,6] is:

$$K_{\mu\nu} - \frac{1}{2} K g_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} \tag{14}$$

Where $K_{\mu\nu}$ is the Ricci tensor and K the corresponding Ricci scalar.

He places his solution in a space $\{ x_1, x_2, x_3, x_4 \}$, with the fourth coordinate referring to time. He considers the space to be quasi-Euclidean, writing (his equation (37)):

$$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon, h_{\mu\nu} \tag{15}$$

He then studies a stationary solution where the $g_{\mu\nu}$ are independent of x_4. He assumes the $g_{\mu\nu}$ are centrally symmetric with respect to the origin of the coordinates ("*zentrisch symmetrisch*"). On page 67, he chooses spherical coordinates:

$$x_1 = r \cos \theta \tag{16}$$

$$x_2 = r \sin \theta \tag{17}$$

$$x_3 = r \sin \theta \sin \varphi \tag{18}$$

$$x_4 = l \tag{19}$$

Taking symmetry into account, he writes the bilinear form (equation (42)):

$$F(r) dr^2 + G(r)(d\theta^2 + \sin^2 \theta \, d\varphi^2) + H(r) dl^2 \tag{20}$$

At this stage, he decides to set:

$$r^* = \sqrt{G(r)} \tag{21}$$

Then he writes:

> *Züren wir in (42) r anstatt r ein und lassen dann wieder das Zeichen * weg**
> (*Let us insert r∗ into (42) instead of t and then drop the asterisk again*)

In doing so, without realizing it, he expresses his solution not in terms of the coordinate

$$r = \sqrt{{x_1}^2 + {x_2}^2 + {x_3}^2} \tag{22}$$

but according to Schwarzschild's intermediate quantity (« *Hilfsgrobe* »):

$$R = (r^3 + \alpha^3)^{1/3} \geq \tag{23}$$

The first to point out this discrepancy was the Canadian mathematician Abrams [7]. Thus, Hilbert's bilinear form becomes, in his equation (43):

$$M(r) + r^2 d\theta^2 + \sin^2 \theta \, d\varphi^2 + W(r) dl^2 \tag{24}$$

4 – Hilbert's legacy: the survival of a purely imaginary time.

After determining the form of his functions M(r) and W(r), he writes (equation (45), p. 70)

$$\text{(25)} \qquad \mathrm{G}(dr, d\theta, d\varphi, dl) = \frac{r}{r-\alpha}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2 + \frac{r-\alpha}{r}\mathrm{d}l^2$$

Then, after writing:

> "... *so ergibt sich aus* (43) *für l=it die gesuchte Maßbestimmung in der von Schwarzschild gefundenen Gestalt*"
>
> *"Thus, from equation (43), for l = it, one obtains the sought-after determination of the metric in the form found by Schwarzschild."*

He writes:

$$\text{(26)} \qquad G(dr, d\theta, d\varphi, dt) = \frac{r}{r-\alpha}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2 - \frac{r-\alpha}{r}dt^2$$

One must recall that before Einstein presented his formulation of special relativity in 1905 [8], the majority of scientists adhered to the interpretation of the French mathematician Henri Poincaré, who in 1902 proposed the idea that the time variable was purely imaginary [9]. Even when Hermann Minkowski developed what would become spacetime in special relativity [10], he retained the idea of imaginary time—though he would soon abandon it [11], adopting Einstein's view that time is ultimately measured in meters.

In 1915, in his article on the advance of Mercury's perihelion [12], Einstein explicitly presents his Gram matrix, on page 832:

> … *der unspünglichen Relativitätstheorie entsprechende Schama gegeben:*
>
> ... *given the original theory of relativity corresponding to the shape*:

$$\text{(27)} \qquad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5 – Everything depends on the meaning given to the word "geodesic"

One may say that at the time, before World War II, most authors (Einstein, Schwarzschild, Droste, Weyl, and others) explicitly opted for the signature (+ - - -), equipping the solution hypersurface with a metric where the length element s, identified with the proper time τ through the relation s = c τ, is real. One might consider that it was Hilbert's influence that suggested the shift (or return) to a signature inheriting the interpretation of time dt as a purely imaginary quantity.By contrast, with Schwarzschild everything is clear from the start, when he writes at the beginning of his article [2]:

(28) $$ds = \sqrt{\sum g_{\mu\nu}\, dx_\mu\, dx_\nu} \qquad \mu = 1, 2, 3, 4$$

The terms are set out very clearly. s is a length, essentially real. As for the constructed curves—the geodesics—they are the shortest paths, materialized by the variational equation:

(29) $$\delta \int ds = \delta \int \sqrt{\sum g_{\mu\nu}\, dx_\mu\, dx_\nu} = 0$$

Thus, Schwarzschild only considers curves that have a real length.But with Hilbert, things change completely when he writes [4], page 68:

> *Der erste Schritt hierzu ist die Aufstellung der Differentialgleichungen der geodätischen Linien durch Variation des Integrals*
>
> (*The first step is to formulate the differential equations of the geodesic lines by varying the integral*)

(30) $$\int \mathrm{M}\left(\frac{\mathrm{dr}}{\mathrm{dp}}\right)^2 + \mathrm{r}^2\left(\frac{\mathrm{d}\theta}{\mathrm{dp}}\right)^2 + \mathrm{r}^2 \sin^2\theta \left(\frac{\mathrm{d}\varphi}{\mathrm{dp}}\right)^2 + \mathrm{H}\left(\frac{\mathrm{dl}}{\mathrm{dp}}\right)^2$$

This integral is Hilbert's action J. The variational calculus is applied to it $\delta J = 0$, leading to the Lagrange equations. But then, what does one call a geodesic?

For Schwarzschild, Einstein, Droste, and Weyl, they are curves along which the real length s is minimized.

For Hilbert, these curves minimize the *square* of that length. The resulting Lagrange equations are the same, but in one case there is a constraint on the domain of validity of the solution, while in the other this constraint disappears.

Later in his article, the fact that his bilinear form $\mathrm{G}\left(\frac{dx_s}{dp}\right)$ may be positive or negative does not bother Hilbert much, since he defines two *real* lengths:

– In the region where the bilinear form $\mathrm{G}\left(\frac{dx_s}{dp}\right)$ is positive, he defines a real length referring to curve segments he calls *segments*.

(31) $$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\, dp$$

– In the region where the bilinear form $G\left(\frac{dx_s}{dp}\right)$ is negative, he defines a second real length referring to curve portions he calls *time lines*.

(32) $$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}\, dp$$

– Finally, when the bilinear form $\mathrm{G}\left(\frac{dx_s}{dp}\right)$is null, it corresponds to “null lines” (*Nulllinien*),

6 – When the choice of Lagrangian determines the topology of the object

This identification of the bilinear form with the Lagrangian appearing in the action will become generalized. One may cite, as an example, the mention in the relatively recent (1992) book by S. Chandrasekhar, *The Mathematical Theory of Black Holes* [13].

After presenting on page 92, in his equation (60), the Schwarzschild metric in the form:

$$\text{(33)} \qquad \mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 - \frac{d\dot{r}^2}{1 - \frac{2M}{r}} - r^2\mathrm{d}\theta^2 - r^2 \sin^2\theta \, \mathrm{d}\varphi^2$$

with signature (+ - - -), he defines on page 96, in equation (80), what he calls "the Lagrangian of the Schwarzschild spacetime":

$$\text{(34)} \qquad L = \frac{1}{2}\left[\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{2M}{r}} - r^2\dot{\theta}^2 - r^2 \sin^2\theta \, \dot{\varphi}^2\right]$$

This allows him to trace the geodesic curves located *"inside the Schwarzschild sphere"*, that is, with a purely imaginary ds:

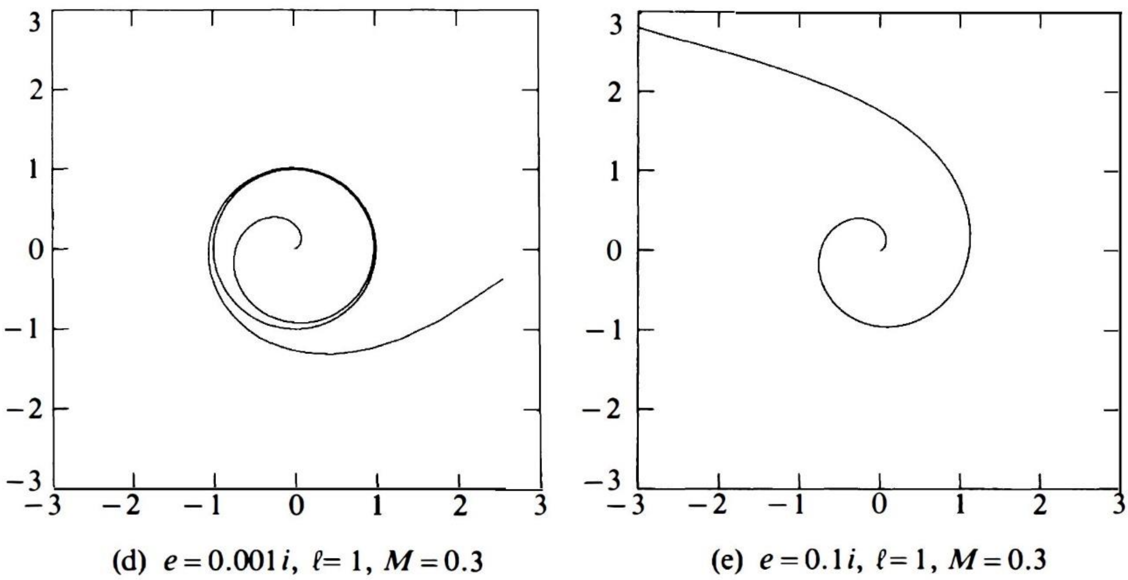

FIG. 7*b*. Various classes of time-like geodesics described by a test particle with $E^2 > 1$: (a), (b), (c): orbits of the first and the second kind with eccentricity $e = 3/2$ and latera recta, 4.5, 2.5, and 1.94 respectively ($M = 3/14$ in the scale along the coordinate axes); (d), (e): unbound orbits with $l = 1$ and with imaginary eccentricities $e = 0.001\,i$ and $0.1\,i$ ($M = 0.3$ in the scale along the coordinate axes).

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Figure 2: Geodesics spiraling *"inside the Schwarzschild sphere"* according to S. Chandrasekhar, from his book [13], p. 121.

It is clear that this choice of Lagrangian, differing from that of Schwarzschild, *"extends the expression of the solution to the entire spacetime"*, which goes hand in hand with the analytic continuation constructed by Kruskal [14]. It is no longer a manifold with boundary, but a space whose topology is $R_+ \times R^3$.

It turns out that when quantities are calculated using the length parameter s, the variation of an action based on the length or its square produces the same system of Lagrange equations. Therefore, when the length is real, when we are within the hypersurface, this system of equations does indeed yield its geodesics. However, it provides what could be called "virtual geodesics," that is, real curves equipped with a purely imaginary length, which are thus outside the hypersurface. In a future article, we will show how, by constructing the Lagrange equations by basing the action on the square of the length, we equip the torus with pseudo-geodesics spiraling "inside" towards its "center" and outwards towards infinity. As a "geometry of the imaginary," we thus demonstrate that the virtual geodesics of the sphere, extending to infinity, are ellipses!

7 – Everything depends on what one considers the physical world to be.

This choice of Lagrangian - a "modern" choice - is at the root of the black hole model. Since it is accompanied by an alteration of the metric signature when crossing the Schwarzschild sphere, the interpretation accepted by all specialists is that in this region ttt becomes a spatial coordinate and r becomes a time coordinate.

As an indication only, let us now examine the various interpretations of the two-dimensional geometric object defined by the bilinear form:

(35)
$$G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2 \dot{\varphi}^2$$

If, like Chandrasekhar, one chooses the Lagrangian:

(36)
$$L = G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2 \dot{\varphi}^2$$

And the action :

(37)
$$J = \int L(r, \dot{r}, \varphi, \dot{\varphi})\, ds$$

then the Lagrange equations yield the differential equation:

(38)
$$\frac{d\varphi}{dr} = \pm \frac{h}{r^2} \frac{1}{\sqrt{\left(1 - \frac{\alpha}{r}\right)\left(1 - \frac{h^2}{r^2}\right)}}$$

Let us assign the constant α the value 1. The following figure shows different solution curves:

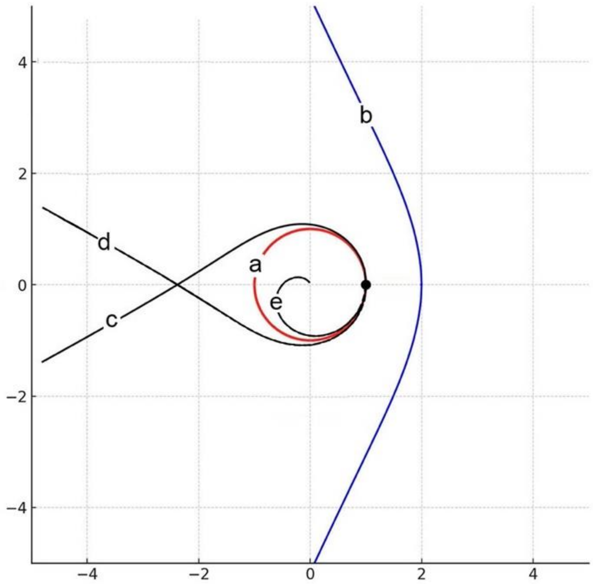


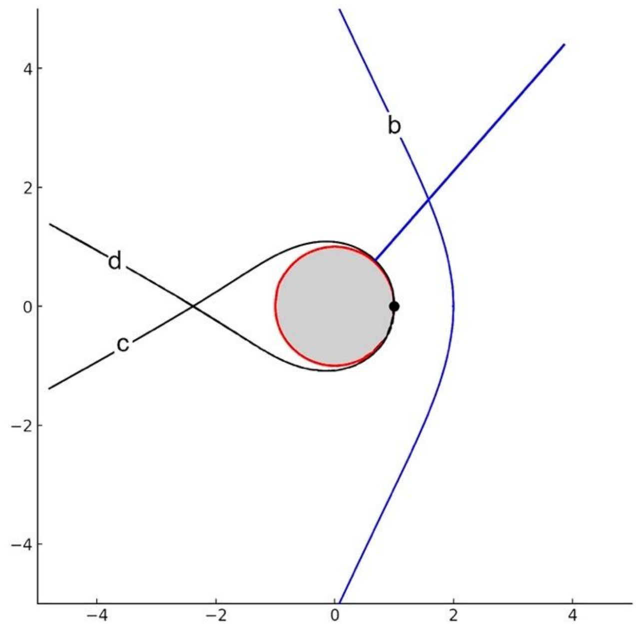
Fig.3 : Solution curves .

(b) : (r > α ; h < α). (c): (r > α ; h = α) . (e) : (r < α ; h < α)

This choice of Lagrangian amounts to choosing an object whose topology is homotopic to R^2.If we define the Lagrangian as:

(39) $$L = \sqrt{G} = \sqrt{\frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2}$$

Then we have two possible topologies, since the drawing of curves inside the circle is now forbidden.



$$Fig.4: \; Non-contractible: The\ object\ cannot\ be\ homotopic\ to\ \ \mathbb{R}^2$$

Either we consider that the geometry is that of a manifold with boundary. It turns out that the object can be embedded in R^3. By adding a third dimension z, and setting:

(40)
$$dr^2 + dz^2 = \frac{dr^2}{1 - \frac{\alpha}{r}}$$

This gives the meridian curve :

(41)
$$z = \pm 2\sqrt{\alpha(r - \alpha)}$$

But we have a third option: that of the two-sheeted covering of this manifold with boundary. This covering is then regular, and the meridian becomes:

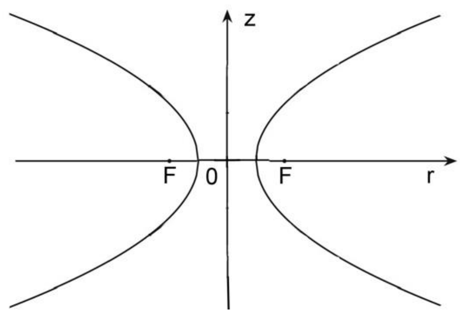


Fig.5 : Meridian of the object viewed as a two-sheeted covering of a manifold with boundary.

Since this object has the particularity of being embeddable in R^3, we can provide a perspective view of it in this three-dimensional representation space:

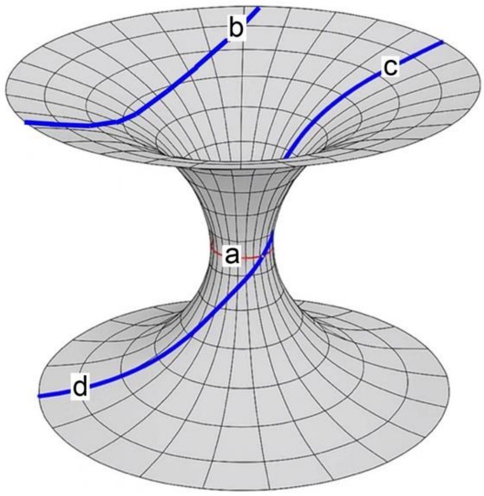


Fig.6 : Perspective view of the Flamm surface, embedded in R^3

It is the surface generated by the rotation of a lying parabola around its axis—a Flamm surface. This allows us to consider that curves (c) and (d) each lie on one of the two sheets joined along a throat circle.

It is immediately evident that this reasoning can be extended to the bilinear form corresponding to the spatial 3D part of the modern form of the Schwarzschild solution.

$$G = \frac{\dot{r}^2}{1 - \frac{\alpha}{r}} + r^2\dot{\theta}^2 + \sin^2\theta\ \dot{\varphi}^2 \qquad (42)$$

When Schwarzschild presents his solution, the object is then the union of two manifolds joined along their common boundary, which is not the Schwarzschild sphere but the surface of the star. Flamm presents its meridian [3]:

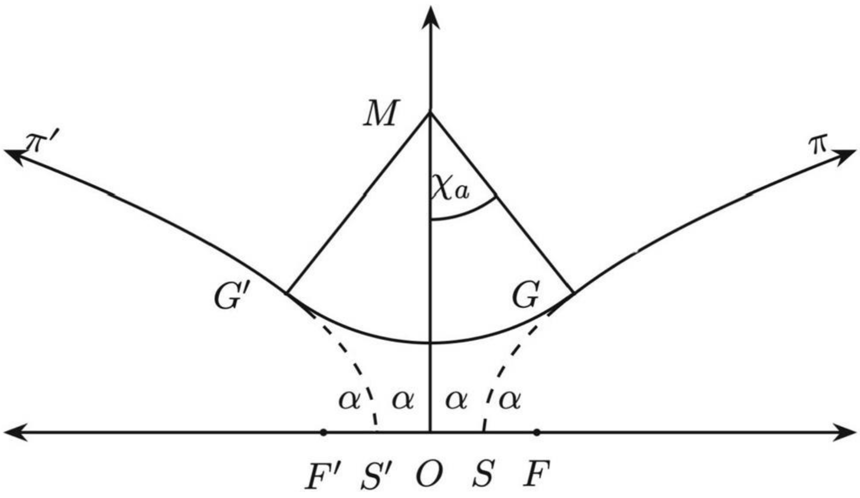


Fig.7 : Meridian of the Schwarzschild hypersurface [3].

Flamm then uses only part of the lying parabolas, which connect via a circular arc. The interior geometry is that of a portion of S3 sphere, with metric:

$$d\sigma^2 = \widehat{R^2}(d\chi^2 + \sin^2\chi\, d\theta^2 + \sin^2\chi\, \sin^2\theta\, d\varphi^2) \tag{43}$$

With:

$$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}} \tag{44}$$

The star's surface corresponds to $\chi = \chi_a$. Schwarzschild and Flamm obviously had no problem with topology. The 3D hypersurface is contractile and unfolds by translation in the time dimension.

The black hole model consists in considering, as a full-fledged geometric object, the one associated with the bilinear form:

$$\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{2M}{r}} - r^2\dot{\theta}^2 - r^2\sin^2\theta\,\dot{\varphi}^2 \tag{45}$$

with the a priori hypothesis of contractibility, that is, of topology $R_+ \times R^3$, which amounts to asserting that what corresponds to values $0 \leq r \leq \alpha$ is part of physics and gives rise to a true singularity at r = 0.

8 – **The Birkhoff theorem** [15].

When a theoretical physicist constructs a mathematical solution to an equation and wants that solution to correspond to a phenomenon situated within the domain of physics, it then appears essential to him that this solution be unique. The Birkhoff theorem is therefore, above all, a uniqueness theorem which immediately prohibits the presence of a cross term in drdt. Put differently, it requires that the solution be not only invariant under time translation—that is, stationary—but also invariant under the change t→−t , that is, static.

In 1934, in his book [16], the mathematician Richard Tolman was the first to note that, mathematically, the general solution to Einstein's equation without source term, in spherical symmetry, includes a cross term in drdt. What is the consequence of this term? It is easy to reveal it by applying the time coordinate change proposed by A. Eddington in 1924 [17]:

(46) $$t = t_E - \frac{\varepsilon\alpha}{c}\ln\left|\frac{r}{\alpha} - 1\right| \text{ with } \varepsilon = \pm 1$$

Then the bilinear form becomes :

(47) $$G = \left(1 - \frac{\alpha}{r}\right)c^2 dt_E^2 - \left(1 + \frac{\alpha}{r}\right)dr^2 - \frac{2\varepsilon\alpha c}{r}dr\,dt_E - r^2(d\theta^2 + \sin^2\theta\ d\varphi^2)$$

We note in passing that this transformation eliminates the coordinate singularity, which was Eddington's goal. Focusing on radial geodesics, P. Koiran [18] analyzed free-fall and escape times. Without a cross term, the null-length geodesics lead to the same velocity evolution as perceived by a distant observer:

(48) $$V_\varphi = \pm c\left(1 - \frac{\alpha}{r}\right)$$

The value is identical for emerging and plunging light rays. This velocity cancels out on the Schwarzschild sphere. In the form resulting from the Eddington transformation, this velocity depends on the direction and sign of ε.

First case: $\varepsilon = -1$. n the case of photons in free fall, they have a constant radial velocity$V_\varphi = -$ c.

Escaping photons have radial velocity :

(49) $$V_\varphi = c\frac{r - \alpha}{r + \alpha}$$

Then this time becomes infinite.

If we choose $\varepsilon = +1$ he values are reversed. This time it is the photons on the escape trajectory that have a constant velocity c. If we refer to the Kerr metric, which also has a cross term in dr dφ it also has two different speeds, depending on whether or not the azimuthal limine ray accompanies the rotational movement. This is known as “frame dragging”, which is reminiscent of Mach's principle, as opposed to the covariance principle. In this way, the solution mentioned above could be associated with “radial frame-dragging”.

Free-fall time calculation($\nu = -1$), or escape time ($\nu = +1$), of a mass with zero velocity at infinity gives :

(50)
$$dt_E = \nu \frac{r + \alpha\varepsilon\nu\sqrt{\frac{\alpha}{r}}}{c(r-\alpha)}\sqrt{\frac{\alpha}{r}}\, dr$$

Whether this time is finite or infinite depends on how the equation behaves in the vicinity of $r = \alpha$:

(51)
$$dt_E \approx \nu \frac{r + \alpha\varepsilon\nu}{c(r-\alpha)}\, dr$$

We see that if $\varepsilon = +1$ this time is finite for plunging radial trajectories, infinite for escape trajectories. Inverse conclusion if $\varepsilon = -1$

To manage this non-uniqueness of the solution, we are forced to abandon the topology $R_+ \times R^3$ to that of a two-sheet covering of a manifold with a (spherical) edge. The first person to consider a two-sheet space-time covering structure was H. Weyl, in 1917 [20] with the aim of attempting a purely topological description of the masses, an approach which was taken up in 1935 by Einstein and Rosen [19]. This concept was also the basis of the wormhole object, supposed to represent a bridge, either between two universes, or between two different regions of space-time. In this case, the two identical metrics represent an abstraction. Free-fall times, expressed as proper times, are finite and brief. It has been deduced that these unstable objects must reform immediately, but this adverb is only relative, since to a distant observer this time would be infinite.

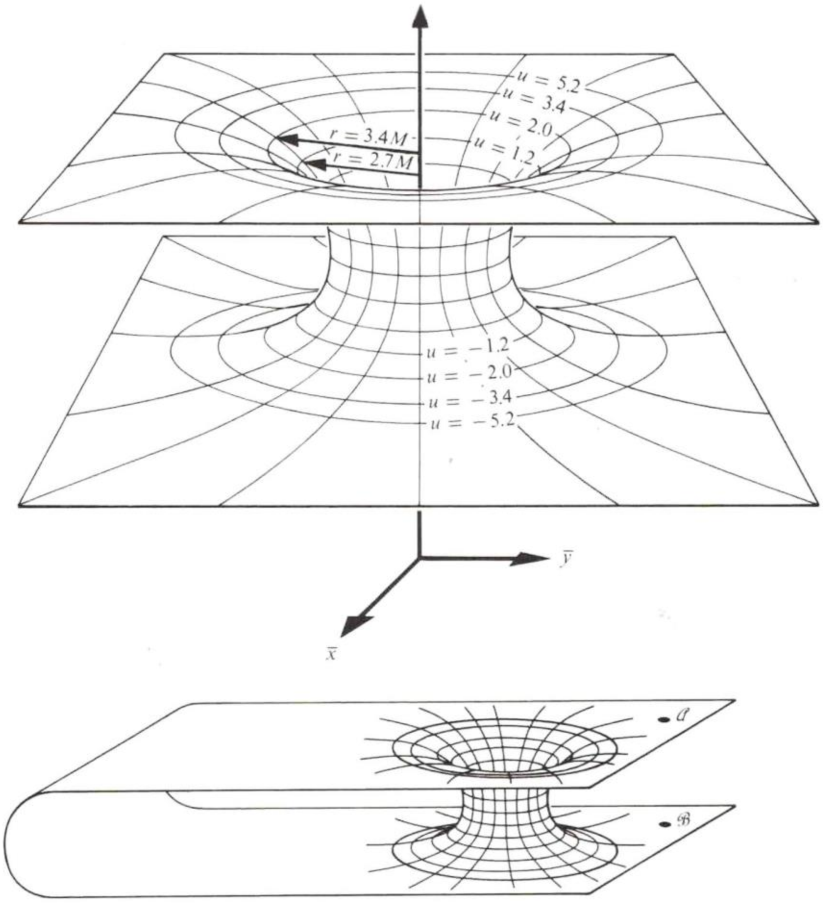


Fig. 8 : Wormholes [14]

If we try to interpret the reformulation of the solution with a cross term and impose continuity in speed, we are led to opt for $\varepsilon = -1$ in our own fold and for $\varepsilon = +1$ in the adjacent fold. If we opt for an "old-fashioned" definition of length and proper time, we have in our fold:

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\alpha c}{r} dr dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{52}$$

In the adjacent fold :

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 + \frac{2\alpha c}{r} dr, dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{53}$$

This results in a very short one-way transit time [17], which would invalidate Oppenheimer and Snyder's hypothesis [1]. Another view [21] is to attribute the inversion of the sign of the cross term to the inversion of the time coordinate in the second sheet, which becomes T-symmetrical to ours. With the help of an extremely simple diagram, it is also shown in [21] that the passage of masses through the throat sphere also generates a PT-symmetry.

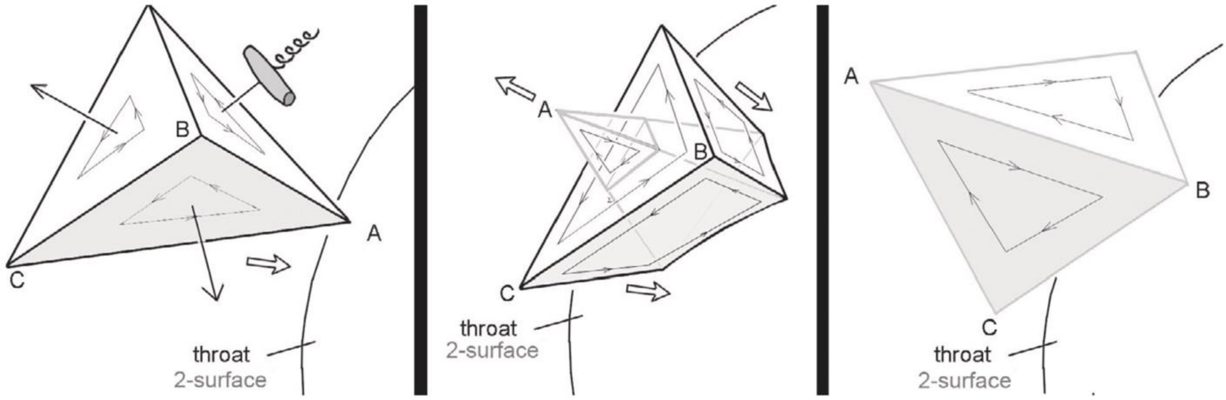


Fig. 9 : P-symmetry when crossing the throat sphere [21]

This figure may be confusing for readers who do not have good 3D vision. The 2D version is easier to visualize.

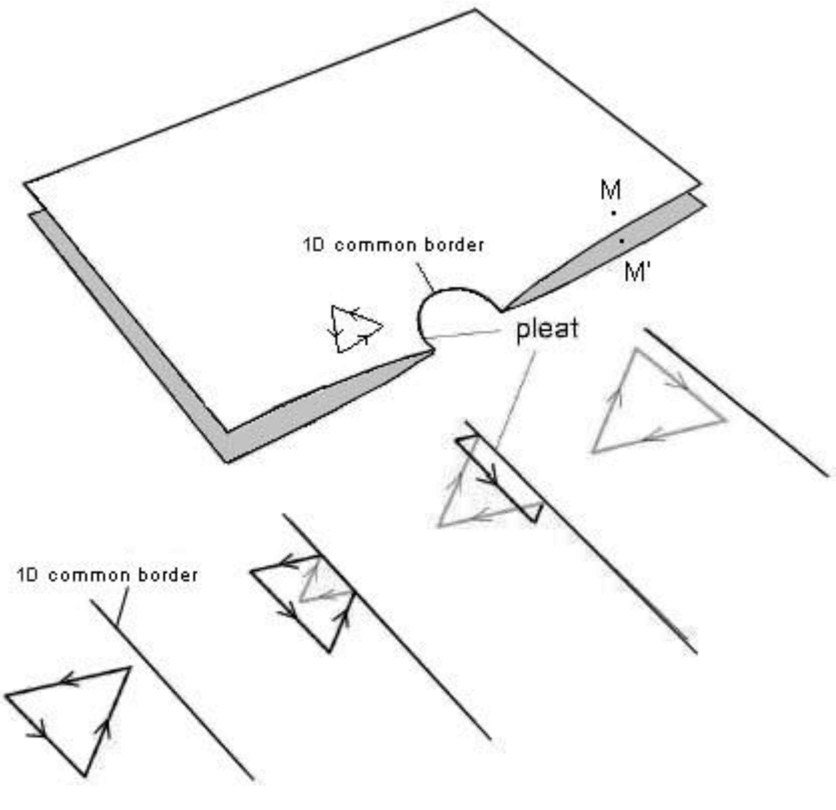


Fig.10 : 2D-P-symmetry when passing the throat circle connecting two 2D spaces.

In 2D, an oriented object can be represented by a triangle with arrows, indicating an arbitrary direction of rotation. In fact, sliding this triangle according to the figure above, it passes over the other layer. The gray figure shows that, with respect to an observer living in the first layer, its orientation has been reversed. It crosses a sphere with a throat S1, that is, a circle. Note that any point M of the first layer is associated with a corresponding point M4, belonging to the second layer. This amounts to saying that these two points have the same polar coordinates. Here, the thing is clear because this 2D surface can be imbedded in our familiar Euclidean 3D representation space. By moving to three dimensions, we cannot produce a drawing and it is difficult to mentally represent two 3D spaces constituting a two-layer covering. What we can keep in mind is that any point M, with coordinates (x, y, z) of the first 3D sheet will have its equivalent M', identified with the same coordinate values. If we were to consider a film, taking place over a time t, the four vertices of the tetrahedron would evolve over time according to coordinates $\left(x^i{}_{(t)}, y^i{}_{(t)}, z^i{}_{(t)}\right)$. The reader can then imagine that the four vertices of the tetrahedron are steel balls sent onto the throat sphere, also made of steel. They bounce off this sphere and, see Figure 9, ball A being the first to touch the surface of the sphere, will also be the first to bounce, which will cause the tetrahedron to turn over. Thanks to this, the reader can then admit that crossing a throat sphere causes the adjacent sheets to be P-symmetric.

This PT-symmetry refers to the extension of the Poincaré group made in [22] and to the idea that this inversion of the time coordinate then leads to the inversion of the mass [23], leading to a reformulation of the pair of metrics, with:

$$\alpha = \frac{2GM}{c^2} \qquad (54)$$

In our own old (attractive field):

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\alpha c}{r} dr dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (55)$$

In the adjacent fold (repulsive field):

$$ds^2 = \left(1 + \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 - \frac{\alpha}{r}\right) dr^2 - \frac{2\alpha c}{r} dr dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (56)$$

9 – **Observational data. The Giant Black Holes Hypothesis.**

Recently, the combination of powerful observing resources has enabled us to obtain images of the hypermassive objects M87*[28] and SgrA*[29].

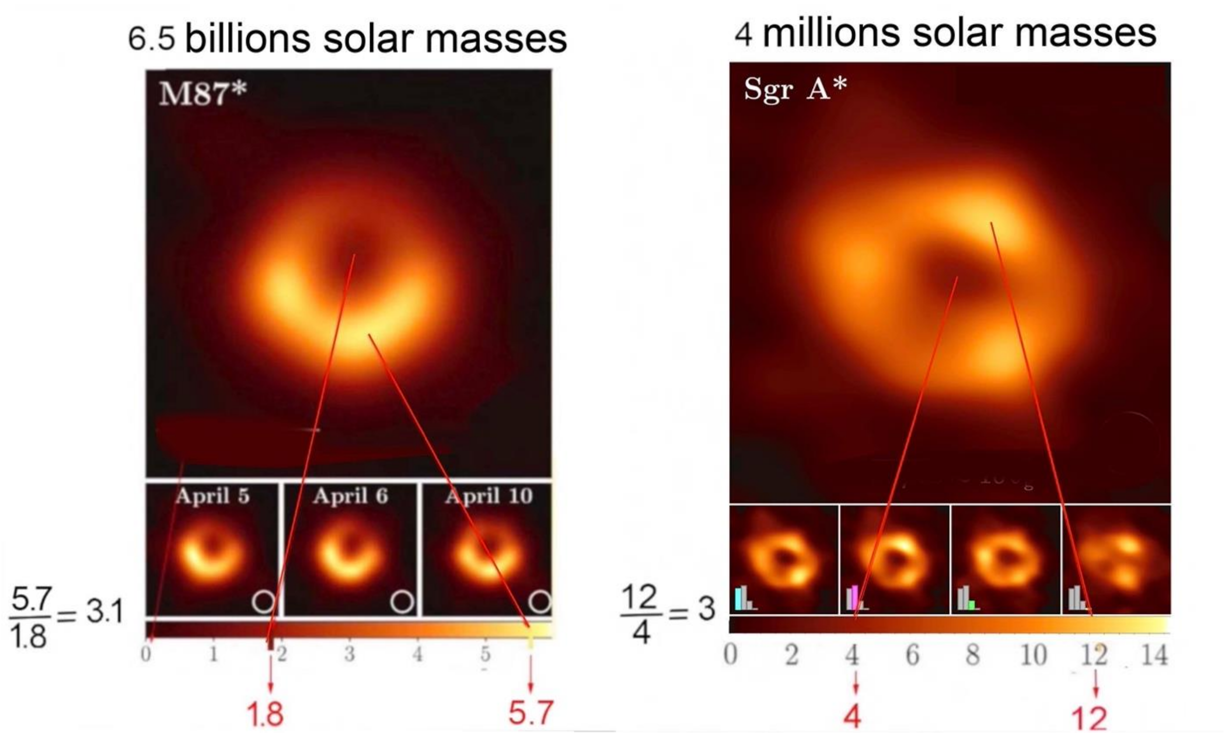


Fig.11 : Images of hypermassive objects M87*and SgrA*

One may wonder what meaning to give to the "equivalent temperature" values appearing on the two color bars. The radio telescopes forming the EHT system capture electromagnetic energy whose wavelength is between 0.3 mm and 1.3 mm. This is therefore a simple measurement of the energy emitted by the different parts of the object and, without prejudging the phenomenon which is at the origin of this emission of radiation, these values correspond to a brightness temperature. The figures obtained are surprisingly close to 3. Strictly speaking, therefore, we can't immediately identify them as giant black holes, for which the gravitational redshift effect would then lead to infinite values, unless the luminosity of the central part can be attributed to a mass of hot gas in the foreground. However, given the vast differences in the masses and brightness temperatures of these objects, this hypothesis is not very credible. There's another aspect to the formation of such giant black holes, by successive accretions. If this were the case, the axis of symmetry of these objects would have a direction uncorrelated with that of the galaxy that hosts it. However, observers agree with the generalization of this correlation between the two axes. If these objects are Giant Black Holes, the emitted radiation does not emanate from the objects themselves, but from their accretion disks and corresponds to the synchrotron phenomenon, that is to say, the radiation emitted by electrons, orbiting in a powerful magnetic field. What we observe is that the central part is less emissive and that there is a region where this power is maximum. Using the color bars of the images we can form the ratio Maximum Brightness Temperature/Minimum Brightness Temperature and starting from these two images we find a value very close to 3. Is this value significant? This is a central question and we cannot affirm it. Indeed, in 2024 new images of M87* were obtained [39]. If the displayed value of the minimum brightness temperature has not changed, the same is not true of the maximum value:

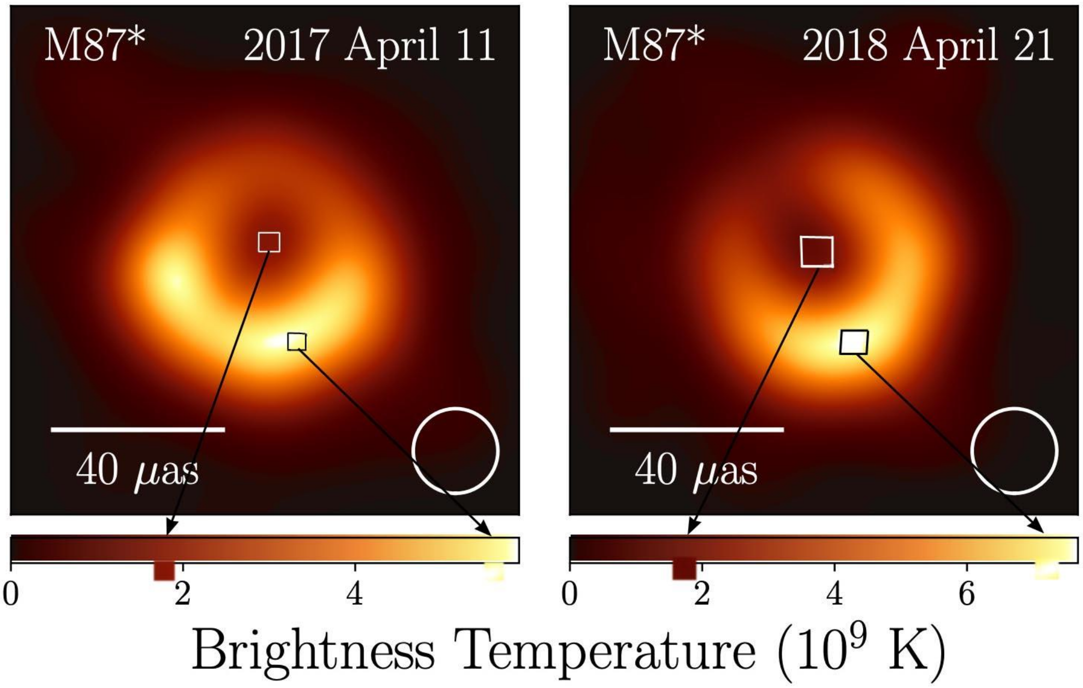


Fig.12 : No variations in the display of extreme brightness temperatures of M87*.

The colors are very similar. The article focuses on determining the diameter of the central dark spot. We will therefore take this assessment of the similarity of the ratios of these maximum brightness temperatures as the minimum value, with all due reservations. Let us therefore assume that the proximity of these two values is reliable and significant. If this is the case, and if these objects are Giant Black Holes, then it is necessary to explain why their central regions do not include completely black areas, with an infinite gravitational redshift. Proponents of Giant Black Holes respond that this emission is due to the presence of hot gas in the foreground. Under these conditions (again, if we can trust these ratio values!), we might find it surprising that this is the case, given that the mass of M87* is 1925 times greater than that of SgrA* while its brightness temperature is, on the contrary, 2.1 times lower. Let us now see what happens with the simulations carried out, assuming that these objects are giant black holes [40]. The image obtained from M87* corresponds to the figure below.

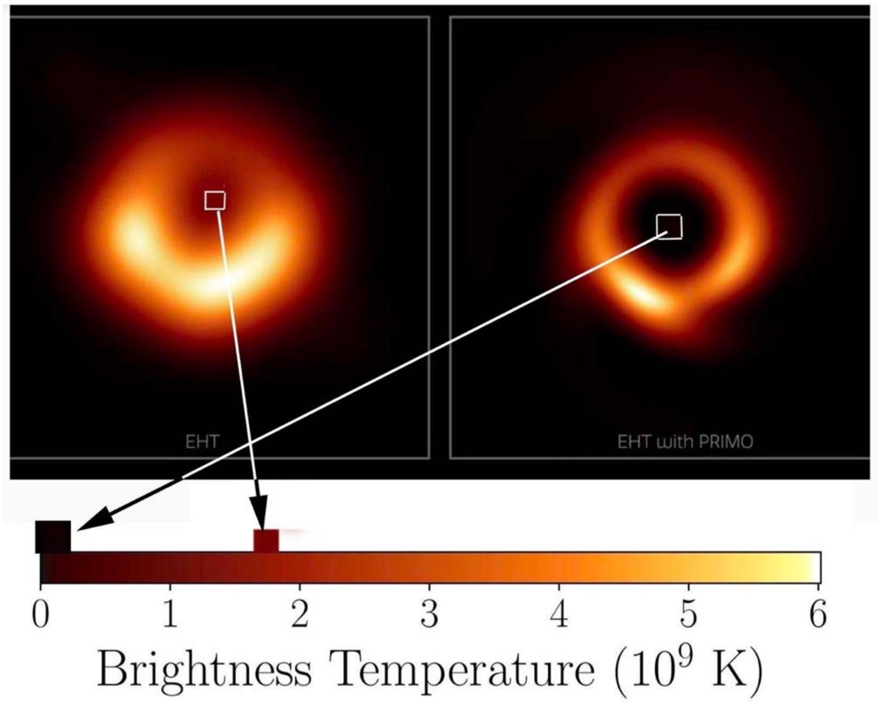


Fig.13 : Attempt to reconstruct the image of M87 [40].

It should be noted that the authors did not consider the brightness value in the central region, as if this meant "this is the Giant Black Hole, as you would see it if we removed the mass of hot gas located in the foreground." Everything that follows depends on the assumption that this number 3, representing the ratio of the maximum brightness temperature to the minimum brightness temperature, is reliable and meaningful. We will only have possible confirmation of this in the relatively distant future. Indeed, these images could only be obtained for two very specific objects. The first is located at the center of our galaxy, and therefore the closest. The second is distant, but M87* is a monster. All the radio telescopes located on Earth, forming the EHT system, do not seem capable of producing images of other objects, which are too distant. For these future images to reach us, we will need to position orbital radio telescopes, a technically complex and costly operation.

Note that if these new images produce Maximum Brightness Temperature/Minimum Brightness Temperature ratios significantly different from 3, then everything that follows will be null and void !

10- **A look back at Karl Schwarzschild's interior metric solution.**

The alternative is to assume that these supermassive objects have a central part darkened by a non-infinite gravitational redshift effect, which would then correspond to the value z = 2. This gravitational redshift then depends on the value of the g_{tt} coefficient of the interior metric, published by Karl Schwarzschild in his second article [24], which was not available in English until 1999 [25]. In its original form:

$$ds^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 c^2 dt^2 - \widehat{R^2}\left(d\chi^2 + \sin^2\chi\, d\theta^2 + \sin^2\chi \sin^2\theta\, d\varphi^2\right) \quad (57)$$

The quantity $\hat{R}$ is the characteristic length:

(58) $$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}}$$

The angle χ allows us to locate the position within the mass. The relationship with the radial variable r is:

(59) $$r = \hat{R}sin$$

The wall of the object r = r_a corresponds to the value χ_a. We then find the more familiar expression:
(60)

(60) $$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1 - \frac{8\pi G\rho r^2}{3c^2}}\right]^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{8\pi G\rho r^2}{3c^2}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right)$$

Which connects with the external metric:

(61) $$ds^2 = \left(1 - \frac{8\pi G\rho}{3c^2 r}\right)c^2 dt^2 - \frac{dr^2}{1 - \frac{8\pi G\rho}{3c^2 r}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right)$$

The time factor is then:

(62) $$f = \frac{3}{2}\sqrt{1 - \frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1 - \frac{8\pi G\rho r^2}{3c^2}}$$

The gravitational redshift effect we want to consider corresponds to f $= \frac{1}{3}$ The ratio $\frac{r_a}{\hat{R}}$ is then equal to $\sqrt{8/9}$. And we have:

(63) $$f = \frac{3}{2}\sqrt{1 - \frac{8}{9}} - \frac{1}{2}\sqrt{1 - \frac{8}{9}} = \frac{1}{3}$$

Thus we have a gravitational redshift:

(64) $$1 + z = \frac{\lambda_{\text{observer}}}{\lambda_{\text{emitter}}} = \frac{\sqrt{g_{tt_{\text{observer}}}}}{\sqrt{g_{tt_{\text{emitter}}}}} = \frac{1}{f} = 3$$

Note that the Classical criticality, arising from the exterior metric, when the Schwarzschild radius catches up with the star's radius r_a, at constant, corresponds to:

(65) $$r_a = R_{critgeom} = \hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}}$$

We would describe this configuration (linked to the external metric, considered as describing the object entirely and in isolation) as:

Geometric criticality

Let us now return to this situation where the gravitational redshift effect z = 2 leads to a brightness temperature ratio of 3. Let us search the literature for curves referring to this situation:

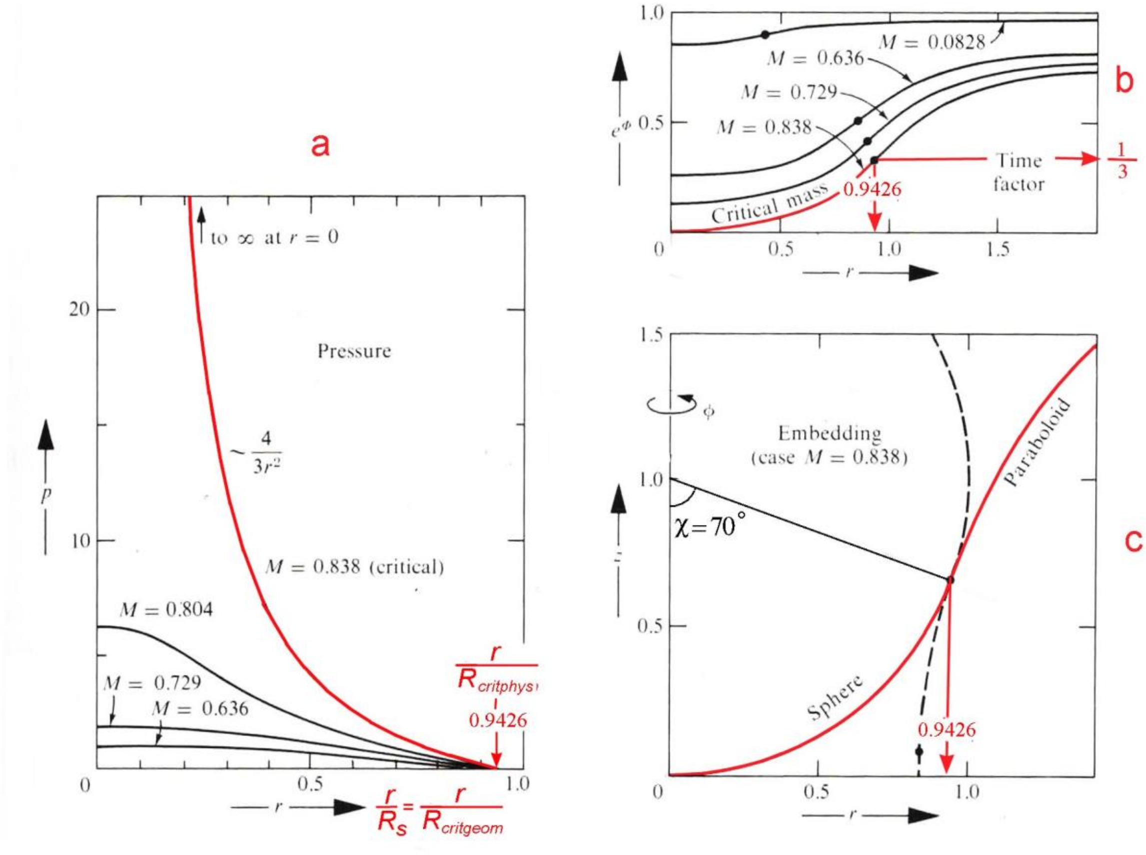


Fig.14 : Physical criticality [26], page 611

We have three curves before us. Figure b, top left, gives the time factor. The physical criticality situation corresponds to:

(66) $$R_{critphys} = \sqrt{\frac{8}{9}}\hat{R} = \sqrt{\frac{8}{9}}R_{critgeom} = \sqrt{\frac{c^2}{3\pi G\rho}} = 0.9426\hat{R}$$

What we will call "geometric critical mass" » $M_{critgeom}$ will be, for a given density ρ:

(67) $$M_{critgeom} = \frac{4\pi(R_{critgeom})^3 \rho}{3} = \frac{4\pi(\hat{R})^3 \rho}{3}$$

This is the "mass of the black hole". It is also the reference mass corresponding to the figures above, noted *M.* The curves indicated in red correspond to a mass:

(68) $$M_{critphys} = \frac{4\pi(R_{critphys})^3 \rho}{3} = 0.838 M_{critgeom}$$

Let's consider the diagram of the formation of a black hole by the capture of matter emitted by a companion star by a neutron star. Considering a scenario with constant density \rho, the mass will regularly increase.

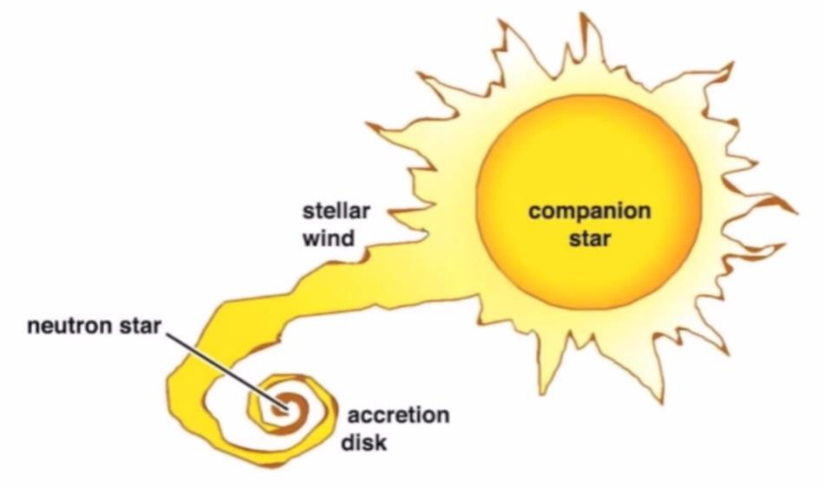


Fig.15 : Capture of matter by a neutron star.

We then classically consider that the neutron star transforms into a black hole when its mass reaches the geometric critical mass $M_{critgeom}$, close to 3 solar masses. However, in this scenario, we ignore physical criticality, for $M_{critphys} < M_{critgeom}$, which is necessarily crossed, with a lower physical critical mass, close to 2.5 solar masses. Note that a similar situation would occur in the more brutal scenario envisaged in 1939 by Oppenheimer and Snyder [1]. Figure 14-c shows the meridian of the object, that is, the particular case of the Flamm meridian, shown in Figure 7, in a situation of physical criticality, which corresponds to:

Figure 14-c gives the meridian of the object, that is to say the particular case of the Flamm meridian, of figure 7, in a situation of physical criticality, which corresponds to:

(69) $$\cos \chi_a = \frac{1}{3} \longrightarrow \chi_a \cong 70°$$

This follows from the original expression given by Schwarzschild in 1916, equation (55) where the time factor is written:

(70) $$f = \frac{3\cos\chi_a - \cos\chi}{2}$$

The center of the object corresponds to the value \ $\chi = 0$. The corresponding meridian curve will be more readable in the figure below:

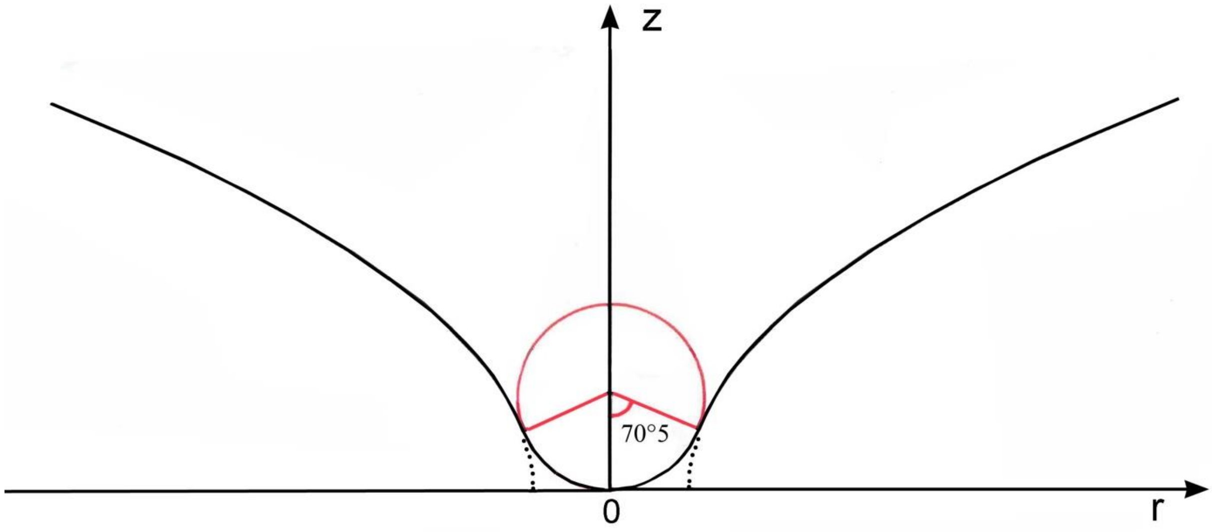


Fig.16 : Meridian in a situation of physical criticality.

The curves giving the time factor are those in Figure 14-b. On the red curve, the large black dot locates the object's wall. The time factor then has the value 1/3 at this point, which is consistent with this attempt to interpret the brightness temperature ratio as a gravitational redshift effect. However, we note that this coefficient is then zero at the center of the star.

For comparison, consider geometric criticality, when the object's radius equals the corresponding Schwarzschild length. The angle χ_a is then \ $\pi/2$ *:*

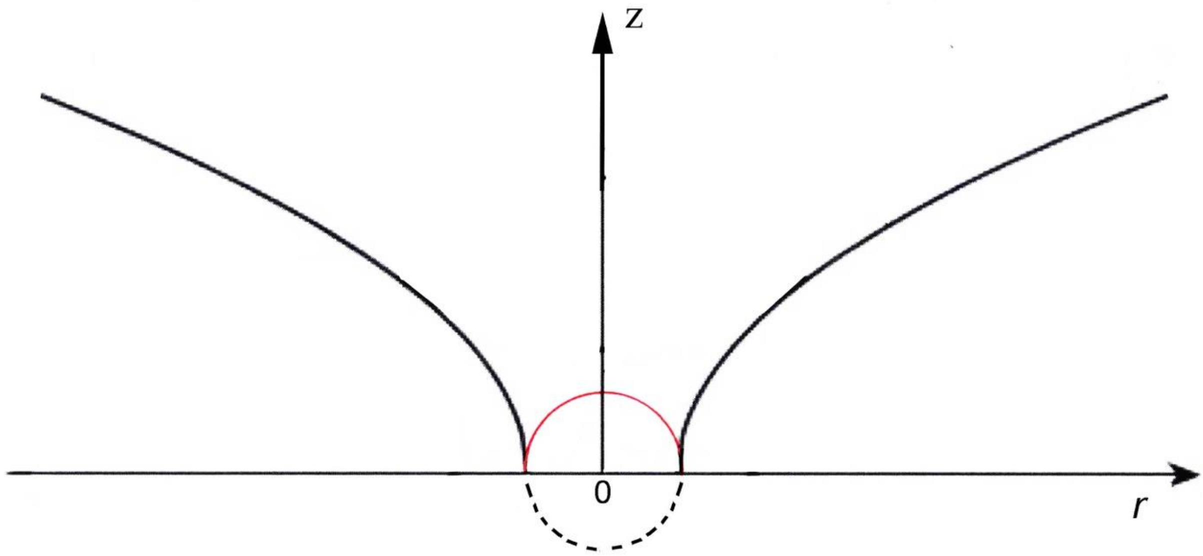
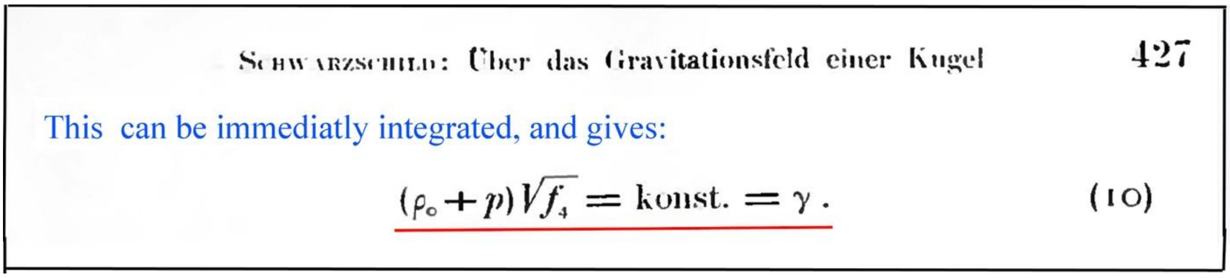


Fig.17 : Meridian in a situation of geometric criticality.

Before the situation of geometric criticality is reached, a prelude to the formation of the black hole, the situation of geometric criticality cannot be avoided, simply by continuity. What happens then in terms of physics? This was known as early as 1916. As we have just seen, it is accompanied by the fact that the coefficient g_{tt} of the metric becomes zero. Let us quote these very important extracts from Karl Schwarzschild's February 1916 article:



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This can be immediatly integrated, and gives:

$$(\rho_0 + p)\sqrt{f_4} = \text{konst.} = \gamma . \qquad (10)$$

Fig.18 : Schwarzschild 1916. Relationship between pressure and metric potential g_{tt}

ρ_o is the density. To ensure homogeneity, we must read $\rho_o {c_o}^2$ \ where c_0 is the value of the speed of light in a vacuum, to which Schwarzschild gives the value unity. f_4 is the metric potential g_{tt}, so $\sqrt{f_4}$ is the time factor. A little further on, we read:

through an elementary calculation the equations (13), (26), (10), (24), (25) transform themselves into the following:

$$f_2 = \frac{3}{\varkappa \rho_0} \sin^2 \chi, \quad f_4 = \left(\frac{3 \cos \chi_a - \cos \chi}{2} \right)^2, \quad f_1 f_2^2 f_4 = 1. \qquad (29)$$

$$\rho_0 + p = \rho_0 \frac{2 \cos \chi_a}{3 \cos \chi_a - \cos \chi} \qquad (30)$$

Fig.19 : Schwarzschild : Evolution of pressure inside the mass.

And Schwarzschild concludes:

to a smaller radius with emission of energy (lowering the temperature through radiation).

4. The velocity of light in our sphere is:

$$v = \frac{2}{3 \cos \chi_a - \cos \chi}, \qquad (44)$$

hence it grows from the value $\frac{1}{\cos \chi_a}$ at the surface to the value : $\frac{2}{3 \cos \chi_a - 1}$ at the center. The value of the pressure $\rho_0 + p$ according to (10) and (30) grows in direct proportion to the velocity of light

At the center of the sphere $(\chi = 0)$ velocity of light and pressure become infinite when $\cos \chi_a = 1/3$, and the fall velocity becomes $\sqrt{8/9}$ of the (naturally measured) velocity of light.

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Hence there is a limit to the concentration, above which a sphere of incompressible fluid can not exist. If one would apply our equations to the value $\cos \chi_a < 1/3$ one would get discontinuities already outside the center of the sphere. One could however find solutions of the problem for larger χ_a which are continuous

Fig. 20 . In 1916 Karl Schwarzschild obtained a variable speed of light within masses [24].

We see therefore that as early as 1916 Karl Schwarzschild had proposed a law of variation of the speed of light within masses, indicating the existence of a configuration of physical criticality such that the pressure and the speed of light became infinite. This remained ignored from 1916 to 1999, for 83 years, until this second article became available in the form of its English translation [25]. Until this date those who called themselves cosmologists had access to Schwarzschild's work only through second-hand texts, noting that neither Hilbert [4], nor Eddigton, who first echoed this general relativity in a book published in 1924, reissued in 1960 [41], mentioned it. No mention of this physical criticality in Tolman's work, 1934 [16]. In 1939 Tolman [42], Oppenheimer and Volkoff [43] joined forces to publish this equation of state in a differential form, which would become the famous TOV equation:

$$\frac{dp}{dr} = -\frac{\left(\rho + \frac{p}{c^2}\right)\left(m + \frac{4\pi G p r^3}{c^4}\right)c^2}{r(r-2m)} \tag{71}$$

About thirty years ago, out of curiosity, we had numerically constructed the solution to this equation, at constant ρ (unpublished), and thus put our finger on this physical criticality, while ignoring that Karl Schwarzschild had already written it down, providing the exact solution:

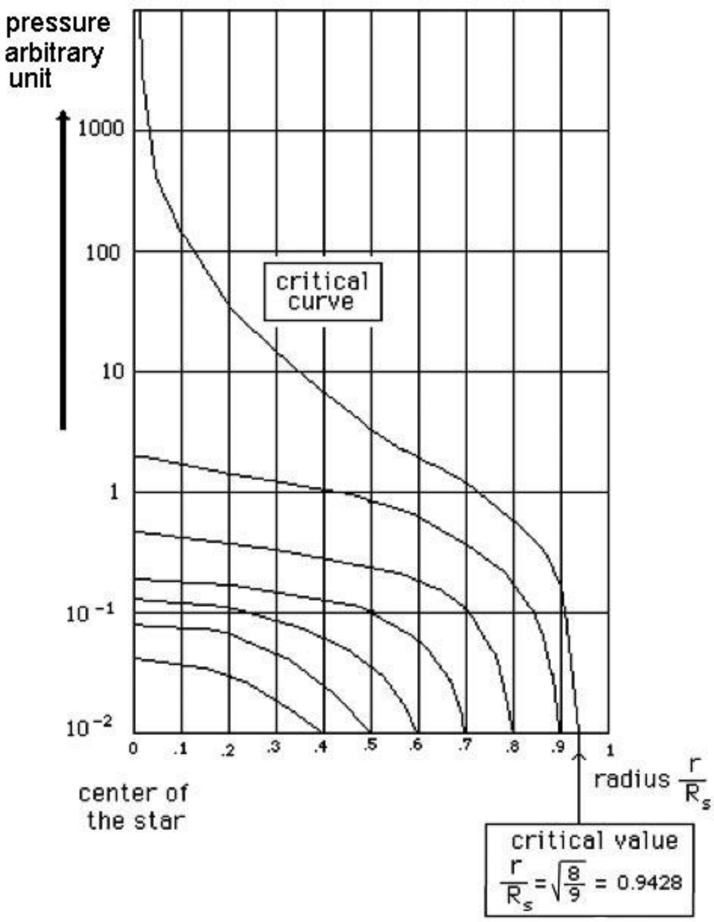

Fig.21 : Physical criticity evidence (1988, unpublished)

It was this observation that triggered all the work of examining the foundations of black hole theory, spread over decades; of which this article represents the conclusion. Returning to the TOV equation, the authors' idea was obviously to try to extend Schwarzchild's work by considering configurations with non-constant density. But, in this form, physical criticality was no longer evident and its existence simply remained ignored by the entire scientific community for decades. It was only in later editions that the curves in Figure 14-a appeared in [26]. If this last reference represents the state of the art in the matter, what do we read about this surge in pressure? Here is this passage:

> *"Incompressibility would imply a speed of sound* $\mathrm{v} = (dp/d\rho)^{1/2}$ *, of unlimited magnitude, therefore in excess of the speed of light, and therefore in contradiction with a central principle of special relativity ("principle of causality") that no physical effect can propagate at a speed* $\mathrm{v} > \mathrm{c}$ *".*

This property of a limitation of the speed of light applies to vacuum. There is no conceptual tool, nor experimental result, which imposes such a principle when electromagnetic waves propagate in matter at very high density. In an ideal gas the pressure is the volume density of kinetic energy of thermal agitation and is based on the value of the root mean square value $< V^2 >$ associated with a thermal agitation speed v.

(72) $$p_m = \frac{\rho\langle V^2\rangle}{3}$$

When considering the state of such a Subcritical Schwarzschild Body (SSB) it can only be that of a fully ionized plasma where the dominant component of pressure is radiation pressure:

(73) $$p_r = \frac{\rho c^2}{3}$$

In a relatively cold (non-relativistic) gas, sound corresponds to the propagation of acoustic waves, from near to far, resulting from collisions between particles with mass. This propagation speed is then of the same order of magnitude as the thermal agitation speed of the elements considered. In the plasma composing an SSB, the density and temperature are then comparable to that which governs in the primitive state of the universe, where the thermal agitation speed then reaches the speed of light. S. Weinberg, in his famous book "The first three minutes", summarized this by writing that at that time the universe "was filled with all kinds of radiation", in the plural. Thus the speed of light, as the speed of propagation of information, replaces the thermal agitation speed of the components. This leads us to focus our attention on this primitive state, knowing that at constant c we are confronted with the paradox of the extreme homogeneity of the medium. Two solutions then present themselves. The mainstream thesis is that of inflation. Another is to envisage an era of "variable constants" [27] such that the cosmological horizon increases as the spatial scale factor a. In 1916, Karl Schwarzschild was the first to envisage a variation in the speed of light within mass, resulting from an exact solution to Einstein's equation with a right-hand side. In the absence of credible ontological arguments, as in [26], one cannot make a choice by retaining only part of an exact solution, considering the other part as incredulous.

But one can just as well consider that this variation in the speed of light within masses, at very high density, as it emerges from this exact solution to Einstein's equation with a right-hand side, belongs to the world of physics. It therefore seems perfectly legitimate to us to conclude that this ratio Maximum brightness temperature/Minimum brightness temperature = 3 (if this data can be considered reliable) fits with a model based on this Schwarzschild Subcritical Body (SSB) model where, at the center, the pressure and the speed of light reach considerable, if not infinite, values, which then allows the pressure forces to counterbalance the force of gravity.

11. **About the stability of such Schwarzschild Subcritical Bodies.**

To address this question of the stability of these SSBs, it would be necessary to have a perturbational, time-dependent solution of the equation, which is not the case today and will have to be done. Returning to the passage from Schwarzschild's article and its translation into English, we see that he considers that if these conditions were exceeded, a singularity could appear, breaking the topology of the solution, that is to say, the total contractibility of space, at all points. Using the word from modern cosmology, we would use the term "wormhole", a concept that suggests a connection between two distinct geometric structures, two sheets of space-time. Before Einstein and Rosen published their own article in 1935 [19], the first person to suggest the existence of such a geometric structure was H. Weyl in 1917 [20]. Of course, with respect to what we are going to consider, his reflection, like that of Einstein and Rosen, only concerns the exterior metric solution, but the general idea is the same, and we will return to this part of his article in more detail. Like Einstein and Rosen, he therefore envisages a description of masses in the form of a topological singularity. We note that he endows his coordinates and his length with the same properties that Einstein and Schwarzschild attributed to him at the time:

(74) $$ds^2 = g_{ik}, dx_i, dx_k > 0$$

The same lack of ambiguity concerns its variable r :

(75) $$r = \sqrt{{x_1}^2 + {x_2}^2 + {x_3}^2}$$

By qualifying the length a as the "gravitational radius", which will later be called the "Schwarzschild radius", he produces, like Flamm, the expression for the meridian:

(76) $$z = \sqrt{8a(r - 2a)}$$

And then he writes:

> This metric characterizes the geometry of a paraboloid of revolution whose equation is written, in Euclidean space with orthogonal coordinates x_1, x_2, z . Its projection on the plane $\mathrm{z} = 0$, whose points are identified by polar coordinates r, ϑ , covers twice the area outside the circle of radius 2a. *But the surface does not project inside this circle.*
> Analytically and continuously, the set of points in real space, identified by the coordinates x_i , which is used to represent the object, *will cover twice the area corresponding to* $\mathrm{r} \geq 2\mathrm{a}$. These two coverings are separated by the sphere r=2a, on which the mass is located, and which is a singular place.

Above, we have indicated in italics two key passages from Weyl's article. The first illustrates the non-contractibility of the object, which goes hand in hand with this dreamlike structure that Weyl was the first to report in physics, but which has its source in Riemann's doctoral thesis, 1851 [30], qui introduit pour la première fois le concept de revètement à deux feuillets d'un manifold, which he later took up again in 1856 [31]. In 1882 the mathematician Felix Klein [32] systematized this work of Riemann, emphasizing the geometry of covered surfaces and automorphic transformations. In [33] Klein presents an extension to automorphic functions, that is to say functions invariant by discrete groups of isometries (Fuchsian, Kleinian groups), directly linked to the coverings of the hyperbolic plane. Weyl is therefore the first to use this concept by applying it to physics. In this perspective, the Janus Cosmological Model [23] extends the use of this geometry and mathematics tool, created more than a century ago, to the entire cosmological model. The elements presented here therefore do not simply reflect a physical interpretation, devoid of foundations. In the rest of the article, the reader will find what is today presented as a representation of this exterior Schwarzschild solution using "isotropic coordinates", but without mentioning the motivation for the approach.

$$d\sigma^2 = \left(1+\frac{a}{2r}\right)^4 (dx_1^2+dx_2^2+dx_3^2), \quad \mathrm{f} = \left(\frac{r-a/2}{r+a/2}\right)^2$$

Referring back to the presentation by Weyl :

In the new coordinates, this metric becomes consistent with a Euclidean metric. The elongation factor being:

$$\left(1+\frac{a}{2r}\right)^2$$

$d\sigma^2$ is regular for all values $r > 0$, f is positive and becomes zero for :

$$r = \frac{a}{2}$$

The circumference of the circle[3] $x_1^2 + x_2^2 = r^2$ is

$$2\pi r\left(1+\frac{a}{2r}\right)^2$$

this function, if we pass *r* in a decreasing way through the values starting from $+\infty$, decreases monotonically up to the value $4\pi a$, which is reached for

[3] This is nothing but the equator of the throat sphere of such new geometric structure.

$$r = \frac{a}{2'}$$

but then starts to increase again as r approaches zéro[4]. According to the view above, here the area

$$r > \frac{a}{2}$$

would correspond to the exterior of the mass point, and

$$r < \frac{a}{2}$$

Inside the mass-point.

If we continue analytically, in this region $r < \frac{a}{2}$ the term :

$$\sqrt{f} = \frac{r - a/2}{r + a/2}$$

becomes negative, in this region of space-time, *so that the time coordinate* (x_4) *and proper time are opposed to it for a stationary observer.*

Here we recognize the first introduction of isotropic coordinates and the "time factor" conceived as the square root $\sqrt{g_{44}} = \sqrt{g_{tt}}$. A quantity that we have called f in our article, as is customary in the literature. We will not make the confusion: in Weyl's article we have $f = g_{tt}$. The same observation applies to the inversion of the time coordinate (x_4 in Weyl), which arises from the simple fact that we cannot make a "half-turn" along a geodesic, that is to say, invert ds. The formulation of Schwarzschild's exterior metric solution, using isotropic coordinates, appears in all books presenting the theory of general relativity. But none of them presents what originally motivated its author, Hermann Weyl, in 1917: an attempt at a purely geometric model of masses, as topological singularities. What we will remember is that this approach leads to a second space-time sheet, where the time coordinate is reversed. The passage through a throat sphere means that this second world sheet is also P-symmetric (see figure 9). Thus this second world sheet is PT-symmetrical with respect to ours ([21],[22],[23]). This detour through Weyl's article shows us that considering the appearance of a topological singularity (suggested by Schwarzschild [24]) leading to a PT-symmetrical space-time sheet is therefore neither artificial nor arbitrary. Furthermore, this reaction of the system would translate, locally, a connection of the two symmetrical PT sheets of the Janus Cosmological Model. In the absence of this time-dependent perturbational solution, we will seek the "response" that the stationary solution would provide when we go beyond the subcritical situation. This leads us to plot the curve giving the time factor in such conditions:

[4] This geometric structure is therefore non-contractile. This circle is nothing but the equator of a family of circles whose area becomes minimal on the throat sphere. This ensures the passage between two space-times, described by Weyl as "exterior" and "interior" of this geometric object which he identifies with a mass.

12– **The suggested plugstar mechanism.**

In the present article we take the same approach as Weyl by exploring what happens to this interior metric, a solution, let us recall, stationary to Einstein's equation with right-hand side. When we exceed what we have called "physical criticality" the time factor becomes negative.

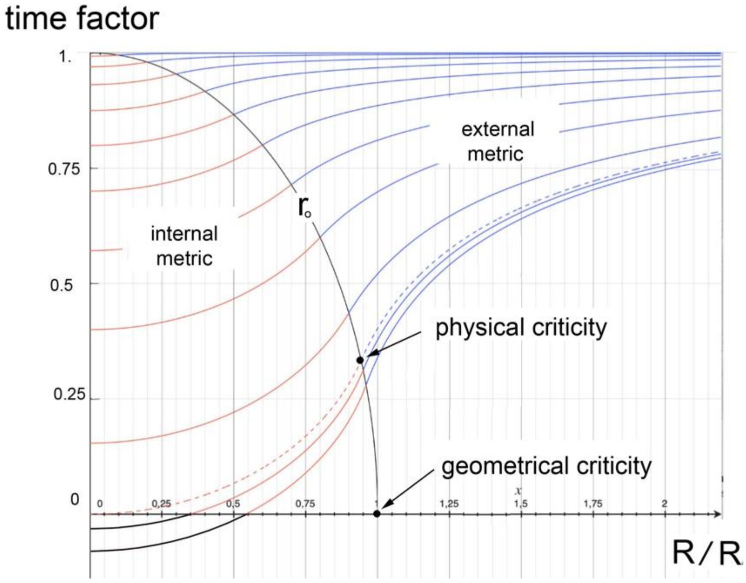

Fig.22 : Time factor

Beyond physical criticality, the time factor f becomes negative:

(77)
$$d\tau = \frac{ds}{c} f dt < 0$$

Doing as Weyl did in 1917 we deduce that this leads to the inversion of the time coordinate t (we can also say that maintaining the sign of ds has the physical meaning that we cannot turn around on a timeline). Like this inversion of the time coordinate (improperly called the "arrow of time") this brings the image of the inversion of the light cone, which turns over like an umbrella in a gust of wind. In its median position the cone transforms into a flat disk:

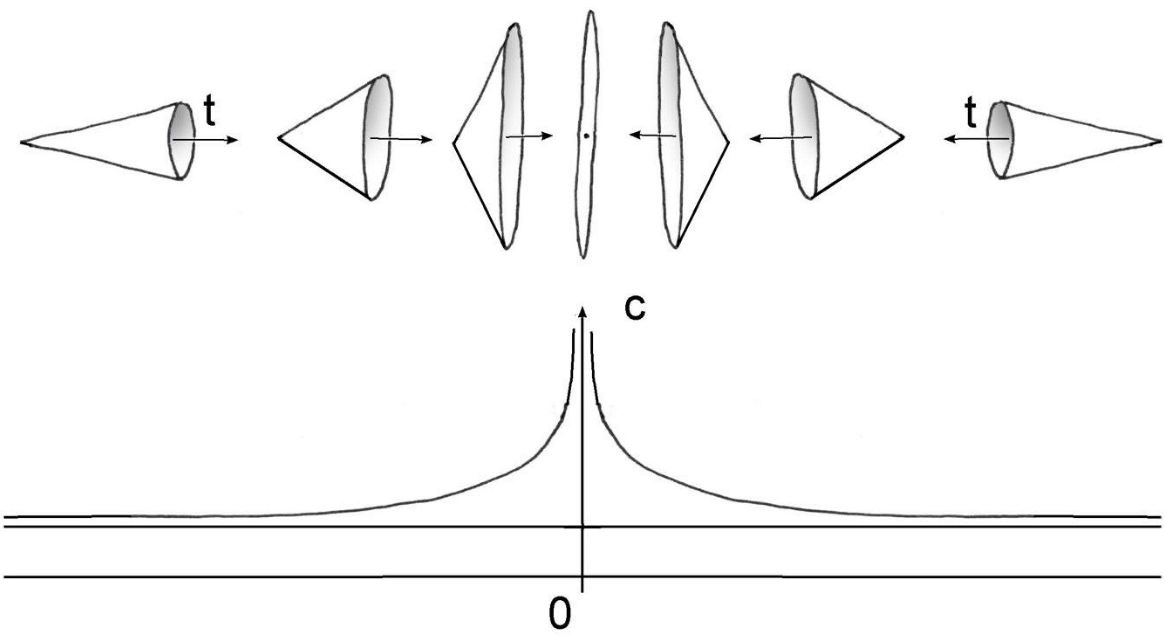

Fig.23 : The image of the light cone turning over.

We must then appeal to dynamic group theory [38] to obtain the physical meaning of this inversion of the time coordinate. All this is detailed in the article [22]. To summarize in a few words: Physicists, classically, only use the restricted Poincaré group, limited to its orthochronous components, a subgroup of the isometry group of Minkowski space. The complete Poincaré group then contains antichronous components, inverting the time coordinate. A first action of the group consists of making its components act on the points of space-time. In symplectic geometry we introduce a second action on the dual of the Lie algebra of the group. This brings up momentum space, which has the structure of a vector space and whose dimension is the same as that of the group: 10. Among its components, we see a scalar appear: energy, and two 3-vectors, momentum and spin, quantities which then emerge as objects of pure geometry. We can also make them appear as an application of Noether's theorem. Energy is then the invariant linked to the subgroup of time translations, momentum to the subgroup of space translations and spin to the subgroup of four-dimensional rotations, arising from the Lorentz group. The action of symplectic geometry adds that of the group on these different quantities. Thus the action of the antichronous components leads to that of energy E, and therefore of mass. This property constitutes the group basis of the Janus model, as a two-sheet covering of a compact and inorientable 4-manifold, this creating the PT-symmetry of the two sheets [22]. The second sheet therefore contains negative masses and negative energy photons which travel according to a second metric $\bar{g}_{\mu\nu}$, while the positive energy masses and photons follow the geodesics resulting from the metric $g_{\mu\nu}$. We therefore have a new geometry, bimetric. The unique equation of general relativity then becomes one of the two equations of a system of two coupled field equations, constructed from an action and satisfying the Bianchi conditions, which is mathematically consistent [22]:

(78) $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi\left(T_{\mu\nu} + K_{\mu\nu}\right)$$

(79) $$\bar{R}\mu\nu - \frac{1}{2} \bar{R} \bar{g}\mu\nu = -\chi(\bar{T}\mu\nu + \bar{K}\mu\nu)$$

χ is the classical Einstein constant. Thus we can say that the Einstein constant of the second sheet is equal and opposite to the first:

(80) $$\bar{\chi} = -$$

$R_{\mu\nu}$, R , $\bar{R}_{\mu\nu}$, $\bar{R}$, are the Ricci tensors and scalars from the two metrics. $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ \ are the field tensors of the positive and negative masses. $K_{\mu\nu}$ and $\bar{K}_{\mu\nu}$ are interaction tensors,

Not to be confused with field tensors!

While the attempt to introduce negative masses into general relativity (by requiring them to follow the geodesics from the single metric $g_{\mu\nu}$) led to the forces:

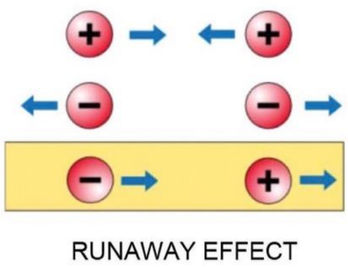

Fig.24 : Force laws from general relativity.

That is to say, to the runaway phenomenon, contradicting the principle of action reaction, the Janus Model provides a different scheme (derived from the relation $\bar{\chi} = -$)

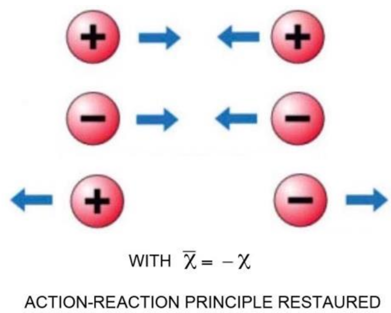

Fig.25 : Force laws from the Janus Cosmological Model[22].

In the positive layer the metric $g_{\mu\nu}$ corresponds, inside, then outside the object, before geometric criticality is reached. These metrics are given by equations (60) and (61). In an approximation The geometry in the adjacent portion of the second layer, in the Janus Cosmological Model (limiting ourselves to the case we can apply the Newtonian approximation) then corresponds to the two metrics $\bar{g}_{\mu\nu}$, interior and exterior:

$$\text{(81)} \qquad ds^2 = \left[\frac{3}{2}\sqrt{1+\frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1+\frac{8\pi G\rho r^2}{3c^2}}\right]^2 c^2dt^2 - \frac{dr^2}{1+\frac{8\pi G\rho r^2}{3c^2}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right)$$

$$\text{(82)} \qquad ds^2 = \left(1+\frac{8\pi G\rho}{3c^2 r}\right)c^2dt^2 - \frac{dr^2}{1+\frac{8\pi G\rho}{3c^2 r}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\;\dot{\varphi}^2\right)$$

So we have the answer to the question: "what happens when we exceed the physical criticality and a small amount of mass is inverted at the center of the object?" This mass, sensitive to the dominant gravitational field, created by the positive mass, is violently ejected. As the Janus model is based on the hypothesis that masses of opposite signs only interact antigravitationally, these negative masses pass through the object freely. The drawing below is a didactic image evoking the repulsion experienced by inverted masses.

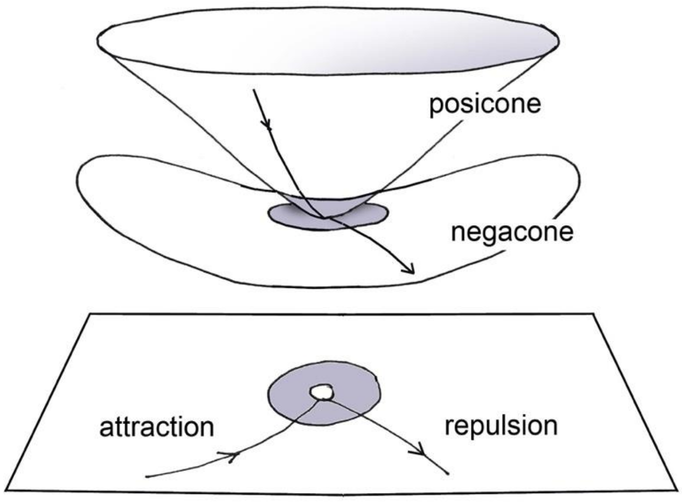

Fig.26 : Didactic 2D image showing the repulsive effect on negative masses.

In the positive sheet, the portion of space containing the positive mass is represented by a spherical cap, which is consistent with the Flamm meridian. Its image, in the negative sheet, is then a portion of a horse saddle. These negative masses are then taken over by the gravitational field of the galaxy and continue their journey to intergalactic space. This is also populated by negative masses, but this time it is primordial antimatter [22] (the Janus model is built, no longer only on a PT-symmetry but on the basis of a CPT-symmetry). There are therefore annihilations between the particles of matter of negative mass ejected from the Supermassive Shwarzschild Body and the particles of antimatter of negative mass populating intergalactic space (and ensuring the confinement of galaxies) [22], these annihilations producing negative energy photons, escaping all observation. We thus obtain a phenomenon that would ensure the self-stability of these SSBs to which we propose to give the name of Plugstars.
In the case of neutron stars, this negative feedback would then limit their mass to around 2.5 solar masses.

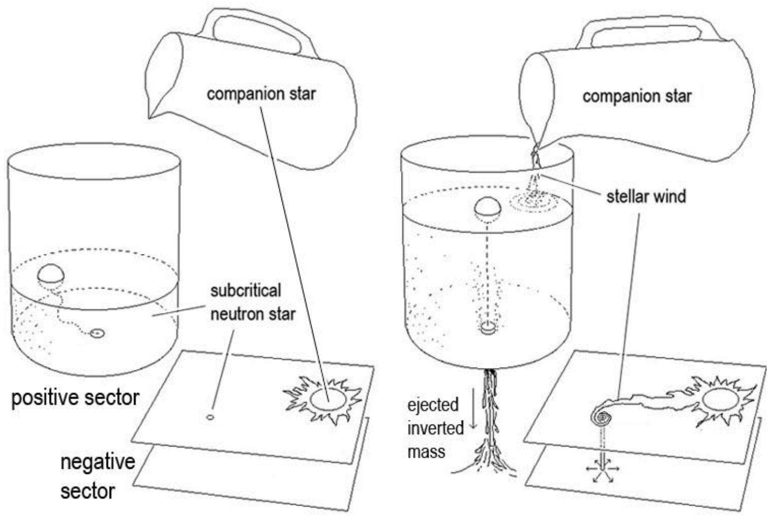

Fig.27: 2D educational image evoking the Plugstar phenomenon and the self-stability of subcritical neutron stars.

But supermassive objects at the centers of galaxies are obviously not giant neutron stars.

Note1: Little attention has been paid to Schwarzschild's interior solution [24], for two reasons. The first is that this article was only available in English in 1999. The second is the criticism Einstein made of it in 1939 [47], in a short commentary. When the object evolves toward physical criticality, at constant density, Schwarzschild finds that the pressure tends toward infinity. But the constancy of density within subcritical objects is not physical. In the core of neutron stars, for example, the density becomes so high that the distance between baryons becomes on the order of their Compton length. Under such conditions, general relativity is no longer sufficient to describe the state of matter. Mastering all the phenomena, including the suggested mass inversion, will only be possible once the cosmological model has been unified, incorporating quantum mechanics, a process whose beginnings are laid by the work of N. Debergh [48, 49, 50, 51]. The phenomenon of excess mass inversion, in the case of object mergers, would be accompanied by a powerful emission of gravitational waves. Integrating this phenomenon into the decoding of signals from the LIGO and Virgo devices could lead to a downward revision of the estimated masses involved, which are immediately identified as black holes.

Note 2: This plugstar model was initially developed based on the steady-state solution, an approach that should be reconsidered using an unsteady, possibly perturbational, solution. However, we can observe, purely as an example, how this central region of the object would evolve when the physical criticality is exceeded, where the inversion of the time factor leads to the inversion of the mass.

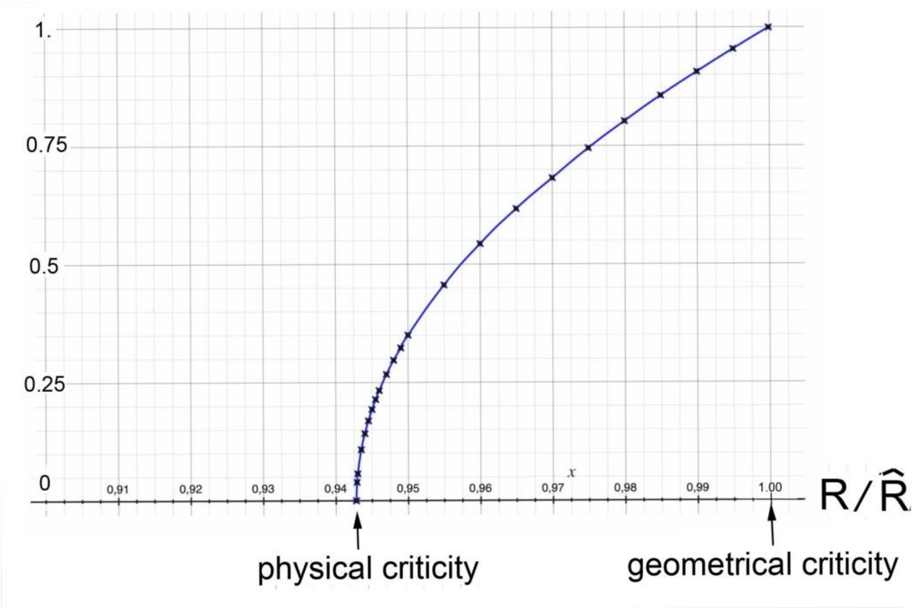

Figure 28 : Suggestion for the window opening speed

We then find that the diameter of this region grows extremely rapidly. We can then imagine that when a subcritical neutron star receives an influx of matter from a companion star, the topological singularity that ensures the removal of an equivalent amount of matter to the center opens and closes extremely rapidly, like a "3D camera diaphragm." This perspective leads us to reconsider the merger of two neutron stars, when the sum of their masses exceeds the critical value and the inversion of the excess mass is accompanied by powerful dissipation in the form of gravitational waves. Finally, a third scenario emerges, associated with the implosion of a massive star. Here again, we would have a mass inversion effect. But what would happen if the inverted, self-attractive mass exceeded the remaining positive mass? This negative mass would not disperse, but would constitute the nucleus of a new object made up of an invisible, spheroidal negative mass, surrounded by a remnant of positive mass in the form of a halo, which would be observable. It is conceivable that this relatively diluted positive mass would lose all energy through radiative dissipation and adapt to the ambient background radiation temperature. A new field of investigation to explore.

13 – **About the origin of such hypermassive objects and the nature of quasars.**

Seyfert galaxies, modulo the difficulty of evaluating the distance, seem to produce a Hubble constant significantly lower than the standard value. If we use the figures for the Hoag galaxy with D = 56 MPc and z = 0.004 we obtain a value of Ho of 21.4 km/s/Mpc. We conjecture that very irregular galaxies should give a value of Ho higher than the standard value. The idea is to consider a phenomenon of expansion turbulence, resulting from joint fluctuations of the metrics, which would have the effect of varying the confinement of the galaxies. In the case of weakening of this field this could go as far as the dislocation of the structure. In the opposite case where there is a strengthening of the gravitational field, this could cause the start of a centripetal density wave. This, destabilizing the gas, would generate the birth of new stars which, during their early youth, would reveal their presence, as in the spiral structure, by illuminating the gas through fluorescence. This is the interpretation that we propose to give to the circular formation present in the Hoag galaxy, as a density wave.

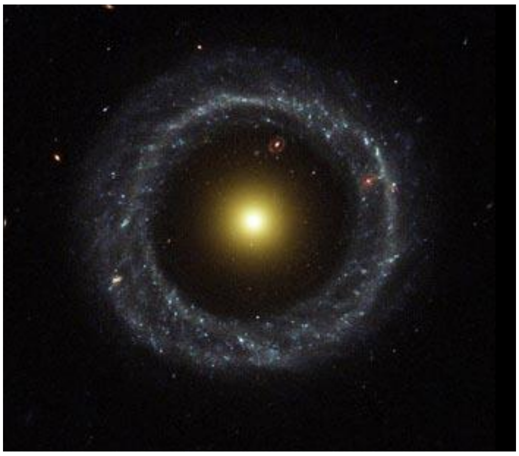

Fig.29 : Hoag galaxy ([45], [46])

Young stars are emissive for times between 5 and 20 million years. Measurements relating to the year give a rotation speed of 250 km/s, but no radial speed. If the ring is a density wave, this is normal, because it is the speed of the wave (comparable to a tsunami with spherical symmetry) and not that of the wave of matter that reacts to its passage. The width of the ring being Hoag's being 7.4 kpc, this allows an evaluation of the radial speed of the wave between 730 and 1470 km/s. It would then reach the center of the galaxy in a time between thirteen and fifty million years. The magnetic Reynolds number being large compared to unity, the wave then brings together the lines of force of the pre-existing magnetic field in the galaxy, which is evaluated at one microgauss. By bringing these lines together, an intense dipole magnetic field is formed in a direction that coincides with the axis of symmetry of the galaxy. The object that forms when the wave is focused at the galactic center is small. Its angular rotation speed, as a solid body, is that which animated the galactic center before the impact, therefore moderate. There is a sudden excursion of density and temperature such that the Lawson conditions are reached in a very large mass. Hence a powerful plasma emission according to the two poles of the magnetic field. This is our vision of the quasar phenomenon. Moreover, this system automatically behaves like a natural accelerator of charged particles, capable of giving them considerable energies and we see there the origin of cosmic rays. From this angle, the object located at the center of our galaxy could be considered as an extinct quasar. This mechanism of joint fluctuations of metrics, which we are currently trying to model, could periodically restart the emission, stimulated by the convergence of new density waves, of very low amplitude, but each time causing an emissive burst. Hence the dashed structure of the M87 jet. We conjecture that these joint fluctuations of metrics would have been very important at the very moment of the formation of galaxies. This is the reason why we observe the presence of these hyperdense objects in the oldest galaxies.

13 – **Conclusion.**

The model of black holes, of all sizes—mini, stellar, giant—has gradually taken its place in the scientific community over the past half-century, to the point of becoming the deus ex machina of astrophysics. Quasars are fantastic sources of radiation emission? This is due to matter falling into a black hole. A question stirs the community of specialists: "What happens to the information when it falls inside black holes? " Gravitational waves are detected? The signals are immediately interpreted as the result of black hole mergers. Very massive objects are identified at the centers of galaxies? As soon as the first images reach us, the journals don't headline: "Images of supermassive objects," but "Images of giant black holes." The argument: we can't see what else they could be.

However, a necessary return to the ontological foundations of this model is required. And it is here that we realize it is merely a chimera, mathematically and geometrically speaking. In 1939, advances in observations raised the question of the fate of massive stars. What happens when these stars collapse onto their iron cores? In 1939, Oppenheimer and Snyder provided the answer. Considering that, according to the standard form of the solution to Einstein's equation without a second member, the free-fall and escape times of a test mass are infinite, as perceived by a distant observer, this allows us to envision describing an eminently unsteady phenomenon using a steady-state solution. The concept of the cosmological horizon then emerges. For this solution to be meaningful, since it refers to the vacuum, all the mass must be contained within a central singularity. We then calculate the spiral trajectories towards it, within what is considered the interior of this Schwarzschild sphere. In this domain, the signs of the terms in the description of the metric are reversed? The explanation is found: it's because the space and time coordinates are reversed. All this because the idea that this Schwarzschild geometry could escape contractibility is not considered by anyone, even though it was perfectly described as early as 1916 by the young mathematician Flamm.

We show how all of this relies both on a topological assumption and on a singular definition of the Lagrange function used for calculating geodesics. Indeed, when the derivatives in this function $\dot{x}^i$ are calculated using the parameter s, the resulting Lagrange equations are the same, whether the action is evaluated based on a length or any power of this quantity, in this case its square. The equations then provide real curves, but equipped with a purely imaginary length, and no one notices that this is simply because these curves lie outside the considered hypersurface. In addition, in 2021, the mathematician Koiran further showed that taking into account a cross term in drdt, depending on its sign, gives either the free-fall time or the escape time a finite value, which invalidates the idea of using this stationary solution. Birkhoff's theorem is then invoked. However, this theorem does not impose the absence of this term; it indicates that it can be reintroduced using a simple change of variable, which is entirely different. If treated as an existence theorem, it then prescribes the coexistence of two solutions (where these terms happen to have opposite signs), which aligns with the reinterpretation of the solution through the Janus Cosmological Model. In this representation of the universe, based on a two-sheet covering, there must indeed be two PT-symmetric solutions for each set of four coordinates.

But then, what is the result of massive stars collapsing in on themselves? What happens to neutron stars whose mass inevitably grows due to the capture of stellar wind emitted by a companion star? What are the supermassive objects M87* and SgrA*? We then focus on the main piece of data leading to observations: the brightness temperature ratio, which turns out to be significantly close to 3. We then observe that this is the value of the gravitational redshift phenomenon that an object described by the second Schwarzschild metric solution would exhibit when it is close to a subcritical situation where the pressure at the center reaches a value that is, if not infinite, at least considerable, thus counterbalancing the force of gravity. Immediately, the objection raised by Einstein as early as 1939 resurfaces. But this objection remained based on an object of constant density. And he concludes, we quote, “It would be necessary, therefore, to introduce a compressible liquid whose question of states excludes the possibility of signals with a speed in excess of the velocity of light.” If we consider the core of neutron stars, the concentration of baryons is such that the distance separating them becomes on the order of their Compton length. Here we touch upon a domain where giving a physical description of matter itself becomes problematic. The question can only be clarified when a common formalism has unified gravitation and quantum mechanics. A mechanism of self-stability is then suggested for objects henceforth designated as plugstars. The slightest criticality exceedance then leads to a local inversion of the term g_{tt} at the center of the object. By using the concepts developed through the Janus Cosmological Model, where the universe is the two-folds cover of a P4 projective, which generates the PT-symmetry of the two, this inversion of the time factor is then interpreted as a local inversion of mass, thanks to the dynamic groups theory [22] with its immediate evacuation from the object. Incidentally, such a mechanism would limit the mass of neutron stars to a value close to 2.5 solar masses. In the event of a neutron star merger, if the sum of their masses exceeds this value, the excess would be immediately reversed, resulting in a strong emission of gravitational waves. According to this new model, the reinterpretation of the detected signals could then lead to a downward revision of the estimated masses involved in these mergers.

The ratio of maximum to minimum brightness temperatures for the two objects M87* and SrgA* is significantly close to 3, while their masses differ by more than three orders of magnitude, which seems to rule out mere chance. If the proposed model is valid, then all future images of such objects will exhibit this same value of 3. Recent studies have shown that the rotational axes of such objects, considered as their emission lobes when they possess them, coincide with the rotational axis of galaxies, which seems to rule out their formation by accretion. The result of the implosion of a centripetal density wave would seem more likely. It is worth noting that Hoag's Galaxy exhibits such an appearance. At this stage, these are obviously only conjectures, but we suggest that the formation of supermassive objects results from joint fluctuations of metrics in the early universe. The periodic recurrence of this phenomenon would reactivate these objects, transforming them into quasars. The object at the center of the Milky Way thus appears as an extinct quasar.

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